Pure Core 1 Past Paper Questions Pack A

Taken from MAME

January 2001

1 Given that $f(x) = x^3 - 4x^2 - x + 4$,

(a) find f(1) and f(2), *(2 marks)*

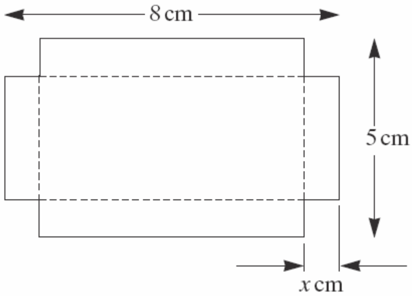
(b) factorise f(x) into the product of three linear factors. *(3 marks)*

2 (a) Express $x^2 - 6x + 7$ in the form $(x + a)^2 + b$, finding the values of a and b. *(2 marks)*

(b) Hence, or otherwise, find the range of values of x for which

$$x^2 - 6x + 7 < 0.$$ *(3 marks)*

5 Small trays are to be made from rectangular pieces of card. Each piece of card is 8 cm by 5 cm and the tray is formed by removing squares of side x cm from each corner and folding the remaining card along the dashed lines, as shown in the diagram, to form an open-topped box.



(a) Explain why $0 < x < 2.5$. *(1 mark)*

(b) Show that the volume, V cm^3, of a tray is given by

$$V = 4x^3 - 26x^2 + 40x.$$ *(3 marks)*

(c) Find the value of x for which $\frac{dV}{dx} = 0$. *(5 marks)*

(d) Calculate the greatest possible volume of a tray. *(1 mark)*

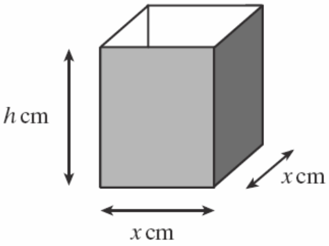
June 2001

3 (a) Express $x^2 + 4x - 5$ in the form $(x + a)^2 + b$, finding the values of the constants a and b. *(2 marks)*

(b) Find the values of x for which $x^2 + 4x - 5 > 0$. *(3 marks)*

4 The cubic polynomial $x^3 + ax^2 + bx + 4$, where a and b are constants, has factors $x - 2$ and $x + 1$. Use the factor theorem to find the values of a and b. *(6 marks)*

6 An open-topped box has height h cm and a square base of side x cm.



The box has capacity V cm^3. The area of its **external** surface, consisting of its horizontal base and four vertical faces, is A cm^2.

(a) Find expressions for V and A in terms of x and h. *(3 marks)*

(b) It is given that $A = 3000$.

(i) Show that $V = 750x - \frac{1}{4}x^3$. *(2 marks)*

(ii) Find the positive value of x for which $\frac{dV}{dx} = 0$, giving your answer in surd form. *(3 marks)*

(iii) Hence find the maximum possible value of V, giving your answer in the form $p\sqrt{10}$, where p is an integer.
[You do not need to show that your answer is a maximum.] *(2 marks)*

January 2002

3 Solve the simultaneous equations

$$y = 2 - x$$
$$x^2 + 2xy = 3.$$ *(5 marks)*

4 The size of a population, P, of birds on an island is modelled by

$$P = 59 + 117t + 57t^2 - t^3,$$

where t is the time in years after 1970.

(a) Find $\frac{dP}{dt}$. *(2 marks)*

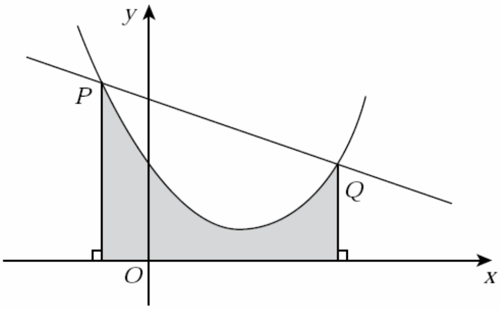
(b) (i) Find the positive value of t for which P has a stationary value. *(3 marks)*

