C1 January 2007 (MA)

Q1) $y = 4x^3 - 1 + 2x^{\frac{1}{2}}$, $x > 0$

$y = 4x^3 - 1 + 2x^{\frac{1}{2}}$

$$\frac{dy}{dx} = 12x^2 + x^{-\frac{1}{2}}$$

$$= \boxed{12x^2 + \frac{1}{\sqrt{x}}}$$

Q2a) $\sqrt{108} = \sqrt{36 \times 3} = \sqrt{36}\sqrt{3} = \boxed{6\sqrt{3}}$

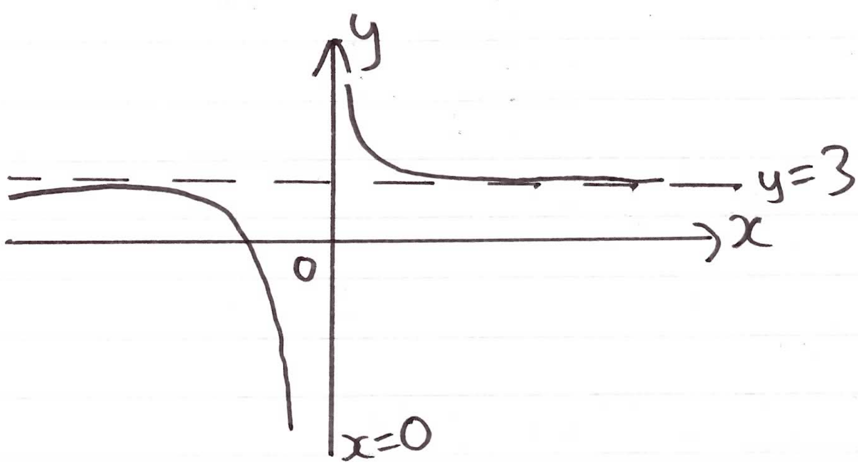
b) $(2 - \sqrt{3})^2 = (2-\sqrt{3})(2-\sqrt{3})$

$$= 4 - 2\sqrt{3} - 2\sqrt{3} + 3$$

$$= \boxed{7 - 4\sqrt{3}}$$

Q3a) $f(x) = \frac{1}{x}$, $x \neq 0$

$y = f(x) + 3$ is a transformation of $f(x)$ up 3 y-coordinates.



$\boxed{\text{Asymptotes at } y = 3 \text{ and } x = 0}$

b) $\frac{1}{x} + 3 = 0$ Since $f(x) = \frac{1}{x}$

$\frac{1}{x} = -3$

$1 = -3x$

$x = -\frac{1}{3}$

$\therefore$ the curve $y = f(x) + 3$ crosses the x-axis at $\boxed{\left(-\frac{1}{3}, 0\right)}$

Q4) $y = x - 2 \Rightarrow$ 1
$y^2 + x^2 = 10 \Rightarrow$ 2

Substitute 1 into 2:

$(x-2)^2 + x^2 = 10$

$(x-2)(x-2) + x^2 = 10$

$x^2 - 4x + 4 + x^2 - 10 = 0$

$2x^2 - 4x - 6 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

Either $x = 3$ or $x = -1$

When $x=3$, substitute into ① for y:

$y=x-2$

$y=3-2$

$\underline{y=1}$

When $x=-1$, substitute into ① for y:

$y=x-2$

$y=-1-2$

$\underline{y=-3}$

Solution set: $\boxed{x=3,\ y=1 \qquad x=-1,\ y=-3}$

Q5) $2x^2-3x-(k+1)=0$ has no real roots

If a quadratic has no real roots, then the discriminant is less than 0

$\therefore b^2-4ac<0$

$(-3)^2-(4)(2)(-k-1)<0$

$9-8(-k-1)<0$

$9+8k+8<0$

$17+8k<0$

$8K < -17$

$$\boxed{K < -\frac{17}{8}}$$

Q6a) $(4+3\sqrt{x})^2 = (4+3\sqrt{x})(4+3\sqrt{x})$

$= 16 + 12\sqrt{x} + 12\sqrt{x} + 9x$

$= \boxed{16 + 24\sqrt{x} + 9x}$

$\therefore \underline{K = 24}$

b) $\int (4+3\sqrt{x})^2 = \int (16 + 24\sqrt{x} + 9x)\,dx$

$= \int (16 + 24x^{\frac{1}{2}} + 9x)\,dx$

$= 16x + \dfrac{24x^{\frac{3}{2}}}{\frac{3}{2}} + \dfrac{9x^2}{2} + C$

$= 16x + 16x^{\frac{3}{2}} + \dfrac{9x^2}{2} + C$

$= \boxed{16x + 16x\sqrt{x} + \dfrac{9x^2}{2} + C}$

Q7a) $f'(x) = 3x^2 - 6 - \frac{8}{x^2}$

$f'(x) = 3x^2 - 6 - 8x^{-2}$

$f(x) = \int (3x^2 - 6 - 8x^{-2})\,dx$

$= \frac{3x^3}{3} - 6x - \frac{8x^{-1}}{-1} + c$

$= x^3 - 6x + 8x^{-1} + c$

$= x^3 - 6x + \frac{8}{x} + c$

We know that (2,1) lies on f(x), so substitute in $x=2$ and $y=1$:

