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GCE

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Core Mathematics C1 (6663)

January 2006

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Mark Scheme (Results)

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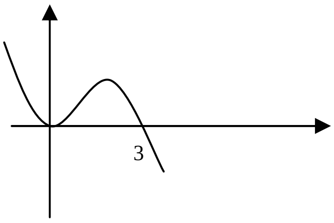
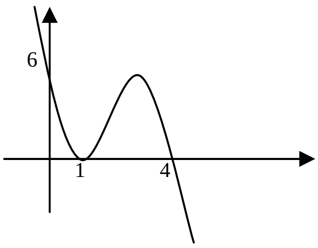
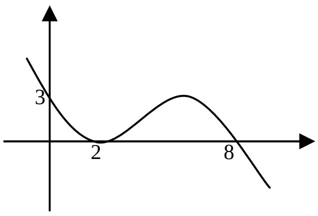
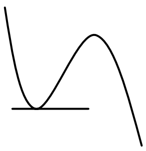
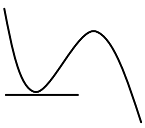
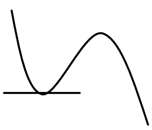
Question number	Scheme		Marks
1.	$x(x^2-4x+3)$	Factor of x. (Allow $(x-0)$)	M1
	$= x(x-3)(x-1)$	Factorise 3 term quadratic	M1 A1
			(3)
			Total 3 marks
	Alternative: $(x^2-3x)(x-1)$ or $(x^2-x)(x-3)$ scores the second M1 (allow ± for each sign), then $x(x-3)(x-1)$ scores the first M1, and A1 if correct. Alternative: Finding factor $(x-1)$ or $(x-3)$ by the factor theorem scores the second M1, then completing, using factor x, scores the first M1, and A1 if correct. Factors "split": e.g. $x(x^2-4x+3) \Rightarrow (x-3)(x-1)$. Allow full marks. Factor x not seen: e.g. Dividing by $x \Rightarrow (x-3)(x-1)$. M0 M1 A0. If an equation is solved, i.s.w.		

Question number	Scheme		Marks
2.	(a) $u_2 = (-2)^2 = 4$		B1
	$u_3 = 1,\ u_4 = 4$	For u_3, ft $(u_2 - 3)^2$	B1ft, B1
			(3)
	(b) $u_{20} = 4$		B1ft
			(1)
			Total 4 marks
	(b) ft only if sequence is "oscillating". Do <u>not</u> give marks if answers have clearly been obtained from wrong working, e.g. $u_2 = (3-3)^2 = 0$ $u_3 = (4-3)^2 = 1$ $u_4 = (5-3)^2 = 4$		

Question number	Scheme	Marks
3.	(a) $y = 5-(2\times 3) = -1$ (or equivalent verification) (*)	B1
		(1)
	(b) Gradient of L is $\frac{1}{2}$	B1
	$y-(-1) = \frac{1}{2}(x-3)$ (ft from a <u>changed</u> gradient)	M1 A1ft
	$x-2y-5=0$ (or equiv. with integer coefficients)	A1
		(4)
		Total 5 marks
	(a) $y-(-1) = -2(x-3) \Rightarrow y = 5-2x$ is fine for B1. Just a table of values including $x=3,\ y=-1$ is insufficient. (b) M1: eqn of a line through (3, –1), with any numerical gradient (except 0 or ∞). For the M1 A1ft, the equation may be in any form, e.g. $\frac{y-(-1)}{x-3} = \frac{1}{2}$. Alternatively, the M1 may be scored by using $y = mx + c$ with a numerical gradient and substituting (3, –1) to find the value of c, with A1ft if the value of c follows through correctly from a <u>changed</u> gradient. Allow $x-2y=5$ or equiv., but must be integer coefficients. The "= 0" can be implied if correct working precedes.	