(ii) Determine whether this stationary value is a maximum or a minimum. *(2 marks)*

(c) (i) State the year when the model predicts that the population will reach its maximum value. *(1 mark)*

(ii) Determine what the model predicts will happen in the year 2029. *(1 mark)*

8 The diagram shows the curve $y = x^2 - 4x + 6$, the points $P(-1, 11)$ and $Q(4, 6)$ and the line PQ.



(a) Show that the length of PQ is $5\sqrt{2}$. *(3 marks)*

(b) Find the equation of the tangent to the curve at Q in the form $y = mx + c$. *(6 marks)*

(c) Find the area of the shaded region in the diagram. *(5 marks)*

June 2002

1 Given that $f(x) = x^3 + 4x^2 + x - 6$:

(a) find $f(1)$ and $f(-1)$; *(2 marks)*

(b) factorise $f(x)$ into the product of three linear factors. *(4 marks)*

3 Find the values of x and y that satisfy the simultaneous equations

$$y = 2 - x^2$$
$$x + 2y = 1$$

(5 marks)

5 (a) (i) Solve $2x^2 + 8x + 7 = 0$, giving your answers in surd form. *(2 marks)*

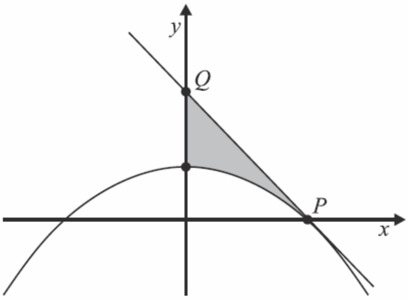
(ii) Hence solve $2x^2 + 8x + 7 > 0$. *(2 marks)*

(b) Express $2x^2 + 8x + 7$ in the form $A(x + B)^2 + C$, where A, B and C are constants. *(3 marks)*

(c) (i) State the minimum value of $2x^2 + 8x + 7$. *(1 mark)*

(ii) State the value of x which gives this minimum value. *(1 mark)*

7 The diagram shows the graph of $y = 12 - 3x^2$ and the tangent to the curve at the point $P(2, 0)$. The region enclosed by the tangent, the curve and the y-axis is shaded.



(a) Find $\int_0^2 (12 - 3x^2)\mathrm{d}x$. *(3 marks)*

(b) (i) Find the gradient of the curve $y = 12 - 3x^2$ at the point P. *(2 marks)*

(ii) Find the coordinates of the point Q where the tangent at P crosses the y-axis. *(2 marks)*

(c) Find the area of the shaded region. *(2 marks)*

November 2002

3 It is given that

$$f(x) = x^3 + 3x^2 - 6x - 8.$$

(a) Find the value of $f(2)$. *(1 mark)*

(b) Use the Factor Theorem to write down a factor of $f(x)$. *(1 mark)*

(c) Hence express $f(x)$ as a product of three linear factors. *(4 marks)*

5 (a) Solve the equation

$$2x^2 + 32x + 119 = 0.$$

Write your answers in the form $p + q\sqrt{2}$, where p and q are rational numbers. *(3 marks)*

(b) (i) Express

$$2x^2 + 32x + 119$$

in the form

$$2(x + m)^2 + n,$$

where m and n are integers. *(2 marks)*

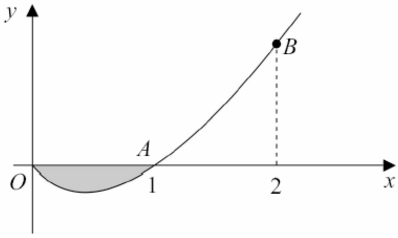
(ii) Hence write down the minimum value of

$$2x^2 + 32x + 119.$$ *(1 mark)*

7 The diagram shows the graph of

$$y = x^3 - x, \qquad x \geqslant 0.$$

The points on the graph for which $x = 1$ and $x = 2$ are labelled A and B, respectively.



