Paper Reference(s)

6663/01

Edexcel GCE

Core Mathematics C1

Silver Level S4

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C1), the paper reference (6663), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 10 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
68	60	52	45	40	35

1. Given that $y = 2x^3 + \frac{3}{x^2}$, $x \neq 0$, find

(*a*) $\frac{dy}{dx}$,

(3)

(*b*) $\int y \ dx$, simplifying each term.

(3)

June 2009

2.

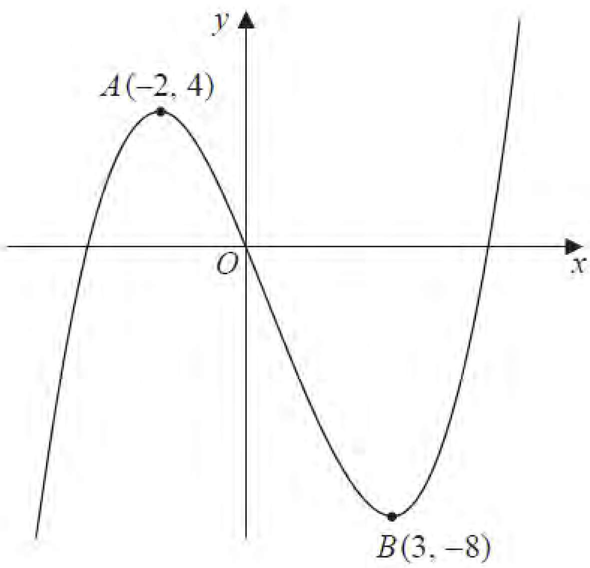

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$. The curve has a maximum point A at (–2, 4) and a minimum point B at (3, –8) and passes through the origin O.

On separate diagrams, sketch the curve with equation

(*a*) $y = 3f(x)$,

(2)

(*b*) $y = f(x) - 4$.

(3)

On each diagram, show clearly the coordinates of the maximum and the minimum points and the coordinates of the point where the curve crosses the *y*-axis.

May 2016

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3. A sequence x_1, x_2, x_3, … is defined by

$$x_1 = 1,$$

$$x_{n+1} = ax_n - 3, \quad n \geq 1,$$

where a is a constant.

(*a*) Find an expression for x_2 in terms of a.

(1)

(*b*) Show that $x_3 = a^2 - 3a - 3$.

(2)

Given that $x_3 = 7$,

(*c*) find the possible values of a.

(3)

June 2008

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4. The straight line L_1 passes through the points (–1, 3) and (11, 12).

(*a*) Find an equation for L_1 in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

The line L_2 has equation $3y + 4x - 30 = 0$.

(*b*) Find the coordinates of the point of intersection of L_1 and L_2 .

(3)

May 2013

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5. Differentiate with respect to x, giving each answer in its simplest form,

(*a*) $(1 - 2x)^2$,

(3)

(*b*) $\dfrac{x^5 + 6\sqrt{x}}{2x^2}$.

(4)

May 2014

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6. The equation $kx^2 + 4x + (5 - k) = 0$, where k is a constant, has 2 different real solutions for x.

(*a*) Show that k satisfies

$$k^2 - 5k + 4 > 0.$$

(3)

(*b*) Hence find the set of possible values of k.

(4)

January 2009

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7. A sequence $a_1,\ a_2,\ a_3, ...$ is defined by

$$a_1 = k,$$

$$a_{n+1} = 3a_n + 5,\quad n \geq 1,$$

where k is a positive integer.

(*a*) Write down an expression for a_2 in terms of k.

(1)

(*b*) Show that $a_3 = 9k + 20$.

(2)

(*c*) (i) Find $\sum_{r=1}^{4} a_r$ in terms of k.

(ii) Show that $\sum_{r=1}^{4} a_r$ is divisible by 10.

(4)

May 2007

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8. On John's 10th birthday he received the first of an annual birthday gift of money from his uncle. This first gift was £60 and on each subsequent birthday the gift was £15 more than the year before. The amounts of these gifts form an arithmetic sequence.

(*a*) Show that, immediately after his 12th birthday, the total of these gifts was £225.

(1)

(*b*) Find the amount that John received from his uncle as a birthday gift on his 18th birthday.

(2)

(*c*) Find the total of these birthday gifts that John had received from his uncle up to and including his 21st birthday.

