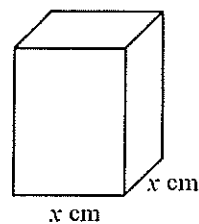


CORE MATHEMATICS (C) UNIT 1 TEST PAPER 7

1. If $x^p = (\sqrt{x})^3$, $x^q = \frac{1}{x^2}$ and $x^r = \frac{x^p}{x^q}$,
- (i) find the value of r , [3]
- (ii) evaluate x^r when $x = 4$. [2]
2. Given that $7 - 4x - x^2 \equiv m - (x + n)^2$,
- (i) find the values of the constants m and n . [4]
- (ii) Deduce the largest value of k for which the equation $7 - 4x - x^2 = k$ has at least one real root. [2]
3. A container, initially empty, is filled with liquid. At time t seconds after filling starts, the volume of liquid is V cm³ where $V = \frac{\pi t}{2}(t + 3)$.
- (i) Find, as a multiple of π , the rate at which V is increasing at the instant when $t = 4$. [4]
- (ii) Find the second derivative of V with respect to t . [2]
4. The line l has equation $y = x + 2$ and the circle C has equation $x^2 + y^2 + 3x - 4y = 1$.
- (i) Find the coordinates of the points where l and C intersect. [6]
- (ii) Determine, by calculation, whether or not l passes through the centre of C . [2]
5. The points P and Q have coordinates $(4, 5)$ and $(-2, -7)$ respectively.
- (i) Find the length of the line segment PQ , giving your answer in the form $k\sqrt{p}$ where p is prime. [3]
- (ii) Find equations for the following straight lines:
- (a) the line through the origin which is parallel to PQ , [2]
- (b) the line through the mid-point of PQ which is perpendicular to PQ . [4]
6. A piece of thin wood, of length 240 cm, is used to make the twelve edges of a rectangular box with a square base of side x cm.
- (i) Show that the volume of the box is $(60x^2 - 2x^3)$ cm³. [4]
- (ii) Find the value of x for which the volume is maximum. [5]
- (iii) Describe the type of box for which the volume is maximum. [1]



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7. (i) Find the three values of x for which $x^3 - x = 0$. [4]
- (ii) Find the coordinates of the stationary points of the function $x^3 - x$. [4]
- (iii) Sketch the graph of $y = f(x)$, where $f(x) = x^3 - x$. [2]
- (iv) On separate diagrams, sketch the curves with equations
(a) $y = f(x - 1)$, (b) $y = -f(x)$,
showing in each case the stationary points and the coordinates of any points of intersection
with the axes. [4]
8. The line l has equation $y = 6 - x$. The curve C has equation $y = x^2 - 4x + 2$.
- (i) Find the coordinates of the point on C at which the tangent is parallel to l . [4]
- (ii) Find the coordinates of the points where l and C intersect. [4]
- (iii) Write down the solution set of the inequality $x^2 - 4x + 2 < 6 - x$. [3]
- (iv) Find the x -coordinates of the points where C crosses the x -axis, giving your answers in
surd form simplified as far as possible. [4]

CORE MATHS 1 (C) TEST PAPER 7 : ANSWERS AND MARK SCHEME

1. (i) $p = 3/2, q = -2$ $r = p - q = 7/2$ B1 B1 B1
(ii) $4^{7/2} = 2^7 = 128$ M1 A1 5
2. (i) $-(x^2 + 4x - 7) = -((x + 2)^2 - 11) = 11 - (x + 2)^2$ $m = 11, n = 2$ M1 M1 A1 A1
(ii) Equation is $(x + 2)^2 = 11 - k$, so largest k is 11 M1 A1 6
3. (i) $V = \frac{\pi^2}{2} + \frac{3\pi}{2}$ $\frac{dV}{dt} = \pi + \frac{3\pi}{2} = \pi\left(4 + \frac{3}{2}\right) = \frac{11\pi}{2}$ when $t = 4$ B1 M1 A1 A1
(ii) Second derivative of $V = \pi$ M1 A1 6
4. (i) $x^2 + (x + 2)^2 + 3x - 4(x + 2) - 1 = 0$ $2x^2 + 3x - 5 = 0$ M1 A1 A1
 $(2x + 5)(x - 1) = 0$ $(1, 3), (-5/2, -1/2)$ M1 A1 A1
(ii) Centre is $(-3/2, 2)$, which does not lie on l M1 A1 8
5. (i) $PQ^2 = 36 + 144 = 180$ $PQ = 6\sqrt{5}$ M1 A1 A1
(ii) (a) Gradient $PQ = 2$, so line is $y = 2x$ M1 A1
(b) Mid-point is $(1, -1)$ Grad = $-1/2$ $y + 1 = -1/2(x - 1)$ B1 B1 M1 A1 9
6. (i) $8x + 4h = 240$ so $h = 60 - 2x$ $V = x^2(60 - 2x) = 60x^2 - 2x^3$ M1 A1 M1 A1
(ii) $dV/dx = 120x - 6x^2 = 0$ when $x = 20$ M1 A1 M1 A1
(iii) Thus the volume is greatest when the box is a cube A1 9
7. (i) $x(x - 1)(x + 1) = 0$ $x = -1, x = 0, x = 1$ M1 A1 A1 A1
(ii) $3x^2 - 1 = 0$ $x = \pm 1/\sqrt{3}$ $y = \mp 2/(3\sqrt{3})$ M1 A1 M1 A1
(iii) Cubic curve through $(-1, 0), (0, 0), (1, 0)$ B2
(iv) (a) Cubic curve through $(0, 0), (1, 0), (2, 0)$ (b) Reflected in x -axis B2 B2 14
8. (i) $2x - 4 = -1$ so $x = 3/2$ Point is $(3/2, -7/4)$ M1 A1 M1 A1
(ii) $x^2 - 3x - 4 = 0$ $(x + 1)(x - 4) = 0$ $(-1, 7), (4, 2)$ B1 M1 A1 A1
(iii) Curve lies below line when $-1 < x < 4$ M1 A1 A1
(iv) $(x - 2)^2 - 2 = 0$ $x = 2 - \sqrt{2}, x = 2 + \sqrt{2}$ M1 A1 A1 A1 15