(a) Find the y-coordinate of B and hence find the equation of the straight line AB, giving your answer in the form

$$ax + by + c = 0.$$ *(4 marks)*

(b) Find, by integration, the area of the shaded region. *(5 marks)*

8 An office worker can leave home at any time between 6.00 am and 10.00 am each morning. When he leaves home x **hours** after 6.00 am ($0 \leqslant x \leqslant 4$), his journey time to the office is y **minutes**, where

$$y = x^4 - 8x^3 + 16x^2 + 8.$$

(a) Find $\frac{dy}{dx}$. *(3 marks)*

(b) Find the **three** values of x for which $\frac{dy}{dx} = 0$. *(4 marks)*

(c) Show that y has a maximum value when $x = 2$. *(3 marks)*

(d) Find the time at which the office worker arrives at the office on a day when his journey time is a maximum. *(2 marks)*

January 2003

3 The numbers x and y satisfy the simultaneous equations

$$y = 2x + 1$$
$$xy = 3$$

(a) Show that

$$2x^2 + x - 3 = 0.$$ *(2 marks)*

(b) Hence solve the simultaneous equations. *(3 marks)*

5 The line

$$2x - y + 6 = 0$$

intersects the coordinate axes at two points, $A(a, 0)$ and $B(0, b)$.

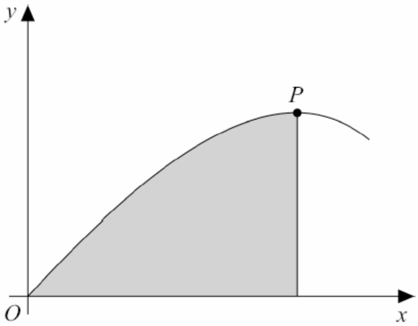
(a) Find the values of a and b. *(2 marks)*

(b) Find the coordinates of M, the midpoint of AB. *(2 marks)*

(c) Find the equation of the line through M perpendicular to AB, giving your answer in the form $y = mx + c$. *(4 marks)*

7 The diagram shows a part of the graph of

$$y = x - 2x^4.$$



(a) (i) Find $\dfrac{dy}{dx}$. *(2 marks)*

(ii) Show that the x-coordinate of the stationary point P is $\frac{1}{2}$. *(2 marks)*

(iii) Find the y-coordinate of P. *(1 mark)*

(b) (i) Find $\int (x - 2x^4)\,dx$. *(2 marks)*

(ii) Hence find the area of the shaded region. *(3 marks)*

8 (a) Express $\dfrac{\sqrt{2}+1}{\sqrt{2}-1}$ in the form $a\sqrt{2} + b$, where a and b are integers. *(4 marks)*

(b) Solve the inequality

$$\sqrt{2}(x - \sqrt{2}) < x + 2\sqrt{2}.$$ *(3 marks)*

2 The function f is defined for all x by

$$f(x) = x^2 + 6x + 7.$$

(a) Express $f(x)$ in the form

$$(x + A)^2 + B,$$

where A and B are constants. *(2 marks)*

(b) Hence, or otherwise, solve the equation

$$f(x) = 0,$$

giving your answers in surd form. *(3 marks)*

4 The point A has coordinates $(2, 3)$ and O is the origin.

(a) Write down the gradient of OA and hence find the equation of the line OA. *(2 marks)*

(b) Show that the line which has equation

$$4x + 6y = 13:$$

(i) is perpendicular to OA; *(2 marks)*

(ii) passes through the midpoint of OA. *(3 marks)*

6 (a) Express each of the following as a power of 3:

