## CORE MATHEMATICS (C) UNIT 1 TEST PAPER 1

1. (i) Given that $2^a = 8^b$, express $a$ in terms of $b$. [2]

   (ii) State the value of $x$ for which $9^x = 3$. [2]

2. Find, in the form $y = mx + c$, the equation of the straight line which passes through the points $(-1, 5)$ and $(2, -7)$. [4]

3. (i) Find the discriminant of the quadratic function $f(x) \equiv x^2 - 3kx + (k^2 + 5)$. [2]

   (ii) Hence find the set of values of $k$ for which the equation $x^2 - 3kx + (k^2 + 5) = 0$ has no real roots for $x$. [3]

4. (i) Find the prime numbers $p$ and $q$ such that $\sqrt{56} = 2\sqrt{p}\sqrt{q}$ where $p < q$. [3]

   (ii) Express $\dfrac{2}{3-\sqrt{3}}$ in the form $a + b\sqrt{3}$, where $a$ and $b$ are rational numbers to be found. [4]

5. Find the values of $x$ and $y$ for which

$$2x + y = 7 \quad \text{and} \quad x^2 + 2y = 19.$$ [7]

6. The circle $C$ has equation $x^2 + y^2 - 4x + 6y - 3 = 0$.

   (i) Find the coordinates of the centre of $C$. [3]

   (ii) Find the radius of $C$. [2]

   (iii) Given that that the point $(p, 1)$ lies on $C$, find the value of $p$. [3]

7. (i) Sketch the graph of $y = -\dfrac{k}{x}$, where $k$ is a positive constant. [2]

   (ii) On the same diagram, sketch the graph of $y = a - \dfrac{k}{x}$, where $a$ is also a positive constant. [2]

   (iii) Find, in terms of $a$ and $k$, the gradient of the graph of $y = a - \dfrac{k}{x}$ at the point where it crosses the $x$-axis. [5]

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8. A function f is defined, for $x > 0$, by

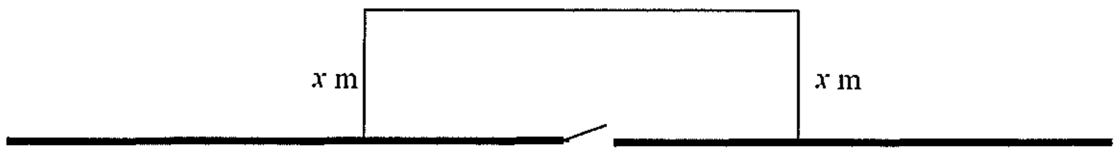
$$f(x) \equiv \frac{x}{2} - \frac{4}{x^2} + 1.$$

(i) Find $f'(x)$ and hence show that $f(x)$ is an increasing function. [4]

(ii) Find $f''(x)$. [2]

(iii) The normal at the point (2, 1) to the curve $y = f(x)$ cuts the $x$ and $y$ axes at $A$ and $B$ respectively. Calculate the length of $AB$, giving your answer in surd form simplified as far as possible. [7]

9. A rectangular playground is bordered on one side by a straight wall with a gate in it. The other three sides are to be formed by a fence of total length $4k$ metres.

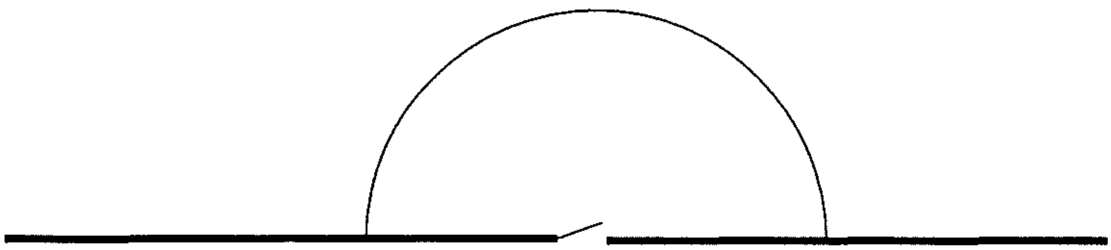



(i) If the two sides perpendicular to the wall are each of length $x$ metres and the area of the playground is $A$ m$^2$, show that

$$A = 2k^2 - 2(x - k)^2.$$ [6]