$1 = (2)^3 - 6(2) + \frac{8}{2} + c$

$1 = 8 - 12 + 4 + c$

$1 = c$

$\therefore \underline{c = 1}$

So, $\boxed{f(x) = x^3 - 6x + \frac{8}{x} + 1}$

b) To find gradient of tangent to $f(x)$ at $(2,1)$, substitute in $x=2$ to $f'(x)$:

$$f'(x) = 3x^2 - 6 - \frac{8}{x^2}$$

When $x=2$, $f'(x) = 3(2)^2 - 6 - \frac{8}{(2)^2}$

$$= 3(4) - 6 - \frac{8}{4}$$

$$= 12 - 6 - 2$$

$$\underline{= 4}$$

So the gradient, m, $= 4$

Equation of tangent : $y - y_1 = m(x - x_1)$

Using $(2,1)$ $\longrightarrow$ $y - 1 = 4(x-2)$

$$y - 1 = 4x - 8$$

$$\boxed{y = 4x - 7}$$

Q8) $y = 4x + 3x^{\frac{3}{2}} - 2x^2, x > 0$

a) $\frac{dy}{dx} = 4 + \frac{9}{2}x^{\frac{1}{2}} - 4x$

$= \boxed{4 + \frac{9\sqrt{x}}{2} - 4x}$

b) Substitute in $x=4, y=8$ into y:

$y = 4x + 3x^{\frac{3}{2}} - 2x^2$

$8 = 4(4) + 3(4)^{\frac{3}{2}} - 2(4)^2$

$8 = 16 + 3(8) - 2(16)$

$8 = 16 + 24 - 32$

$\underline{8 = 8}$

$\therefore$ the point $P(4,8)$ lies on C.

c) when $x = 4$, $\frac{dy}{dx} = 4 + \frac{9(\sqrt{4})}{2} - 4(4)$

$= 4 + 9 - 16$

$\underline{= -3}$

$\therefore$ the gradient of the normal at P is $\underline{\frac{1}{3}}$.

Equation of normal: $y - y_1 = m(x - x_1)$

Using $P(4,8)$ $\rightarrow$ $y - 8 = \frac{1}{3}(x - 4)$

$3(y-8) = 1(x-4)$

$3y - 24 = x - 4$

$\boxed{3y = x + 20}$

d) Normal at P cuts x-axis at Q.

When normal cuts x-axis, $y=0$

So, $3(0) = x + 20$

$0 = x + 20$

$x = -20$

$\therefore$ Q is at $(-20, 0)$

Also, P is at $(4,8)$

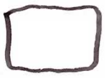
Length $PQ = \sqrt{(x_1 - x_2)^2 + (y_2 - y_1)^2}$

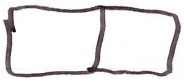
$= \sqrt{(4-(-20))^2 + (8-0)^2}$

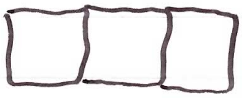
$= \sqrt{24^2 + 8^2}$

$= \sqrt{640} = \sqrt{64 \times 10} = \sqrt{64}\sqrt{10}$

$= \boxed{8\sqrt{10} \text{ units}}$

Q9) Row 1  - 4 sticks

Row 2  - 7 sticks

Row 3  - 10 sticks

a) 4 sticks in Row, so first term, $a = 4$

Common difference, $d = 3$

$$U_n = a + (n-1)d$$

$$U_n = 4 + (n-1)3$$

$$U_n = 4 + 3n - 3$$

$$\boxed{U_n = 3n + 1}$$

b) $$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{10} = \frac{10}{2}(2(4) + (10-1)3)$$

$$= 5(8 + (9)3)$$

$$= 5(8 + 27)$$

$$= 5(35)$$

$$\boxed{= 175}$$

c) Ann started with 1750 sticks.
She has enough to complete k rows, but not $(k+1)$ rows.