(i) $\sqrt{3}$; *(1 mark)*

(ii) $\dfrac{3^x}{\sqrt{3}}$. *(1 mark)*

(b) Hence, or otherwise, solve the equation

$$\frac{3^x}{\sqrt{3}} = \frac{1}{3}.$$ *(3 marks)*

8 The function f is defined for all values of x by

$$f(x) = x^3 - 7x^2 + 14x - 8.$$

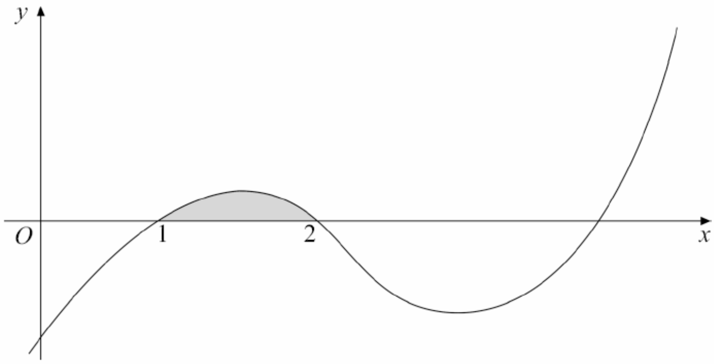
It is given that $f(1) = 0$ and $f(2) = 0$.

(a) Find the values of $f(3)$ and $f(4)$. *(2 marks)*

(b) Write $f(x)$ as a product of **three** linear factors. *(2 marks)*

(c) The diagram shows the graph of

$$y = x^3 - 7x^2 + 14x - 8.$$



(i) Find $\frac{dy}{dx}$. *(3 marks)*

(ii) State, giving a reason, whether the function f is increasing or decreasing at the point where $x = 3$. *(2 marks)*

(iii) Find $\int (x^3 - 7x^2 + 14x - 8)\,dx$. *(3 marks)*

(iv) Hence find the area of the shaded region enclosed by the graph of $y = f(x)$, for $1 \leqslant x \leqslant 2$, and the x-axis. *(3 marks)*

November 2003

2 (a) Solve the equation

$$2x^2 - 12x + 17 = 0,$$

giving your answers in surd form. *(3 marks)*

(b) Show that the equation

$$2x^2 - 12x + 21 = 0$$

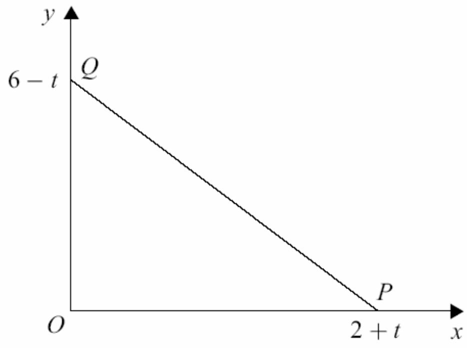
has no real roots. *(2 marks)*

(c) Find the value of p for which the equation

$$2x^2 - 12x + p = 0$$

has equal roots. *(2 marks)*

4 The diagram shows the points $O(0,0)$, $P(2+t,0)$ and $Q(0,6-t)$, where $0 \leqslant t \leqslant 6$.



(a) The area of the triangle OPQ is A. Show that

$$A = 6 + 2t - \tfrac{1}{2}t^2.$$
(2 marks)

(b) (i) Find $\dfrac{dA}{dt}$. *(2 marks)*

(ii) Show that A has a stationary value when $t = 2$. *(1 mark)*

(c) In the case when $t = 2$, find:

(i) the coordinates of P and Q; *(1 mark)*

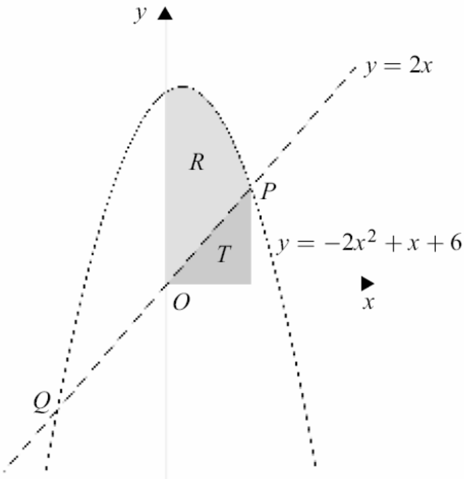
(ii) the gradient of the line PQ; *(1 mark)*

(iii) the equation of the line PQ. *(2 marks)*

7 The diagram shows the graphs of

$$y = 2x \quad \text{and} \quad y = -2x^2 + x + 6,$$

intersecting at two points P and Q.