(3)

When John had received n of these birthday gifts, the total money that he had received from these gifts was £3375.

(*d*) Show that $n^2 + 7n = 25 \times 18$.

(3)

(*e*) Find the value of n, when he had received £3375 in total, and so determine John's age at this time.

(2)

May 2016

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9. (*a*) Factorise completely $x^3 - 6x^2 + 9x$

(3)

(*b*) Sketch the curve with equation

$$y = x^3 - 6x^2 + 9x$$

showing the coordinates of the points at which the curve meets the x-axis.

(4)

Using your answer to part (*b*), or otherwise,

(*c*) sketch, on a separate diagram, the curve with equation

$$y = (x - 2)^3 - 6(x - 2)^2 + 9(x - 2)$$

showing the coordinates of the points at which the curve meets the x-axis.

(2)

June 2009

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10. Given that $f(x) = 2x^2 + 8x + 3$,

(*a*) find the value of the discriminant of $f(x)$.

(2)

(*b*) Express $f(x)$ in the form $p(x + q)^2 + r$ where p, q and r are integers to be found.

(3)

The line $y = 4x + c$, where c is a constant, is a tangent to the curve with equation $y = f(x)$.

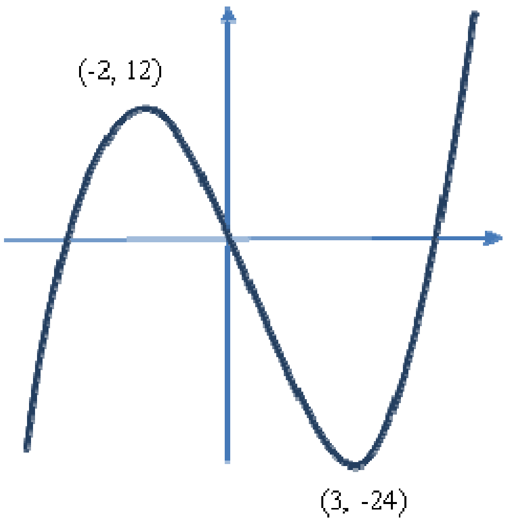
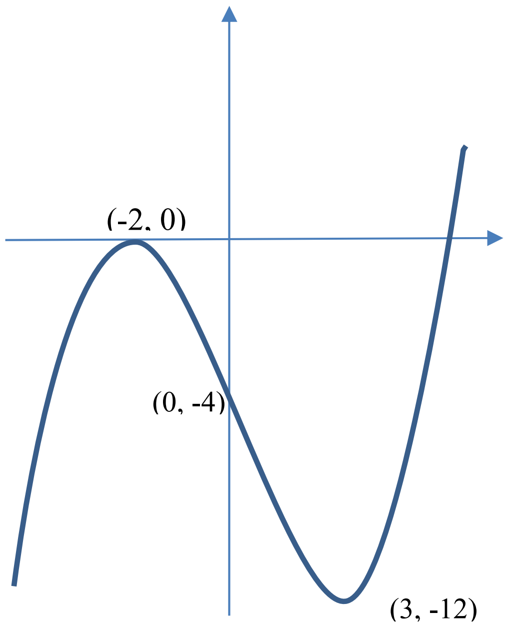
(*c*) Calculate the value of c.

(5)

May 2014

TOTAL FOR PAPER: 75 MARKS

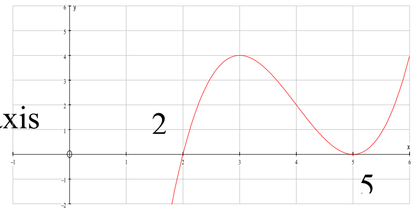
END

Question number	Scheme	Marks
1 (a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 6x^2 - 6x^{-3}$	M1 A1 A1 **(3)**
(b)	$\frac{2x^4}{4} + \frac{3x^{-1}}{-1}(+C)$	M1 A1
	$\frac{x^4}{2} - 3x^{-1} + C$	A1 **(3)** **[6]**
2 (a)		B1 B1 **(2)**
(b)		M1 A1 A1 **(3)** **[5]**

