

$$1a) \quad 16^{1/2} = \sqrt{16} = 4$$

$$b) \quad 16^{-3/2} = \frac{1}{16^{3/2}} = \frac{1}{(16^{1/2})^3} = \frac{1}{(\sqrt{16})^3} = \frac{1}{4^3} = \frac{1}{64}$$

$$2i) \quad y = 5x^3 + 7x + 3$$

$$a) \quad \frac{dy}{dx} = 15x^2 + 7$$

$$b) \quad \frac{d^2y}{dx^2} = 30x$$

$$\begin{aligned} ii) \quad & \int \left( 1 + 3\sqrt{x} - \frac{1}{x^2} \right) dx \\ &= \int 1 + 3x^{1/2} - 1x^{-2} dx \\ &= x + \frac{6}{3} x^{3/2} + x^{-1} + \underline{\underline{C}} \\ &= x + 2x^{3/2} + \frac{1}{x} + C \end{aligned}$$

$$3) \quad kx^2 + 12x + k = 0$$

$$\text{divide by } k \quad x^2 + \frac{12}{k}x + 1 = 0$$

roots are equal = same

can only be from  $1 \times 1$

$$\text{roots} = (x+1)^2$$

$$(x+1)(x+1) = x^2 + 2x + 1$$

equating coeffs of  $x$

$$2 = \frac{12}{k}$$

$$k = \frac{12}{2} = 6 \quad \underline{\underline{k=6}}$$

$$4) \quad x + y = 2 \quad \dots \textcircled{1}$$

$$x^2 + 2y = 12 \quad \dots \textcircled{2}$$

from  $\textcircled{1}$   $y = 2 - x$  sub into  $\textcircled{2}$

$$x^2 + 2(2 - x) = 12$$

$$x^2 + 4 - 2x = 12$$

$$x^2 - 2x - 8 = 0$$

$$(x - 4)(x + 2) = 0$$

$$x = 4 \quad \text{or} \quad x = -2$$

when  $x = 4$

$$y = 2 - (4) = -2$$

when  $x = -2$

$$y = 2 - (-2) = 4$$

|     |         |          |                 |
|-----|---------|----------|-----------------|
| 1st | 2nd     | 3rd      | $r^{\text{th}}$ |
| $a$ | $a + d$ | $a + 2d$ | $a + (r - 1)d$  |

$$a + (r - 1)d = 2r - 5$$

$$a + dr - d = 2r - 5$$

equating coeffs of  $r$

$$d = 2$$

$$a + 2r - 2 = 2r - 5$$

first =  $a = -3$

second =  $a + d = -3 + 2$

=  $-1$

$$= \boxed{-3, -1, 1}$$

third =  $a + 2d = -3 + 4$

=  $1$

b) common difference =  $\underline{\underline{a = 2}}$

c)  $\sum_{r=1}^n (2r-5) = n(n-4)$

$$S_n = \frac{1}{2}n [2a + (n-1)d]$$

$$a = -3 \quad d = 2$$

$$S_n = \frac{1}{2}n [-6 + (n-1)2]$$

$$= \frac{1}{2}n [-6 + 2n - 2]$$

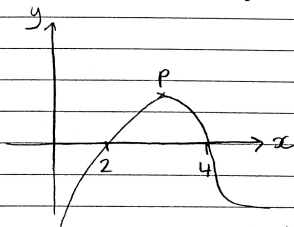
$$= \frac{1}{2}n [-8 + 2n]$$

$$= -4n + n^2$$

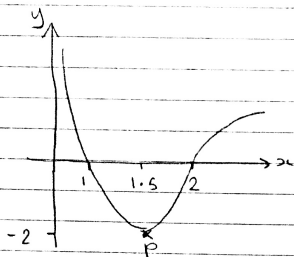
$$= n(n-4)$$

/ reflection in x-axis

6) a)  $y = -f(x) \Rightarrow$  (all y coords  $\times -1$ )  
 $(2,0) (4,0) (3,2)$



b)  $y = f(2x) \Rightarrow$  all  $x$  coords  $\times \frac{1}{2}$   
 $(1, 0) (2, 0) (1.5, -2)$



$$y = 4x^2 + \frac{5-x}{x}$$

$$y = 4x^2 + \frac{5}{x} - \frac{x}{x}$$

$$y = 4x^2 + 5x^{-1} - 1$$

$$\frac{dy}{dx} = 8x - 5x^{-2}$$

when  $x=1$  at  $p$

$$\frac{dy}{dx} = 8(1) - 5(1)^{-2} = 8 - 5 = \underline{\underline{3}}$$

c) when  $x=1$

$$y = 4(1)^2 + \frac{5-(1)}{(1)} = 4 + 4 = 8$$

$$y - y_1 = m(x - x_1)$$

$$y - 8 = 3(x - 1)$$

$$y = 3x - 3 + 8$$

$$y = 3x + 5$$



c)