(a) Show that P has x-coordinate $\frac{3}{2}$ and find the x-coordinate of Q. *(4 marks)*

(b) Calculate the area of the shaded triangle T. *(2 marks)*

(c) (i) Find $\int (-2x^2 + x + 6)\,dx$. *(3 marks)*

(ii) Hence find the area of the shaded region R. *(3 marks)*

January 2004

3 It is given that

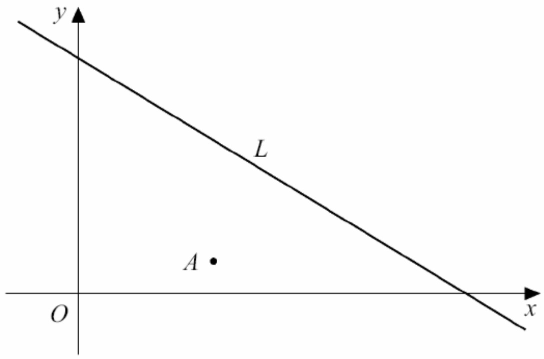
$$f(x) = x^3 + 4x^2 - 3x - 18\,.$$

(a) Find the value of $f(2)$. *(1 mark)*

(b) Use the Factor Theorem to write down a factor of $f(x)$. *(1 mark)*

(c) Hence express $f(x)$ as a product of three linear factors. *(4 marks)*

5 The diagram shows a line L which represents a pipeline, and a point A which is to be connected to the pipeline by the shortest possible connection.



The equation of the line L is

$$2x + 3y = 24,$$

and A is the point $(4, 1)$.

(a) Find the gradient of the line L. *(2 marks)*

(b) Hence write down the gradient of a line perpendicular to L. *(1 mark)*

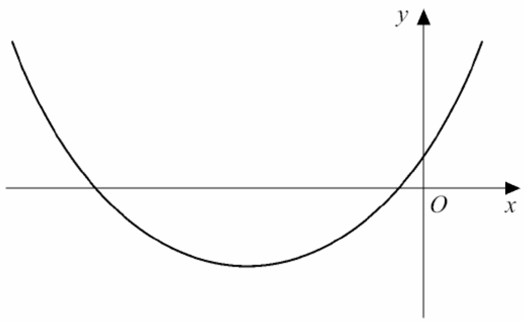
(c) Show that the line through A perpendicular to L has equation

$$3x - 2y = 10.$$ *(2 marks)*

(d) Hence calculate the coordinates of the point of intersection of the two lines. *(3 marks)*

(e) Find the length of the shortest possible connection from A to the pipeline. *(2 marks)*

7 The diagram shows the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = x^2 + 6x + 1$.



(a) Express $\mathrm{f}(x)$ in the form $(x + m)^2 + n$, where m and n are integers. *(2 marks)*

(b) Solve the equation $\mathrm{f}(x) = 0$, giving your answers in the form $p + q\sqrt{2}$, where p and q are integers. *(3 marks)*

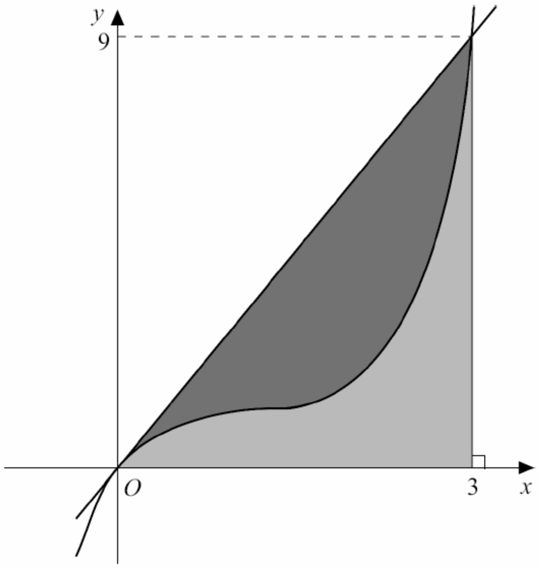
(c) Solve the inequality $\mathrm{f}(x) < 0$. *(1 mark)*