Question number	Scheme	Marks
3 (a)	$[x_2 =]\, a-3$	B1
		(1)
(b)	$[x_3 =]\ ax_2 - 3$ or $a(a-3)-3$	B1
	$= a(a-3)-5,\quad = a^2-3a-3$ **(*)**	A1 cso
		(2)
(c)	$a^2-3a-3=7$	
	$a^2-3a-10=0$ or $a^2-3a=10$	M1
	$(a-5)(a+2)=0$	M1
	$a=5$ or -2	A1
		(3)
		[6]
4 (a)	(-1, 3) , (11, 12)	
	$m = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{12-3}{11-(-1)}, = \dfrac{3}{4}$	M1 A1
	$y-3 = \frac{3}{4}(x+1)$ or $y-12 = \frac{3}{4}(x-11)$ or $y = \frac{3}{4}x + c$ with attempt at substitution to find c	M1
	$4y-3x-15=0$	A1
		(4)
(b)	Solve equation from part (a) and L_2 simultaneously to eliminate one variable	M1
	$x=3$ or $y=6$	A1
	Both $x=3$ **and** $y=6$	A1
		(3)
		[7]

Question number	Scheme	Marks
5 (a)	$(1-2x)^2 = 1-4x+4x^2$	M1
	$\frac{d}{dx}(1-2x)^2 = \frac{d}{dx}(1-4x+4x^2) = -4+8x$ o.e.	M1A1
		(3)
	Alternative method using chain rule: Answer of -4 (1 – 2 x)	M1M1A1
		(3)
(b)	$\frac{x^5+6\sqrt{x}}{2x^2} = \frac{x^5}{2x^2} + 6\frac{\sqrt{x}}{2x^2}, = \frac{1}{2}x^3 + 3x^{-\frac{3}{2}}$	M1 A1
	Attempts to differentiate $x^{-\frac{3}{2}}$ to give k	M1
	$= \frac{3}{2}x^2 - \frac{9}{2}x^{-\frac{5}{2}}$ o.e.	A1
	Quotient Rule (May rarely appear) – See note below	**(4)**
		[7]
6 (a)	$b^2-4ac>0 \Rightarrow \quad 16-4k(5-k)>0$ or equiv., e.g. $16>4k(5-k)$	M1 A1
	So $k^2-5k+4>0$ (Allow any order of terms, e.g. $4-5k+k^2>0$)(*)	A1 cso
		(3)
(b)	<u>Critical Values</u> $(k-4)(k-1)=0 \quad k = \ldots.$	M1
	k = 1 or 4	A1
	Choosing "outside" region	M1
	<u>$k<1$ or $k>4$</u>	A1
		(4)
		[7]

Question number	Scheme	Marks
7 (a)	$(a_2 =)3k+5$ [must be seen in part (a) or labelled $a_2 =$]	B1
		(1)
(b)	$(a_3 =)3(3k+5)+5$	M1
	$= 9k+20$	A1 cso
		(2)
(c)(i)	$a_4 = 3(9k+20)+5 \quad (= 27k+65)$	M1
	$\sum_{r=1}^{4} a_r = k+(3k+5)+(9k+20)+(27k+65)$	M1
(ii)	$= 40k + 90$	A1
	$= 10(4k+9)$ (or explain why divisible by 10)	A1ft
		(4)
		[7]
8 (a)	$60 + 75 + 90 = 225^*$ or $S_3 = \frac{3}{2}\left(120 + (3-1)(15)\right) = 225^*$	B1 *
		(1)
(b)	$t_9 = 60 + (n-1)15 = (£)180$	M1 A1
		(2)
(c)	$S_n = \frac{n}{2}\left(120 + (n-1)(15)\right)$ or $S_n = \frac{n}{2}\left(60 + 60 + (n-1)(15)\right)$	M1
	$S_n = \frac{12}{2}\left(120 + (12-1)(15)\right)$	A1
	$= (£)1710$	A1
		(3)
(d)	$3375 = \frac{n}{2}\left(120 + (n-1)(15)\right)$	M1
	$6750 = 15n\left(8 + (n-1)\right) \Rightarrow 15n^2 + 105n = 6750$	A1
	$n^2 + 7n = 25 \times 18$ *	A1*
		(3)
(e)	$n = 18 \Rightarrow$ Aged 27	M1 A1
		(2)
		[11]