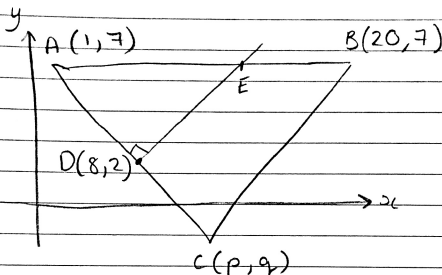
$$y = 3x + 5 \quad \text{at } x\text{-axis } y = 0$$

$$3x + 5 = 0$$

$$3x = -5$$

$$x = -\frac{5}{3}$$

8)



$$\text{midpoint} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\frac{1+p}{2} = 8$$

$$\frac{7+q}{2} = 2$$

$$1+p = 16$$

$$7+q = 4$$

$$\underline{\underline{p = 15}}$$

$$\underline{\underline{q = -3}}$$

b)  $C(-15, -3)$

$$m \text{ of } \overrightarrow{AD} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{7 - 2}{1 - 8} = \frac{5}{-7}$$

$$m \text{ of } l = \frac{7}{5} \text{ as perpendicular}$$

$$y - y_1 = m(x - x_1) \quad \text{at } D(8, 2)$$

$$y - 2 = \frac{7}{5}(x - 8) \Rightarrow 5y - 10 = 7x - 56$$

$$5y = 7x - 46$$

$$-7x + 5y + 46 = 0$$

c) line AB =  $y = 7$

Sub in to

$$-7x + 5y + 46 = 0$$

$$\bullet -7x + 5(7) + 46 = 0$$

$$-7x + 35 + 46 = 0$$

$$-7x + 81 = 0$$

$$-7x = -81$$

$$x = \frac{-81}{-7}$$

$$\boxed{x = \frac{81}{7}}$$

35

46

81

mdC =  $\frac{dy}{dx} = (3x-1)^2$

at point (1, 4)

$$\frac{dy}{dx} = (3(1)-1)^2 = 2^2 = 4$$

$$\frac{dy}{dx} \text{ at normal} = -\frac{1}{4}$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{1}{4}(x - 1)$$

$$4y - 16 = -x + 1$$

$$4y = -x + 17$$

b)  $\frac{dy}{dx} = (3x-1)^2 = (3x-1)(3x-1) = 9x^2 - 6x + 1$

$$\int \frac{dy}{dx} = y = \int 9x^2 - 6x + 1 \, dx$$

$$y = \frac{9}{3}x^3 - \frac{6}{2}x^2 + x + c$$

point P(1, 4) is on C so when  $x = 1$   $y = 4$

$$y = \frac{9}{3}x^3 - \frac{6}{2}x^2 + x + c$$

$$y = 3x^3 - 3x^2 + x + c$$

$$4 = 3(1)^3 - 3(1)^2 + 1 + c$$

$$4 = 3 - 3 + 1 + c$$

$$\underline{\underline{c = 3}}$$

$$y = 3x^3 - 3x^2 + x + 3$$

c) parallel to  $y = 1 - 2x$   
means same gradient  $= -2$

↙ squared always  $\neq 0$

$$\text{tangent to } C \text{ gradient} = (3x - 1)^2 \geq 0$$

$$-2 < 0$$

$\therefore$  cannot have same gradient  $\therefore$  cannot be parallel.

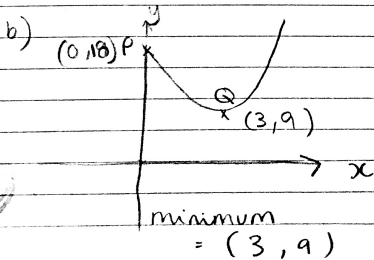
$$10) f(x) = x^2 - 6x + 18$$

↙ complete the square

$$f(x) = \left(x - \frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 18$$

$$= (x - 3)^2 - 9 + 18$$

$$= (x - 3)^2 + 9$$



meet y-axis at

$$x = 0$$

$$y = 18$$

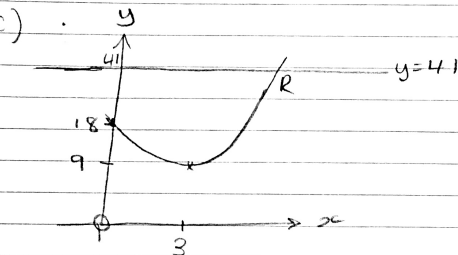
minimum

$$(x - 3)^2 = 0$$

$$x = 3$$

$$y = 3^2 - 6(3) + 18$$

$$y = 9 - 18 + 18 = 9$$



$$41 = x^2 - 6x + 18$$

$$0 = x^2 - 6x - 23$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{6 \pm \sqrt{36 - 4 \times -23}}{2}$$

$$\begin{array}{r} \times 23 \\ 4 \\ \hline 92 \end{array}$$

$$\frac{6 \pm \sqrt{36 + 92}}{2}$$

$$\begin{array}{r} 36 \\ + 92 \\ \hline 128 \end{array}$$

$$\frac{6 \pm \sqrt{128}}{2}$$

16

$$\begin{array}{r} 032 \\ 4 \overline{)128} \end{array}$$

$$\frac{6 \pm \sqrt{4 \times 32}}{2}$$

$$= \frac{6 \pm \sqrt{4 \times 16 \times 2}}{2} = \frac{6 \pm 2 \times 4\sqrt{2}}{2}$$

$$3 \pm 4\sqrt{2}$$

$$x > 0$$

$$\therefore R = \underline{3 + 4\sqrt{2}}$$