8 The diagram shows the straight line

$$y = 3x$$

and the curve

$$y = x^3 - 3x^2 + 3x\,.$$



(a) (i) Differentiate $x^3 - 3x^2 + 3x$. *(2 marks)*

(ii) Find the coordinates of the stationary point on the curve

$$y = x^3 - 3x^2 + 3x\,.$$ *(3 marks)*

(b) (i) Find $\int (x^3 - 3x^2 + 3x)\mathrm{d}x$. *(3 marks)*

(ii) Show that the areas of the two shaded regions are equal. *(3 marks)*

June 2004

1 The numbers x and y satisfy the simultaneous equations

$$2x - y = 1$$

$$x^2 + y = 2\,.$$

(a) Show that

$$x^2 + 2x - 3 = 0\,.$$ *(1 mark)*

(b) Hence solve the simultaneous equations. *(3 marks)*

5 (a) Simplify the expression $(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})$. *(1 mark)*

(b) It is given that

$$k = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}.$$

(i) Express k in the form $p + q\sqrt{6}$, where p and q are integers. *(3 marks)*

(ii) Express $\frac{1}{k}$ in the form $r + s\sqrt{6}$, where r and s are integers. *(2 marks)*

6 A curve is defined by

$$y = x^3 - 3x^2 + 6x.$$

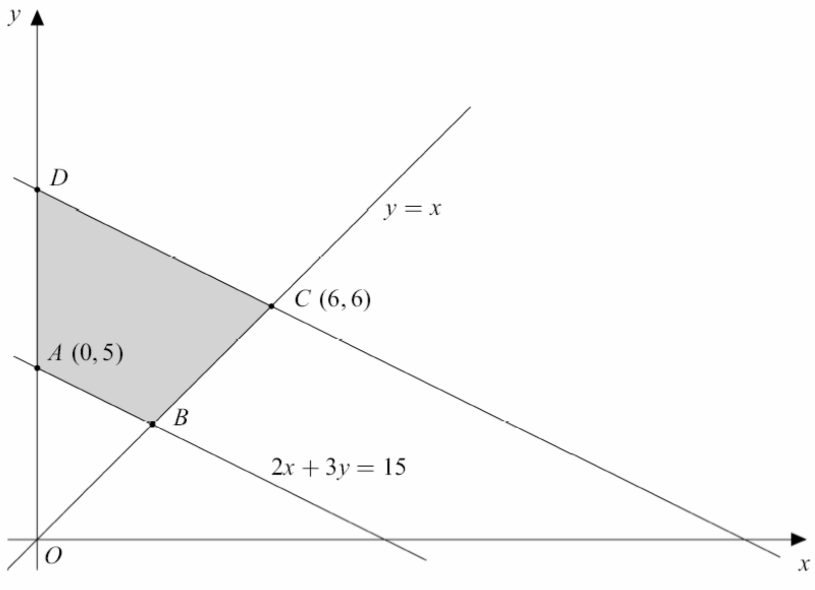
(a) (i) Find $\int (x^3 - 3x^2 + 6x)\,\mathrm{d}x$. *(2 marks)*

(ii) Hence find $\int_1^3 (x^3 - 3x^2 + 6x)\,\mathrm{d}x$. *(3 marks)*

(b) (i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. *(2 marks)*

(ii) Show that y is an increasing function of x for all values of x. *(3 marks)*

7 The diagram shows a trapezium $ABCD$. The vertices A and C have coordinates $(0, 5)$ and $(6, 6)$ respectively. The sides AB and BC have equations $2x + 3y = 15$ and $y = x$ respectively.



Find:

(a) the coordinates of B; *(2 marks)*

(b) the equation of the side CD, which is parallel to AB; *(3 marks)*

(c) the coordinates of D, which lies on the y-axis; *(1 mark)*

(d) the area of $ABCD$. *(4 marks)*