CORE MATHEMATICS (C) UNIT 1 TEST PAPER 9

1. Find, in its simplest form, the exact value of $\dfrac{3}{2+2\sqrt{2}} - \dfrac{3}{\sqrt{2}}$. [4]

2. Differentiate with respect to x:

   (i) $(2x + 3)(2 - 3x)$, (ii) $(\sqrt{x})^3$. [5]

3. The vertices of a triangle are P (–2, 2), Q (0, 6) and R (8, t).

   Given that PQ is perpendicular to QR, find

   (i) the value of t, [3]

   (ii) the area of triangle PQR. [3]

4. (i) Given that $x = \dfrac{1}{y^2}$, express $\dfrac{18}{y^4}$ in terms of x. [1]

   (ii) Hence find the real value of y for which $\dfrac{18}{y^4} + \dfrac{1}{y^2} = 4$. [5]

5. Determine by calculation whether or not the line $y + 2x = 1$ is a normal to the curve $y = 9 - x^2$. [7]

6. Given that $f(x) \equiv (x + 3)^2$, sketch the following graphs on separate diagrams. In each case show the coordinates of the minimum point and any points where the graph intersects the x and y axes.

   (i) $y = f(x)$, [2]

   (ii) $y = 2f(x)$, [2]

   (iii) $y = f(x - 3)$. [2]

   (iv) $y = f(x) + 3$. [2]

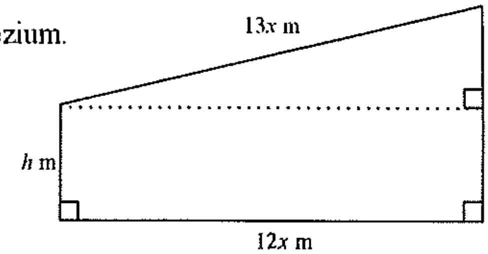
7. Given that $f(x) \equiv (5x^{-1} + 3x^{-2})^2 - 4$ where $x > 0$,

   (i) find the value of x for which $f(x) = 0$. [6]

   (ii) Find an equation of the tangent to the curve $y = f(x)$ at the point where $x = 1$. [6]

8. The diagram shows a plot of land in the shape of a trapezium.

   The perimeter of the plot of land is 200 m.



   (i) Show that $h = 100 - 15x$. [3]

   (ii) Show that the area of the plot is $150(a - (x - b)^2)$ m, where a and b are integers to be found. [6]

   (iii) Explain why this area is maximum when $x = b$, and hence state the largest area of the plot as x varies. [3]

9. The points A, B and C have coordinates $(-3, 4)$, $(7, 4)$ and $(5, 8)$ respectively.

   The straight lines l_1 and l_2 are the perpendicular bisectors of AB and BC respectively.

   (i) Find equations of l_1 and l_2. [6]

   (ii) Find the coordinates of the point P where l_1 and l_2 intersect. [3]

   (iii) Hence find an equation of the circle which passes through A, B and C. [3]

CORE MATHS 1 (C) TEST PAPER 9 : ANSWERS AND MARK SCHEME

1.	$\frac{3\sqrt{2}-3(2+2\sqrt{2})}{\sqrt{2}(2+2\sqrt{2})} = \frac{-3\sqrt{2}-6}{2\sqrt{2}+4} = \frac{-3(\sqrt{2}+2)}{2(\sqrt{2}+2)} = -\frac{3}{2}$	M1 A1 M1 A1	4
2.	(i) d/dx $(6-5x-6x^2) = -5-12x$	B1 M1 A1	
	(ii) d/dx $(x^{3/2}) = \frac{3}{2}x^{1/2} = \frac{3}{2}\sqrt{x}$	M1 A1	5
3.	(i) $4/2 \times (t-6)/8 = -1$ $t-6=-4$ $t=2$	M1 A1 A1	
	(ii) $PR = 10$, height = 4 so area = 20	B1 M1 A1	6
4.	(i) $18/y^4 = 18x^2$	B1	
	(ii) $18x^2 + x - 4 = 0$ $(2x+1)(9x-4) = 0$	M1 A1 A1	
	$x = -1/2$ or $4/9$ Must have $x > 0$, so $y = 3/2$	M1 A1	6
5.	Line meets curve where $9 - x^2 = 1 - 2x$ $x^2 - 2x - 8 = 0$	M1 A1	
	$(x+2)(x-4) = 0$ $x = -2, x = 4$	M1 A1	
	Gradient of curve $= -2x = 4, -8$ at these points, $\neq$ ½ so not a normal	M1 A1 A1	7
6.	(i) Minimum at (–3, 0), cuts axes at (–3, 0) and (0, 9)	B2	
	(ii) Minimum at (–3, 0), cuts axes at (–3, 0) and (0, 18)	B2	
	(iii) Minimum at (0, 0), cuts axes only at (0, 0)	B2	
	(iv) Minimum at (–3, 3), cuts axes at (0, 12)	B2	8
7.	(i) $f(x) = 0$ when $\frac{5}{x}+\frac{3}{x^2} = \pm 2$ $2x^2-5x-3=0$ or $2x^2+5x+3=0$	M1 A1 A1	
	$(2x+1)(x-3) = 0$ or $(x+1)(2x+3) = 0$ $x > 0$, so $x = 3$	M1 A1 A1	
	(ii) $f(x) = 25x^{-2} + 30x^{-3} + 9x^{-4} - 4$ so $f'(x) = -50x^{-3} - 90x^{-4} - 36x^{-5}$	B1 M1 A1	
	At $x = 1$, $f(x) = 60$ and $f'(x) = -176$ $y - 60 = -176(x-1)$	B1 M1 A1	12
8.	(i) Height of triangle $= 5x$ (5, 12, 13), so $2h + 30x = 200$ $h = 100 - 15x$	B1 M1 A1	
	(ii) Area $= 30x^2 + 12x(100-15x) = 1200x - 150x^2 = 150(8x - x^2)$	M1 A1 A1	
	$= 150(16 - (x-4)^2)$ $a = 16, b = 4$	M1 A1 A1	
	(iii) $(x-4)^2 = 0$ when $x = 4$ and > 0 otherwise, so area is max. at 2400 m^2	B1 M1 A1	12
9.	(i) l_1 is $x = 2$ Mid-point of BC is (6, 6)	B2 B1	
	Gradient of $BC = -2$ so l_2 is $y - 6 = ½(x-6)$	B1 M1 A1	
	(ii) $y = ½(-4) + 6 = 4$ $P = (2, 4)$	M1 A1 A1	
	(iii) Radius = 5 $(x-2)^2 + (y-4)^2 = 25$	B1 M1 A1	12