Question number	Scheme		Marks
9 (a)	$x(x^2-6x+9)$		B1
	$= x(x-3)(x-3)$		M1 A1
			(3)
(b)		Shape 	B1
		<u>Through</u> origin (<u>not</u> touching)	B1
		Touching x-axis only once	
		Touching at (3, 0), or 3 on x-axis	B1
		[Must be on graph not in a table]	B1ft
			(4)
(c)		Moved horizontally (either way)	M1
		(2, 0) and (5, 0), or 2 and 5 on x-axis	A1
			(2)
			[9]

Question number	Scheme	Marks
10 (a)	Discriminant = $b^2 - 4ac = 8^2 - 4\times 2\times 3, = 40$	M1 A1
		(2)
(b)	$2x^2 + 8x + 3 = 2(x^2 + \ldots\ldots\ldots\ldots)$ or p=2	B1
	$= 2((x+2)^2 \pm \ldots)$ or $q = 2$	M1
	$= 2(x+2)^2 - 5$ or $p = 2$, $q = 2$ and $r = -5$	A1
		(3)
(c)	Method 1A: Sets derivative "$4x+8$" $= 4 \Rightarrow x =$, $x = -1$	M1 A1
	Substitute $x = -1$ in $y = 2x^2 + 8x + 3$ $(\Rightarrow y = -3)$	dM1
	Substitute $x = -1$ and $y = -3$ in $y = 4x + c$ or into $(y + 3) = 4(x + 1)$ and expand	dM1
	$c = 1$ or writing $y = 4x + 1$	A1cso
		(5)
	Method 1B: Sets derivative "$4x+8$" $= 4 \Rightarrow x =$, $x = -1$	M1 A1
	Substitute $x = -1$ in $2x^2 + 8x + 3 = 4x + c$	dM1
	Attempts to find value of c	dM1
	$c = 1$ or writing $y = 4x + 1$	A1cso
		(5)
	Method 2: Sets $2x^2 + 8x + 3 = 4x + c$ and collects x terms together	M1
	Obtains $2x^2 + 4x + 3 - c = 0$ or equivalent	A1
	States that $b^2 - 4ac = 0$	dM1
	$4^2 - 4\times 2\times (3 - c) = 0$ and so $c =$	dM1
	$c = 1$	A1cso
		(5)
	Method 3: Sets $2x^2 + 8x + 3 = 4x + c$ and collects x terms together	M1
	Obtains $2x^2 + 4x + 3 - c = 0$ or equivalent	A1
	Uses $2(x+1)^2 - 2 + 3 - c = 0$ or equivalent	dM1
	Writes $-2 + 3 - c = 0$	dM1
	So $c = 1$	A1cso
		(5)
		[10]

Examiner reports

Question 1
This question was answered very well with most candidates knowing and applying the rules for differentiation and integration correctly although the use of the notation for this topic, especially the integration sign, is still poor.

Most differentiated $2x^3$ correctly in part (a) and although many wrote $\frac{3}{x^2}$ as $3x^{-2}$, some still thought the derivative was $-6x^{-1}$ or $+6x^{-3}$. Similar problems arose with the integration in part (b) and some lost marks through failing to simplify their expressions and of course others forgot the $+c$.

Question 2
Most answers were correct with clear, well labelled sketches.

In part (a) the majority gained full marks. Errors were in drawing the graph of $y = f(3x)$ rather than the required graph or errors in their multiplication of 4 or –8 by 3.

In part (b), most drew the correct graph here with maximum and minimum in the correct positions. Errors were either in forgetting to mark at which point the graph crossed the y-axis or in assuming all graphs must go through the origin.

Question 3
The notation associated with sequences given in this form still causes difficulties for some candidates and as a result parts (a) and (b) were often answered less well than part (c). A common error in the first two parts was to leave an x in the expression but most of those who could handle the notation gave clear and accurate answers. There were the usual errors in part (c), with $a^2 - 3a - 4 = 0$ appearing quite often and it was encouraging to see most candidates factorising their quadratic expression confidently as a means of solving the equation. A few candidates still use a trial and improvement approach to questions of this type and they often stopped after finding just one solution and gained no credit.

Question 4
In part (a) most candidates used a correct method to find the gradient of L_1. The equation of L_1 was usually found by using $y = mx + c$ or $y - y_1 = m(x - x_1)$. This was done well by the majority of candidates although there were sometimes errors in substitution and candidates should be encouraged to quote formulae before using them. Some did not convert their equation to the required form with integer coefficients. Some incorrect answers were due to failures in dealing correctly with the signs or by not multiply each term by 4 (or 12).

