CORE MATHEMATICS (C) UNIT 1 TEST PAPER 2

1. Write down the exact values of

   (i) $16^{\frac{3}{4}}$, (ii) $\left(\frac{2}{3}\right)^{-2}$, (iii) $4^{-\frac{1}{2}}$. [3]

2. (i) Find the set of real values of k for which the equation $x^2 - kx + k = 0$ has no real roots. [4]

   (ii) State the number of real roots of the equation $x^2 - 4x + 4 = 0$. [2]

3. Solve the simultaneous equations $xy = 1, \quad 3x + 2y = 5$. [6]

4. The line l_1 passes through $(-2, 5)$ and is parallel to the line $4x + 3y = 0$.
   The line l_2 also passes through $(-2, 5)$ and is perpendicular to the line $4x + 3y = 0$.
   Find equations of l_1 and l_2, expressing each answer in the form $ax + by + c = 0$ where a, b, c are integers. [7]

5. Two quantities P and t are related by the equation $P = \frac{k}{\sqrt{t}}$, where k is a positive constant.

   (i) Find, in terms of k, the rate of change of P with t when $t = k^2$. [4]

   Q also varies with t, such that $Q = \frac{\sqrt{t} + \sqrt{k}}{6}$.

   (ii) Show that when $P = Q$, $t + \sqrt{k}\sqrt{t} - 6k = 0$. [3]

   (iii) By substituting $t = x^2$, or otherwise, find, in terms of k, the value of t for which $P = Q$. [4]

6. In this question, $f(x) \equiv x^2 - 6x + 11$.

   (i) Express $f(x)$ in the form $(x - p)^2 + q$ and hence find the minimum value of $f(x)$. [3]

   (ii) If the line of symmetry of the graph of $y = f(x)$ has equation $x = a$, state the value of a. [2]

   (iii) With this value of a, the graph of $y = f(x)$ is translated a units in the negative x-direction.
   Find the equation of the resulting graph, giving your answer in a form without brackets. [3]

   (iv) Sketch these two graphs on the same diagram and find the coordinates of the point where they intersect. [5]

7. The equation of a curve is $x^2y = x - 6$.

   The normal to the curve at the point where $x = -2$ meets the x-axis at P.

   (i) Find the coordinates of P. [9]

   (ii) Find the coordinates of the point on the curve at which the gradient is 0. [4]

8. The points A and B have coordinates $(-2, 4)$ and $(6, -2)$ respectively.

   (i) Find the coordinates of the mid-point of AB. [2]

   (ii) Find the length of AB. [2]

   (iii) Find the equation of the circle which has AB as a diameter, giving your answer in the form $x^2 + y^2 + ax + by + c = 0$ where a, b and c are integers. [3]

   (iv) Find an equation of the tangent at A to the circle. [4]

   The point P lies on the circle, and the straight line AP has gradient 2.

   (v) State the gradient of BP. [2]

CORE MATHS 1 (C) TEST PAPER 2 : ANSWERS AND MARK SCHEME

1.	(i) $2^3 = 8$ (ii) $(3/2)^2 = 9/4$ (iii) $1/\sqrt{4} = 1/2$	B1 B1 B1	3
2.	(i) No real roots if $k^2 - 4k < 0$ $k(k-4) < 0$ $0 < k < 4$	B1 M1 A1 A1	
	(ii) Here $k = 4$, so there is one (repeated) real root	M1 A1	6
3.	$y = (5 - 3x)/2$ $x(5 - 3x) = 2$ $3x^2 - 5x + 2 = 0$	B1 M1 A1	
	$(3x - 2)(x - 1) = 0$ $x = 2/3, y = 3/2$ or $x = 1, y = 1$	M1 A1 A1	6
4.	Gradient of $l_1 = -4/3$, so l_1 is $y - 5 = -4/3\ (x + 2)$ $4x + 3y - 7 = 0$	M1 A1 A1	
	Gradient of $l_2 = 3/4$, so l_2 is $y - 5 = 3/4\ (x + 2)$ $3x - 4y + 26 = 0$	B1 M1 A1 A1	7
5.	(i) $P = kt^{-1/2}$ $dP/dt = -\frac{1}{2} kt^{-3/2}$ When $t = k^2$, $dP/dt = -1/(2k^2)$	B1 M1 A1 A1	
	(ii) When $P = Q$, $6k = t + \sqrt{kt}$ $t + \sqrt{k}\sqrt{t} - 6k = 0$	M1 A1 A1	
	(iii) $(\sqrt{t} + 3\sqrt{k})(\sqrt{t} - 2\sqrt{k}) = 0$ $\sqrt{t} > 0$ so $\sqrt{t} = 2\sqrt{k}$ $t = 4k$	M1 A1 M1 A1	11
6.	(i) $f(x) = (x - 3)^2 + 2$ Minimum is $f(x) = 2$	M1 A1 A1	
	(ii) Line of symmetry is $x = 3$, so $a = 3$	M1 A1	
	(iii) New graph is $f(x + 3)$, so it is $y = x^2 + 2$	M1 A1 A1	
	(iv) Sketches	B1 B1	
	Intersect where $x^2 - 6x + 11 = x^2 + 2$ $x = 3/2$ $(3/2, 17/4)$	M1 A1 A1	13
7.	(i) $y = \frac{1}{x} - \frac{6}{x^2}$ $\frac{dy}{dx} = -\frac{1}{x^2} + \frac{12}{x^3} = -\frac{7}{4}$ when $x = -2$	B1 M1 A1 A1	
	$y = -2$, so normal is $y + 2 = 4/7\ (x + 2)$ When $y = 0$, $x = 3/2$	B1 M1 A1 A1 A1	
	(ii) When $-\frac{1}{x^2} + \frac{12}{x^3} = 0$, $x = 12$ Point is $(12, 1/24)$	M1 A1 M1 A1	13
8.	(i) Mid-point is $(2, 1)$	B1 B1	
	(ii) $AB^2 = 64 + 36$ $AB = 10$	M1 A1	
	(iii) $(x - 2)^2 + (y - 1)^2 = 25$ $x^2 + y^2 - 4x - 2y - 20 = 0$	M1 A1 A1	
	(iv) Gradient of radius to $A = -3/4$, so gradient of tangent $= 4/3$	B1 B1	
	Tangent is $y - 4 = 4/3\ (x + 2)$	M1 A1	
	(v) Angle $ABP = 90°$ (angle in semicircle), so gradient of $BP = -1/2$	M1 A1	13