Question number	Scheme		Marks
4.	(a) $\frac{\mathrm{d}y}{\mathrm{d}x} = 4x + 18x^{-4}$	M1: $x^2 \to x$ or $x^{-3} \to x^{-4}$	M1 A1
			(2)
	(b) $\frac{2x^3}{3} - \frac{6x^{-2}}{-2} + C$	M1: $x^2 \to x^3$ or $x^{-3} \to x^{-2}$ or $+C$	M1 A1 A1
			(3)
	$\left(= \frac{2x^3}{3} + 3x^{-2} + C\right)$	First A1: $\frac{2x^3}{3} + C$	
		Second A1: $-\frac{6x^{-2}}{-2}$	
			Total 5 marks
	In both parts, accept any correct version, simplified or not. Accept $4x^1$ for $4x$. <u>$+ C$ in part (a) instead of part (b):</u> Penalise only once, so if otherwise correct scores M1 A0, M1 A1 A1.		

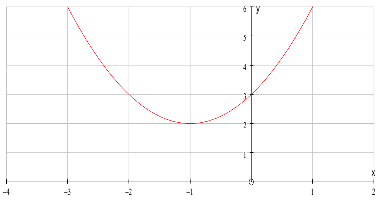
Question number	Scheme	Marks
5.	(a) $3\sqrt{5}$ (or $a = 3$)	B1
		(1)
	(b) $\frac{2(3+\sqrt{5})}{(3-\sqrt{5})} \times \frac{(3+\sqrt{5})}{(3+\sqrt{5})}$	M1
	$(3-\sqrt{5})(3+\sqrt{5}) = 9-5$ (= 4) (Used as or intended as denominator)	B1
	$(3+\sqrt{5})(p \pm q\sqrt{5}) = ...$ 4 terms $(p \neq 0, q \neq 0)$ (Independent)	M1
	or $(6+2\sqrt{5})(p \pm q\sqrt{5}) = ...$ 4 terms $(p \neq 0, q \neq 0)$	
	[Correct version: $(3+\sqrt{5})(3+\sqrt{5}) = 9+3\sqrt{5}+3\sqrt{5}+5$, or double this.]	
	$\frac{2(14+6\sqrt{5})}{4} = 7+3\sqrt{5}$ 1st A1: b = 7, 2nd A1: c = 3	A1 A1
		(5)
		Total 6 marks
	(b) 2nd M mark for attempting $(3+\sqrt{5})(p+q\sqrt{5})$ is generous. Condone errors.	

Question number	Scheme	Marks
6.	(a) 	
	(See below)	M1
	Clearly through origin (or (0, 0) seen)	A1
	3 labelled (or (3, 0) seen)	A1 (3)
	(b) 	
	Stretch parallel to *y*-axis	M1
	1 and 4 labelled (or (1, 0) and (4, 0) seen)	A1
	6 labelled (or (0, 6) seen)	A1 (3)
	(c) 	
	Stretch parallel to *x*-axis	M1
	2 and 8 labelled (or (2, 0) and (8, 0) seen)	A1
	3 labelled (or (0, 3) seen)	A1 (3)
		Total 9 marks
	(a) M1: 	
	(b) M1:  with at least two of: (1, 0) unchanged (4, 0) unchanged (0, 3) changed	
	(c) M1:  with at least two of: (1, 0) changed (4, 0) changed (0, 3) unchanged	
	<u>Beware</u>: Candidates may sometimes re-label the parts of their solution.	

