

C1 January 2006 (MA)

$$Q1) x^3 - 4x^2 + 3x = x(x^2 - 4x + 3)$$

$$= \boxed{x(x-3)(x-1)}$$

$$Q2a) U_2 = (1-3)^2 = (-2)^2 = \underline{4}$$

$$U_3 = (4-3)^2 = (1)^2 = \underline{1}$$

$$U_4 = (1-3)^2 = (-2)^2 = \underline{4}$$

b) We notice that for U_n , when n is even, $U_n = 4$, and when n is odd, U_n is 1.

$$\therefore \boxed{U_{20} = 4}$$

$$Q3a) y = 5 - 2x$$

$$y = 5 - 2(3) = \underline{-1}$$

Substitute in
 $x=3$

$\therefore P(3, -1)$ lies on the curve

$$b) y = 5 - 2x \Rightarrow y = -2x + 5$$

Gradient of line is -2 .

$\therefore$ Gradient of perpendicular line is $\frac{1}{2}$

$$\text{Equation: } y - (-1) = \frac{1}{2}(x-3) \Rightarrow y+1 = \frac{1}{2}x - \frac{3}{2}$$

$$\boxed{x - 2y - 5 = 0}$$

$$Q4a) \quad y = 2x^2 - \frac{6}{x^3}, \quad x \neq 0$$

$$y = 2x^2 - 6x^{-3}$$

$$\frac{dy}{dx} = 4x + 18x^{-4}$$

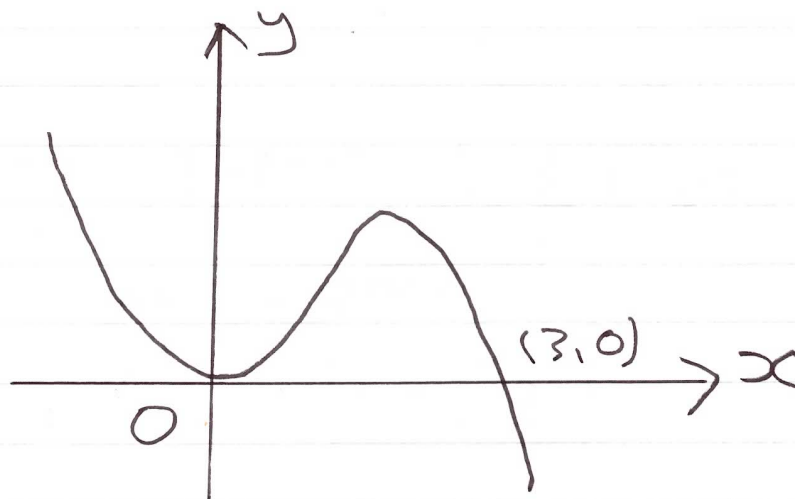
$$= \boxed{4x + \frac{18}{x^4}}$$

$$\begin{aligned} b) \quad \int (2x^2 - 6x^{-3}) dx &= \frac{2x^3}{3} - \frac{6x^{-2}}{-2} + C \\ &= \frac{2x^3}{3} + 3x^{-2} + C \\ &= \boxed{\frac{2x^3}{3} + \frac{3}{x^2} + C} \end{aligned}$$

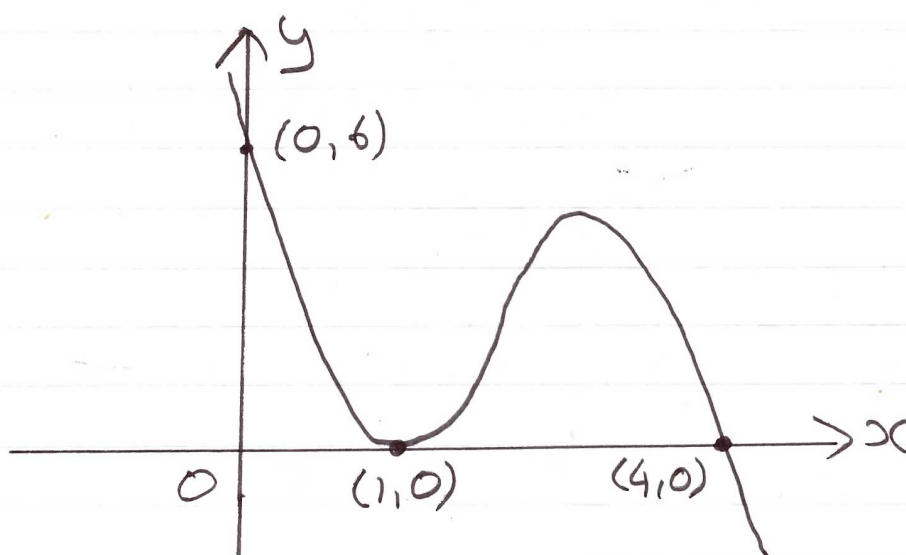
$$Q5a) \quad \sqrt{45} = \sqrt{9 \times 5} = \sqrt{9} \sqrt{5} = \boxed{3\sqrt{5}}$$