So, $S_k < 1750$

$$\frac{k}{2}(2(4) + (k-1)3) < 1750$$

$$\frac{k}{2}(8 + 3k - 3) < 1750$$

$$\frac{k}{2}(3k+5) < 1750$$

$$\frac{3k^2}{2} + \frac{5k}{2} < 1750$$

$$\frac{3k^2}{2} + \frac{5k}{2} - 1750 < 0$$

$$3k^2 + 5k - 3500 < 0$$

$$\therefore \boxed{(3k-100)(k+35) < 0}$$

d) Either $3k = 100$ or $k = -35$

Since no. of rows, k has to be positive,

$$3k = 100 \Rightarrow k = \frac{100}{3} = 33.\dot{3}$$

$$\boxed{k = 33}$$

Q10a)

i) $y = x^2(x-2)$

$y = x^3 - 2x^2$

When $\underline{x=0,\ y=0}$

When $y=0,\ x^3-2x^2=0$

$x^2(x-2)=0$

$\underline{x=0 \text{ or } x=0 \text{ or } x=2}$

ii) $y = x(6-x)$

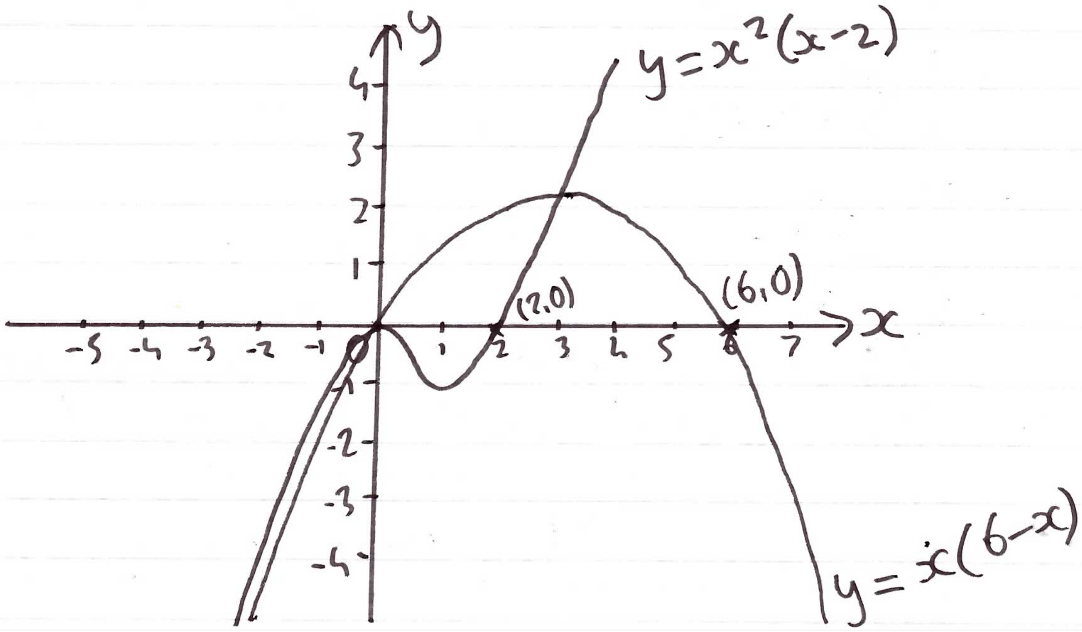
$y = 6x - x^2$

When $\underline{x=0,\ y=0}$

When $y=0,\ 6x-x^2=0$

$x(6-x)=0$

$\underline{x=0 \text{ or } x=6}$



b) To find the coordinates of the points where the graphs intersect, solve simultaneous eqs

$$y = x^3 - 2x^2 \quad (1)$$
$$y = 6x - x^2 \quad (2)$$

Substitute (1) into (2)

$$x^3 - 2x^2 = 6x - x^2$$

$$x^3 - x^2 - 6x = 0$$

$$x(x^2 - x - 6) = 0$$

<u>Either $x = 0$ or $x^2 - x - 6 = 0$</u>

For $x^2 - x - 6 = 0$, $(x-3)(x+2) = 0$

<u>Either $x = 3$ or $x = -2$</u>

Substitute into (2) for y:

When $x = 0$, $y = 6(0) - 0^2 \ \underline{= 0}$

When $x = 3$, $y = 6(3) - 3^2 = 18 - 9 \ \underline{= 9}$

When $x = -2$, $y = 6(-2) - (-2)^2 = -12 - 4 \ \underline{= -16}$

The coordinates of the points where the graphs intersect are:

$$\boxed{(0,0),\ (3,9),\ \text{and}\ (-2,-16)}$$