In solving the simultaneous equations in part (b), a variety of methods were seen with varying degrees of success. Those using substitution often made errors in the arithmetic and/or algebra. Those candidates using elimination were generally more successful. Some candidates equated the = 0 forms of the straight lines to form another equation in x and y. A common incorrect method was to substitute values into the equations, e.g. $x = 0$ or $y = 0$ or points given in part (a).

Question 5
This question provided some discrimination in part (b) and 53.7% gained full marks on the whole question.

A very high proportion of candidates scored full marks in part (a). Most expanded the brackets correctly and then differentiated with only a small minority using the chain rule, which is on an A2 module. Of those candidates who did not score full marks this was mainly because of an incorrect expansion of the brackets or an incorrect method for differentiation without expanding. Frequently the $2x$ was squared to $2x^2$ or there were often sign errors. Occasionally the constant of 1 was squared to 2 and there were a surprising number of misreads with the original expression often being written as $(1 + 2x)$.

Part (b) was completed reasonably well by most candidates. Of those who did not score full marks, most did not manage to achieve the expression in the necessary correct form of $\frac{x^3}{2}+3x^{-\frac{1}{2}}$. The most common error was to multiply the numerator by $2x^{-2}$ rather than by $\frac{1}{2}x^{-2}$ and some added or subtracted $2x^{-2}$ creating a third term. Even those who divided both terms by $2x^2$ made errors to obtain, for example, $2x^2$ for the first term or $3x^{-\frac{1}{2}}$ for the second. Most who achieved $-\frac{3}{2}$ as a power correctly differentiated it to $-\frac{5}{2}$.

Common errors were due to the misconception that the numerator and denominator could be differentiated separately .There were quite a number of candidates who, having done the necessary work to change the expression into a suitable form for differentiation, then completely failed to do the differentiation. A few candidates (who had covered A2 material) attempted to treat the expression as a quotient and differentiate it directly. This approach had mixed success with the necessary formula often misquoted or misapplied.

Question 6
Candidates who understood the demands of this question usually did well, while others struggled to pick up marks. In part (a), those who correctly used the discriminant of the original equation often progressed well, but it was sometimes unclear whether they knew the condition for different real roots. An initial statement such as "$b^2-4ac>0$ for different real roots" would have convinced examiners. Irrelevant work with the discriminant of k^2-5k+4 was sometimes seen.

In part (b) by the vast majority of candidates scored two marks for finding the correct critical values, although it was disappointing to see so many resorting to the quadratic formula. It was surprising, however, that many did not manage to identify the required set of values of k. The inappropriate statement "$1>k>4$" was sometimes given as the final answer, rather than "$k<1$ or $k>4$".

Question 7

Many of the comments made on the June 2006 paper would apply here too. Many candidates were clearly not familiar with the notation and a number used arithmetic series formulae to find the sum in part (c) although this was less common than in June 2006.

Apart from those candidates who had little idea about this topic most were able to answer parts (a) and (b) correctly. In part (c) many attempted to find a_4 using the recurrence relation and those who were not tempted into using the arithmetic series formulae often went on to attempt the sum and usually obtained $40k + 90$ which they were easily able to show was divisible by 10. Some lost marks for poor arithmetic $30k + 90$ and $40k + 80$ being some of the incorrect answers seen.

Question 8

Listing methods alone were quite rare in this question. Many produced very good solutions but a large number of candidates failed to match John's age to the arithmetic sequence. It was pleasing to see that, on the whole, candidates realised when to use u_n and when to use S_n.

In part (a) correct solutions were equally split between adding the first three terms and using the sum formula. A very common error was to evaluate the 12th term as 225.

In part (b) a correct formula was invariably used for u_n and most gained full marks. However, many forfeited the marks by using $n = 18$. Occasionally $n = 8$ was used and merited a method mark only.

In part (c) Almost all candidates were able to pick up the first mark but again many lost subsequent marks as result of using an incorrect value for n (usually 21, 22, 20 or 11). Most candidates favoured using $S_n = \frac{n}{2}(2a + (n-1)d)$ but a few preferred to work out the last term, and used $S_n = \frac{n}{2}(a + l)$. Some candidates missed out on full marks for this part because of arithmetic mistakes: 6×285 and $120 + 165$ caused particular problems.