$$\begin{aligned} b) \quad \frac{2(3+\sqrt{5})}{3-\sqrt{5}} &= \frac{6+2\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} \\ &= \frac{(6+2\sqrt{5})(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})} \\ &= \frac{18+6\sqrt{5}+6\sqrt{5}+10}{9+3\sqrt{5}-3\sqrt{5}-5} \\ &= \frac{28+12\sqrt{5}}{4} \\ &= \boxed{7+3\sqrt{5}} \end{aligned}$$

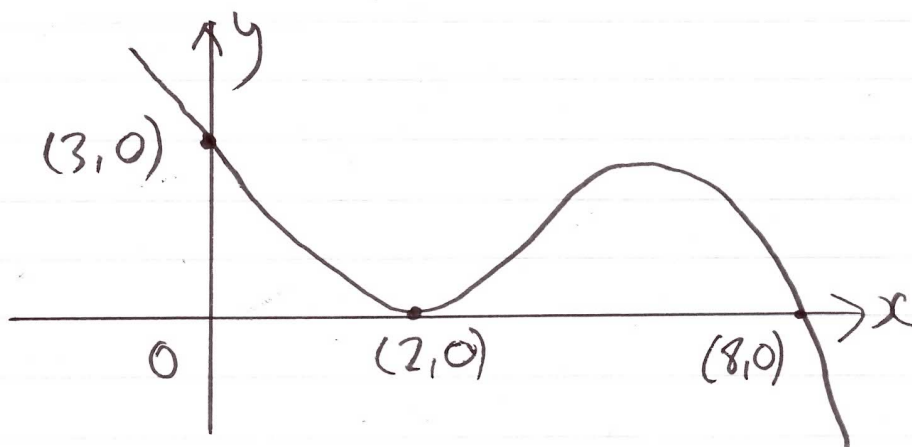
Q6a) $y = f(x+1)$: transformation of x -coordinates to the left by 1:



b) $y = 2f(x)$: multiply y -coordinates by 2:



c) $y = f(\frac{1}{2}x)$ - multiply x -coordinates by 2:



Q7a) $a = 500, d = 200$

$$S_n = \frac{n}{2} (2a + (n-1)d)$$

$$S_2 = \frac{2}{2} (2(500) + (2-1)200)$$

$$= 1(1000 + 200)$$

$$= \boxed{\pounds 1200}$$

b) $a = 500, d = 200$

$$U_n = a + (n-1)d$$

$$U_8 = 500 + (7)200$$

$$= \boxed{\pounds 1900}$$

c) $S_n = \frac{n}{2} (2a + (n-1)d)$

$$S_8 = \frac{8}{2} (2(500) + (8-1)d)$$

$$= 4(1000 + 1400)$$

$$= 4 \times 2400$$

$$= \boxed{\pounds 9600}$$

$$d) \quad 32000 = \frac{1}{2} n (2(500) + (n-1)200)$$

$$64000 = n(1000 + 200n - 200)$$

$$64000 = n(800 + 200n)$$

$$64000 = 800n + 200n^2$$

$$200n^2 + 800n - 64000 = 0$$

$$n^2 + 4n - 320 = 0$$

$$(n+20)(n-16) = 0$$

Since n has to be positive, $n=16$

$\therefore$ Alice is 26 when she receives her last allowance

$$Q8) \quad f'(x) = 3 + \frac{5x^2 + 2}{x^{1/2}}, \quad x > 0$$

$$f'(x) = 3 + \frac{5x^2}{x^{1/2}} + \frac{2}{x^{1/2}}$$

$$f'(x) = 3 + 5x^{\frac{3}{2}} + 2x^{-\frac{1}{2}}$$

$$f(x) = 3x + \frac{5x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$f(x) = 3x + 2x^{\frac{5}{2}} + 4x^{\frac{1}{2}} + C$$

