CORE MATHEMATICS (C) UNIT 1 TEST PAPER 5

1. Find the complete set of values of x for which $x^2 \geq 5x + 84$. [4]

2. (i) Given that $4^{3x+1} = 8^{y+1}$, express y in terms of x. [3]

   (ii) Find the value of x for which $4^{3x+1} = 64$. [2]

3. The diagram shows the graph of $y = f(x)$,

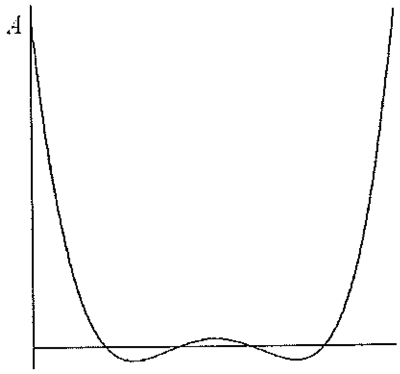
   where $f(x) \equiv (x-1)(x-2)(x-3)(x-4)$.

   The graph crosses the y-axis at A.

   (i) Write down the coordinates of A. [1]

   (ii) Sketch the following graphs, clearly showing the coordinates of the points where they cross the axes:

   (a) $y = -f(x)$, (b) $y = f(x+2)$. [4]



4. (i) Find the integers a and b such that $\dfrac{\sqrt{3}-2}{\sqrt{3}+2} = a\sqrt{3} + b$. [5]

   (ii) Hence or otherwise, solve for x the equation

$$(1-x)\sqrt{3} = 2(x+1).$$ [2]

5. Find $\dfrac{dy}{dx}$ for each of the following:

   (i) $y = (x-4)^2$, (ii) $y = \dfrac{1}{\sqrt[3]{x}}$, (iii) $y = \dfrac{x^4 - 2x}{x^3}$. [9]

6. A is the point $(-3, 4)$ and B is the point $(k, -10)$. M is the mid-point of AB.

   The straight line through A and B has equation $2x + y + 2 = 0$. Find

   (i) the value of k, [2]

   (ii) the length of AB, in its simplest surd form. [2]

   (iii) the coordinates of M, [2]

   (iv) the equation of the line through M perpendicular to AB, in the form $ax + by + c = 0$. [4]

CORE MATHEMATICS 1 (C) TEST PAPER 5 Page 2

7. The circle C_1 has equation $x^2 + y^2 - 10x - 12y - 20 = 0$.
   (i) Find the centre and the radius of C_1. [4]

   The circle C_2 has radius 5 cm and centre at (5, 10).
   (ii) Verify that the two circles touch, and find the coordinates of the point which lies on both circles. [4]
   (iii) State the equation of the common tangent to the two circles. [2]

8. In this question, $f(x) \equiv 4x^3 + 10x^2 + 5x$.
   (i) Factorise f(x). [1]
   (ii) Solve the equation f(x) = 0, giving any irrational roots as surds in their simplest form. [4]

   A and B are the points on the graph of $y = f(x)$ at which the gradient is 5.
   (iii) Find the x-coordinates of A and B. [5]

9. Given that $x^2 + 18x + 100 \equiv (x + p)^2 + q$,
   (i) find the values of the constants p and q. [3]
   (ii) Deduce that the equation $x^2 + 18x + 100 = 0$ has no real roots. [2]
   (iii) Sketch the graph of $y = x^2 + 18x + 100$. [2]
   (iv) If the equation $x^2 + 18x + 100 = t$ has at least one real root, find the set of possible values of t. [2]
   (v) State the value of t for which $x^2 + 18x + 100 = t$ has a repeated root, and find this root. [3]

CORE MATHS 1 (C) TEST PAPER 5 : ANSWERS AND MARK SCHEME

1.	$(x + 7)(x - 12) \geq 0$	$x \leq -7, x \geq 12$	M1 A1 A1 A1	4
2.	(i) $2^{6x+2} = 2^{3y+3}$ $3y = 6x - 1$	$y = 2x - 1/3$	M1 A1 A1	
	(ii) Here $y = 1$, so $x = 2/3$		M1 A1	5
3.	(i) A is (0, 24) (ii) Graph (a) reflected in x-axis,		B1 B2	
	(b) translated –2 units in x direction: through (–1, 0), (0, 0), (1, 0), (2, 0)		B2	5
4.	(i) $\sqrt{3} - 2 = (a\sqrt{3} + b)(\sqrt{3} + 2) = 3a + 2b + (2a + b)\sqrt{3}$		M1 A1	
	$2a + b = 1, 3a + 2b = -2$	$a = 4, b = -7$	M1 A1 A1	
	(ii) $(\sqrt{3} + 2)x = \sqrt{3} - 2$, so $x = 4\sqrt{3} - 7$		M1 A1	7
5.	(i) $y = x^2 - 8x + 16$	$dy/dx = 2x - 8$	B1 M1 A1	
	(ii) $y = x^{-1/3}$	$dy/dx = -{}^1/_3\, x^{-4/3}$	B1 M1 A1	
	(iii) $y = x - 2x^{-2}$	$dy/dx = 1 + 4/x^3$	B1 M1 A1	9
6.	(i) $2k - 10 + 2 = 0$	$k = 4$	M1 A1	
	(ii) $AB^2 = 7^2 + 14^2$	$AB = 7\sqrt{5}$	M1 A1	
	(iii) $M = (½, -3)$		B1 B1	
	(iv) Gradient of $AB = -2$	Gradient of perp. = ½	B1 B1	
	$y + 3 = ½(x - ½)$	$2x - 4y - 13 = 0$	M1 A1	10
7.	(i) $(x - 5)^2 + (y - 6)^2 = 81$	Centre (5, 6), radius 9	M1 A1 A1 A1	
	(ii) $x = 5$ is common diameter and (5, 15) is on both circles, so they touch there		B1 M2 A1	
	(iii) Common tangent is $y = 15$		B2	10
8.	(i) $f(x) = x(4x^2 + 10x + 5)$		B1	
	(ii) $x = 0$ or $x = \dfrac{-10 \pm \sqrt{20}}{8} = \dfrac{-5 \pm \sqrt{5}}{4}$		B1 M1 A1 A1	
	(iii) $12x^2 + 20x + 5 = 5$ $4x(3x + 5) = 0$	$x = 0, x = -5/3$	M1 A1 M1 A1 A1	10
9.	(i) $p = 9, q = 19$		M1 A1 A1	
	(ii) $(x + 9)^2 + 19 > 0$ for all real x		B2	
	(iii) Quadratic graph with minimum at (–9, 19)		B2	
	(iv) $t \geq 19$		M1 A1	
	(v) $t = 19$; then $x = -9$		M1 A1 A1	12