Even candidates who struggled to pick up marks in parts (a), (b) and (c) due to the incorrect value for n, generally managed part (d) well. The majority equated their summation formula in terms of n to the value given, and most, with considerable effort in some cases, managed to simplify their expressions. The most common reason for losing the final mark was to omit a connecting statement between 3375 or 6750 and the final 25×18.

In part (e) it was disappointing to see so many ignore the given factors and embark on the quadratic formula, thus struggling with the square root of 1849. Some candidates factorised incorrectly to give an answer of 25 (discarding 18 for being negative). Most appreciated their value of -25 was a false solution of the equation in the context given, but once again many candidates failed to distinguish between the term number 18 and the age 27. Other common incorrect answers were 28, 29 and 17.

Question 9

It seemed clear that many candidates did not appreciate the links between the 3 parts of this question and there was much unnecessary work carried out: differentiating to find turning points and tables of values to help draw, not sketch, the curves.

Part (a) was usually answered well but not all the successful candidates started by taking out the factor of x, rather they tried to use the factor theorem to establish a first factor. Whilst techniques from C2 (or higher units) **may** be used in C1 they are not required and the "best" approach will not use them.

A correct factorisation in (a) should have made the sketch in (b) straightforward. Most drew a cubic curve (but some had a negative cubic not a positive one) and usually their curve either touched or passed through the origin. The most common non-cubic curve was a parabola passing through (0, 0) and (3,0). Part (c) looked complicated but those who spotted that they were sketching $f(x - 2)$ had few problems in securing both marks. Many candidates though embarked upon half a page or more of algebraic manipulation to no avail - this part was only worth 2 marks and a little thought may have helped them realise that such an approach was unlikely to be the correct one.

Question 10

Full marks was scored by 25% of the candidates while 32% scored 4 marks or fewer out of the 10 available. This question provided good discrimination.

In part (a) the discriminant was found correctly by most candidates. There were some arithmetic errors, and a few included a square root with the $b^2 - 4ac$ or used the whole quadratic formula. A few confused discriminant with derivative and differentiated.

Part (b) proved a significantly tougher challenge and although a good percentage of candidates achieved full marks, there were frequent errors. Most candidates seemed to know that completing the square was required, however many struggled to deal with the coefficient of x^2 not being unity. Bracketing errors were common and a large minority could not even get as far as $2(x + 2)^2 + k$.

Part (c) was again done with varying success. Most candidates opted to either use differentiation or set $2x^2 + 8x + 3 = 4x + c$. Those candidates who used differentiation were more likely to achieve a full solution that was fully correct though there were many who did not realize that they needed to set the gradient of the curve equal to the gradient of the line; commonly it was equated either to zero or the constant in the gradient function was erroneously believed to be the required value of c. Those who set $2x^2 + 8x + 3 = 4x + c$ fared less well, with many candidates not knowing what to do after they had rearranged their equation to collect together the x terms.

Statistics for C1 Practice Paper Silver Level S4

				Mean score for students achieving grade:							
Qu	**Max score**	**Modal score**	**Mean %**	**ALL**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
1	6		81	4.84		5.72	5.51	5.29	4.95	4.45	2.99
2	5	5	88	4.40	4.92	4.88	4.74	4.61	4.43	4.18	3.19
3	6		83	4.97		5.98	5.85	5.62	5.13	4.39	2.33
4	7	7	75	5.28	6.89	6.70	6.22	5.81	5.25	4.65	3.20
5	7		79	5.51	6.83	6.62	6.19	5.83	5.45	4.87	3.39
6	7		64	4.50		6.56	5.50	4.53	3.78	3.13	1.73
7	7		69	4.85		6.49	5.87	5.38	4.73	3.95	2.09
8	11	11	70	7.65	9.96	9.50	8.47	7.73	7.10	6.31	4.52
9	9		67	6.00		8.02	7.08	6.46	5.73	4.87	2.88
10	10		63	6.25	9.51	9.06	7.49	6.16	5.13	4.34	2.84
	75		**72.33**	**54.25**	**38.11**	**69.53**	**62.92**	**57.42**	**51.68**	**45.14**	**29.16**