$$f(x) = 3x + 2x^2\sqrt{x} + 4\sqrt{x} + C$$

Using the point (1, 6):

$$6 = 3(1) + 2(1)^2(\sqrt{1}) + 4\sqrt{1} + C$$

$$6 = 3 + 2 + 4 + C$$

$$\underline{C = -3}$$

$$\therefore \boxed{f(x) = 3x + 2x^2\sqrt{x} + 4\sqrt{x} - 3}$$

Q9a) When curve cuts x-axis, $y=0$

$$\therefore (x-1)(x^2-4) = 0$$

$$(x-1)(x-2)(x+2) = 0$$

$$\therefore \frac{x\text{-coordinate of P is } -2}{x\text{-coordinate of Q is } 2}$$

b) $y = (x-1)(x^2-4)$

$$y = x^3 - 4x - x^2 + 4$$

$$y = x^3 - x^2 - 4x + 4$$

$$\boxed{\frac{dy}{dx} = 3x^2 - 2x - 4}$$

c) At $x = -1$, $\frac{dy}{dx} = 3(-1)^2 - 2(-1) - 4$

$$= 3 + 2 - 4$$

$$\underline{= 1}$$

Equation of tangent: $y - y_1 = m(x - x_1)$

Using $(-1, 6)$: $y - 6 = 1(x - (-1))$

$$y - 6 = x + 1$$

$$\boxed{y = x + 7}$$

d) Since the gradient of the tangent at R is equal, $3x^2 - 2x - 4 = 1$

$$3x^2 - 2x - 5 = 0$$

$$(3x - 5)(x + 1) = 0$$

$$x = \frac{5}{3} \text{ or } x = -1$$

We already know that $(-1, 6)$ lies on a tangent to the curve, so find the coordinates when $x = \frac{5}{3}$:

$$y = (x - 1)(x^2 - 4)$$

$$y = \left(\frac{5}{3} - 1\right)\left(\left(\frac{5}{3}\right)^2 - 4\right)$$

$$y = \left(\frac{2}{3}\right)\left(\frac{25}{9} - 4\right)$$

$$y = \left(\frac{2}{3}\right)\left(-\frac{11}{9}\right)$$

$$y = -\frac{22}{27}$$

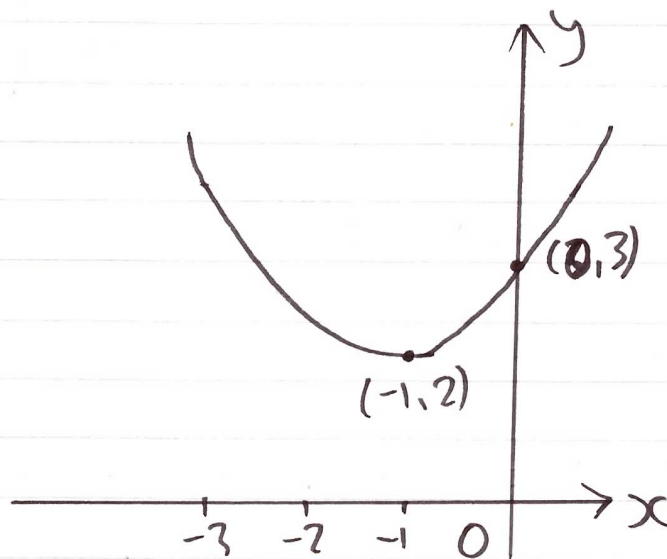
$\therefore$ coordinates of R are

$$\boxed{\left(\frac{5}{3}, -\frac{22}{27}\right)}$$

$$\begin{aligned} \text{Q10a) } x^2 + 2x + 3 &= (x+1)^2 - 1 + 3 \\ &= (x+1)^2 + 2 \end{aligned}$$

$$\therefore \boxed{a=1 \text{ and } b=2}$$

b) sketch of $y = x^2 + 2x + 3$:



$x=0, y=3$
 • Turning point
 at $(-1, 2)$
 [from (a)]

$$\text{c) } b^2 - 4ac = 2^2 - 4(1)(3) = \underline{-8}$$

$\therefore$ there are no real roots at $y=0$, and so the curve doesn't cross the x -axis.

d) $x^2 + Kx + 3 = 0$ has no real roots, so $b^2 - 4ac < 0$

$$K^2 - 4(1)(3) < 0$$

$$K^2 - 12 < 0$$

$$K^2 < 12$$

$$\boxed{-2\sqrt{3} < K < 2\sqrt{3}}$$