(ii) Deduce that the playground has its largest area when $x = k$. State its area in this case. [3]

(iii) Show that a larger area is obtained if the playground is bounded by a semicircular fence, also of length $4k$ metres. [6]



# CORE MATHS 1 (C) TEST PAPER 1 : ANSWERS AND MARK SCHEME

| | | | |
|---|---|---|---|
| 1. | (i) $8^b = (2^3)^b = 2^{3b}$, so $a = 3b$ | M1 A1 | |
| | (ii) $9^{1/2} = 3$, so $x = ½$ | M1 A1 | 4 |
| 2. | Gradient = $-4$    $y - 5 = -4(x + 1)$    $y = -4x + 1$ | B1 M1 A1 A1 | 4 |
| 3. | (i) Discriminant = $9k^2 - 4(k^2 + 5) = 5k^2 - 20$ | M1 A1 | |
| | (ii) $5k^2 - 20 < 0$    $k^2 < 4$    $-2 < k < 2$ | M1 A1 A1 | 5 |
| 4. | (i) $\sqrt{56} = \sqrt{4 \times 2 \times 7} = 2\sqrt{2}\sqrt{7}$    $p = 2, q = 7$ | M1 A1 A1 | |
| | (ii) $\dfrac{2}{3-\sqrt{3}} = \dfrac{2(3+\sqrt{3})}{(3-\sqrt{3})(3+\sqrt{3})} = \dfrac{6+2\sqrt{3}}{6} = 1 + \dfrac{1}{3}\sqrt{3}$    $a = 1, b = \dfrac{1}{3}$ | M1 M1 A1 A1 | 7 |
| 5. | $y = 7 - 2x$    $x^2 + 2(7 - 2x) = 19$    $x^2 - 4x - 5 = 0$ | B1 M1 A1 | |
| | $(x + 1)(x - 5) = 0$    $x = -1, y = 9$ or $x = 5, y = -3$ | M1 M1 A1 A1 | 7 |
| 6. | (i) $(x - 2)^2 + (y + 3)^2 = 16$    Centre $(2, -3)$ | M1 A1 A1 | |
| | (ii) Radius = $\sqrt{16} = 4$ | M1 A1 | |
| | (iii) $(p - 2)^2 = 0$    $p = 2$ | M1 A1 A1 | 8 |
| 7. | (i) Graph in second and fourth quadrants with axes as asymptotes | B2 | |
| | (ii) Graph in (i) translated $a$ units upwards | B2 | |
| | (iii) $dy/dx = k/x^2$    When $x = k/a$, gradient = $a^2/k$ | B2 M1 A1 A1 | 9 |
| 8. | (i) $f'(x) = \dfrac{1}{2} + \dfrac{8}{x^3} > 0$ for $x > 0$, so f($x$) is increasing | M1 A1 M1 A1 | |
| | (ii) $f''(x) = -\dfrac{24}{x^4}$ | M1 A1 | |
| | (iii) At (2, 1), gradient = 3/2 so normal has gradient $-2/3$ | M1 A1 | |
| | Normal is $y - 1 = -2/3(x - 2)$, cutting axes at (0, 7/3), (7/2, 0) | M1 A1 A1 | |
| | $AB^2 = 49/9 + 49/4 = 49(13/36)$ so $AB = 7\sqrt{13}/6$ | M1 A1 | 13 |
| 9. | (i) Length = $4k - 2x$, so area = $x(4k - 2x) = -2(x^2 - 2kx) = -2((x - k)^2 - k^2)$ | B1 M1 A1 M1 | |
| | $= 2k^2 - 2(x - k)^2$ | A1 A1 | |
| | (ii) This is largest when $2(x - k)^2 = 0$, i.e. when $x = k$    Then area = $2k^2$ | M1 A1 A1 | |
| | (iii) Semicircular arc of length $4k$ has radius $4k/\pi$, so area = $½\,\pi\,(4k/\pi)^2$ | B1 M1 A1 | |
| | $= (8/\pi)k^2$ . Now $2\pi \approx 6.2 < 8$, so $8/\pi > 2$, hence area is larger | M1 A1 A1 | 15 |