Question number	Scheme	Marks
7.	(a) 500 + (500 + 200) = 1200 or $S_2 = \frac{1}{2}2\{1000+200\} = 1200$ (*)	B1 (1)
	(b) Using a = 500, d = 200 with n = 7, 8 or 9 $a+(n-1)d$ or "listing"	M1
	$500+(7\times 200)$ = (£)1900	A1 (2)
	(c) Using $\frac{1}{2}n\{2a+(n-1)d\}$ or $\frac{1}{2}n\{a+l\}$, or listing and "summing" terms	M1
	$S_8 = \frac{1}{2}8\{2\times 500+7\times 200\}$ or $S_8 = \frac{1}{2}8\{500+1900\}$, or all terms in list correct	A1
	= (£) 9600	A1 (3)
	(d) $\frac{1}{2}n\{2\times 500+(n-1)\times 200\} = 32000$ M1: General S_n, equated to 32000	M1 A1
	$n^2+4n-320=0$ (or equiv.) M1: Simplify to 3 term quadratic	M1 A1
	$(n+20)(n-16)=0$ $n = ...$ M1: Attempt to solve 3 t.q.	M1
	n = 16, Age is 26	A1cso,A1cso
		(7)
		Total 13 marks
	(b) Correct answer with no working: Allow both marks. (c) <u>Some</u> working must be seen to score marks: Minimum working: 500 + 700 + 900 +…(+ 1900) =… scores M1 (A1). (d) Allow ≥ or > throughout , apart from "Age 26". A common <u>misread</u> here is 3200. This gives n = 4 and age 14, and can score M1 A0 M1 A0 M1 A1 A1 with the usual misread rule. <u>Alternative:</u> (Listing sums) (500, 1200, 2100, 3200, 4500, 6000, 7700, 9600,) 11700, 14000, 16500, 19200, 22100, 25200, 28500, 32000. List at least up to 32000 M3 All values correct A2 n = 16 (perhaps implied by age) A1cso Age 26 A1cso If there is a mistake in the list, e.g. 16th sum = 32100, possible marks are: M3 A0 A0 A0 <u>Alternative:</u> (Trial and improvement) Use of S_n formula with n = 16 (and perhaps other values) M3 Accurately achieving 32000 for n = 16 A3 Age 26 A1	

Question number	Scheme		Marks
8.	$\dfrac{5x^2+2}{x^{\frac{1}{2}}} = 5x^{\frac{3}{2}} + 2x^{-\frac{1}{2}}$	M1: One term correct.	M1 A1
		A1: Both terms correct, and no extra terms.	
	$\text{f}(x) = 3x + \dfrac{5x^{\frac{5}{2}}}{\left(\frac{5}{2}\right)} + \dfrac{2x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}$ $(+C)$	(+ C not required here)	M1 A1ft
	$6 = 3 + 2 + 4 + C$	Use of x = 1 and y =6 to form eqn. in C	M1
	$C = -3$		A1cso
	$3x + 2x^{\frac{5}{2}} + 4x^{\frac{1}{2}} - 3$	(simplified version required)	A1 (ft C)
			(7)
	[or: $3x + 2\sqrt{x^5} + 4\sqrt{x} - 3$ or equiv.]		
			Total 7 marks
	For the integration: M1 requires evidence from just one term (e.g. $3 \rightarrow 3x$), but not just "+C". A1ft requires correct integration of at least 3 terms, with at least one of these terms having a fractional power. For the final A1, follow through on C only.		

Question number	Scheme	Marks
9.	(a) -2 (P), 2 (Q) (± 2 scores B1 B1)	B1, B1
		(2)
	(b) $y = x^3 - x^2 - 4x + 4$ (May be seen earlier) Multiply out, giving 4 terms	M1
	$\frac{dy}{dx} = 3x^2 - 2x - 4$ (*)	M1 A1cso
		(3)
	(c) At $x = -1$: $\frac{dy}{dx} = 3(-1)^2 - 2(-1) - 4 = 1$	
	Eqn. of tangent: $y - 6 = 1(x - (-1))$, $y = x + 7$ (*)	M1 A1cso
		(2)
	(d) $3x^2 - 2x - 4 = 1$ (Equating to "gradient of tangent")	M1
	$3x^2 - 2x - 5 = 0$ $(3x - 5)(x + 1) = 0$ $x = \ldots$	M1
	$x = \frac{5}{3}$ or equiv.	A1
	$y = \left(\frac{5}{3} - 1\right)\left(\frac{25}{9} - 4\right)$, $= \frac{2}{3} \times \left(-\frac{11}{9}\right) = -\frac{22}{27}$ or equiv.	M1, A1
		(5)
		Total 12 marks
	(b) <u>Alternative:</u> Attempt to differentiate by product rule scores the <u>second</u> M1: $\frac{dy}{dx} = \{(x^2 - 4) \times 1\} + \{(x - 1) \times 2x\}$ Then multiplying out scores the <u>first</u> M1, with A1 if correct (cso). (c) M1 requires full method: Evaluate $\frac{dy}{dx}$ and use in eqn. of line through $(-1, 6)$, (n.b. the gradient need not be 1 for this M1). <u>Alternative:</u> Gradient of $y = x + 7$ is 1, so solve $3x^2 - 2x - 4 = 1$, as in (d)… M1 to get $x = -1$. A1cso (d) 2nd and 3rd M marks are dependent on starting with $3x^2 - 2x - 4 = k$, where k is a constant.	

Question number	Scheme		Marks	
10.	(a) $x^2 + 2x + 3 = (x+1)^2, \; +2$	(a = 1, b = 2)	B1, B1	(2)
	(b) 	"U"-shaped parabola	M1	
		Vertex in correct quadrant (ft from (–a, b))	A1ft	
		(0, 3) (or 3 on y-axis)	B1	(3)
	(c) $b^2 - 4ac = 4 - 12 = -8$		B1	
	Negative, so curve does not cross x-axis		B1	(2)
	(d) $b^2 - 4ac = k^2 - 12$	(May be within the quadratic formula)	M1	
	$k^2 - 12 < 0$	(Correct inequality expression in any form)	A1	
	$-\sqrt{12} < k < \sqrt{12}$	(or $-2\sqrt{3} < k < 2\sqrt{3}$)	M1 A1	(4)
			Total 11 marks	

(b) The B mark can be scored independently of the sketch.
(3, 0) shown on the y-axis scores the B1, but if not shown on the axis, it is B0.

(c) "…. no real roots" is insufficient for the 2nd B mark.
"…. curve does not touch x-axis" is insufficient for the 2nd B mark.

(d) 2nd M1: correct solution method for <u>their</u> quadratic inequality,
e.g. $k^2 - 12 < 0$ gives k <u>between</u> the 2 critical values $\alpha < k < \beta$,
whereas $k^2 - 12 > 0$ gives $k < \alpha,\; k > \beta$.
"$k > -\sqrt{12}$ and $k < \sqrt{12}$" scores the final M1 A1, but
"$k > -\sqrt{12}$ or $k < \sqrt{12}$" scores M1 A0,
"$k > -\sqrt{12}$, $k < \sqrt{12}$" scores M1 A0.

N.B. $k < \pm\sqrt{12}$ does not score the 2nd M mark.
$k < \sqrt{12}$ does not score the 2nd M mark.

$\leq$ instead of <: Penalise only once, on first occurrence.

GENERAL PRINCIPLES FOR C1 MARKING

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm p)^2 \pm q \pm c$, $p \neq 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but will be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may gain no credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Misreads

(See the next sheet for a simple example).

A misread must be consistent for the whole question to be interpreted as such.

These are not common. In clear cases, please deduct the first 2 A (or B) marks which would have been lost by following the scheme. (Note that 2 marks is the maximum misread penalty, but that misreads which alter the nature or difficulty of the question cannot be treated so generously and it will usually be necessary here to follow the scheme as written).

Sometimes following the scheme as written is more generous to the candidate than applying the misread rule, so in this case use the scheme as written.

MISREADS

Question 8. $5x^2$ misread as $5x^3$

8. $\dfrac{5x^3+2}{x^{\frac{1}{2}}} = 5x^{\frac{5}{2}} + 2x^{-\frac{1}{2}}$ M1 A0

$f(x) = 3x + \dfrac{5x^{\frac{7}{2}}}{\left(\frac{7}{2}\right)} + \dfrac{2x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} \ (+C)$ M1 A1ft

$6 = 3 + \dfrac{10}{7} + 4 + C$ M1

$C = -\dfrac{17}{7}$, $\quad f(x) = 3x + \dfrac{10}{7}x^{\frac{7}{2}} + 4x^{\frac{1}{2}} - \dfrac{17}{7}$ A0, A1
