# A-LEVEL

# Mathematics

## Pure Core 1 – MPC1
## Mark scheme

6360
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Version/Stage: Final V1.0

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

**Key to mark scheme abbreviations**

| | |
|---|---|
| M | mark is for method |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |
| √or ft or F | follow through from previous incorrect result |
| CAO | correct answer only |
| CSO | correct solution only |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| A2,1 | 2 or 1 (or 0) accuracy marks |
| $-x$ EE | deduct $x$ marks for each error |
| NMS | no method shown |
| PI | possibly implied |
| SCA | substantially correct approach |
| c | candidate |
| sf | significant figure(s) |
| dp | decimal place(s) |

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

| Q | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| 1 (a)(i) | Grad $AB = \frac{-5-2}{3--1}$ OE | M1 | | correct unsimplified eg $\frac{2--5}{-1-3}$ |
| | $= -\frac{7}{4}$ | A1 | 2 | |
| (ii) | $y--5 = \text{'their grad'}\ (x-3)$ <br> $y-2 = \text{'their grad'}\ (x--1)$ | M1 | | either pair of coordinates used correctly and attempt to find $c$ if using $y=mx+c$ |
| | $y-2=-\frac{7}{4}(x+1)$ <br> $y+5=-\frac{7}{4}(x-3)$ <br> $y=-\frac{7}{4}x+\frac{1}{4}$ | A1 | | OE, any form of correct equation with $--$ simplified to $+$ |
| | $7x+4y=1$ | A1 | 3 | integer coefficients & in this form |
| (b)(i) | $(M)\quad (1,-1.5)$ | B1 | 1 | condone $x=1,\ y=-\frac{3}{2}$ |
| (ii) | Perp grad = $\frac{4}{7}$ | B1√ | | perp grad = –1/ 'their' grad $AB$ |
| | $y--\frac{3}{2}=\text{'their'}\frac{4}{7}(x-1)$ | M1 | | ft 'their $M$' but must have attempted perpendicular gradient |
| | $y+\frac{3}{2}=\frac{4}{7}(x-1)$ | A1 | 3 | **any** correct form with $--$ simplified to $+$ <br> eg $8x-14y=29$ ; $y=\frac{4}{7}x+c,\ c=-\frac{29}{14}$ |
| (c) | $(AC^2=)\quad (k--1)^2+(2k+3-2)^2$ | M1 | | $(k+1)^2+(2k+1)^2$ |
| | $k^2+2k+1+4k^2+4k+1=13$ <br> $5k^2+6k-11=0$ | A1 | | |
| | $(5k+11)(k-1)=0$ | A1 | | correct factors or correct use of formula as far as $\frac{-6\pm\sqrt{256}}{10}$ |
| | $\Rightarrow k=1,\quad k=-\frac{11}{5}$ | A1 | 4 | |
| | **Total** | | **13** | |

**(a) (i)** **NMS** grad $AB = -\frac{7}{4}$ earns 2 marks.

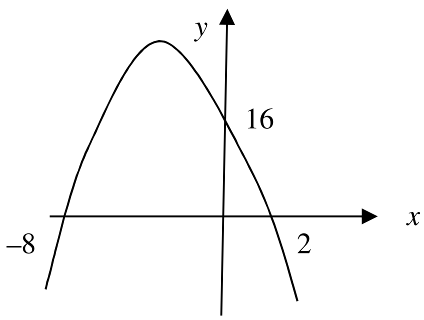
**(ii)** must simplify $y--5$ to $y+5$ or $x--1$ to $x+1$ for first **A1**

Condone $8y+14x=2$ etc for final **A1**, but not $7x+4y-1=0$ etc

**(b)(ii)** If their gradient of $AB$ is $m$, then use of $-m$ or $1/m$ can earn **M1**. For **A1**, $1/\left(\frac{7}{4}\right)$ , $\frac{14.5}{7}$ etc must be simplified.

| Q | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| 2 | $\frac{15+7\sqrt{3}}{9+5\sqrt{3}} \times \frac{9-5\sqrt{3}}{9-5\sqrt{3}}$ | M1 | | writing correct quotient and multiplying by correct conjugate of denominator |
| | (Numerator =) $135-75\sqrt{3}+63\sqrt{3}-105$ | A1 | | $30-12\sqrt{3}$ |
| | (Denominator = $81-45\sqrt{3}+45\sqrt{3}-75$ ) $=6$ | B1 | | must be seen as denominator |
| | $\left(\frac{30-12\sqrt{3}}{6}=\right)$ $5-2\sqrt{3}$ | A1cso | 4 | units (cm) need not be given |
| | **Alternative**<br>$(9+5\sqrt{3})(m+n\sqrt{3})$<br>$=9m+15n+5m\sqrt{3}+9n\sqrt{3}$<br>$9m+15n=15, \quad 5m+9n=7$<br>$m=5 \quad , \quad n=-2$<br>$5-2\sqrt{3}$ | <br><br>(M1)<br>(A1)<br>(A1)<br>(A1) | | <br><br>must be correct<br>both equations correct<br>either correct |
| | **Total** | | 4 | |
| | No marks if candidate uses $\frac{9+5\sqrt{3}}{15+7\sqrt{3}}$<br><br>Condone multiplication by $9-5\sqrt{3}$ instead of $\frac{9-5\sqrt{3}}{9-5\sqrt{3}}$ for **M1 only if** subsequent working shows multiplication by **both** numerator and denominator – otherwise **M0**.<br><br>May use alternative conjugate $\frac{15+7\sqrt{3}}{9+5\sqrt{3}} \times \frac{5\sqrt{3}-9}{5\sqrt{3}-9}$ **M1** numerator $=12\sqrt{3}-30$ **A1** denominator = –6 **B1**<br><br>Ignore any incorrect units | | | |

| Q | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| 3 (a)(i) | $\left(\frac{dy}{dx}=\right)$ $10x^4+20x^3$ | M1<br>A1 | 2 | one term correct<br>all correct ( no $+c$ etc) |
| (ii) | $\left(\frac{d^2y}{dx^2}=\right)$ $40x^3+60x^2$ | B1√ | 1 | ft their $\frac{dy}{dx}$ |
| (b)(i) | $\left(\frac{dy}{dx}=\right)$ $10-20 \quad =-10$ | B1√ | | correctly sub $x=-1$ into their $\frac{dy}{dx}$ and evaluated correctly |
| | $\frac{dy}{dx}<0$ (therefore $y$ is) decreasing | E1√ | 2 | Must state "decreasing" **and** $\frac{dy}{dx}<0$<br>ft 'therefore $y$ is increasing' **and** reason if their value of $\frac{dy}{dx}>0$ |
| (ii) | (When $x=-1$) $y=2$ | B1 | | |
| | $y-'their'2='their\ grad'(x--1)$<br>but must be tangent and not normal | M1 | | ft ' their' value of $\frac{dy}{dx}$ when $x=-1$ and ' their' $y$-coordinate |
| | $y-2=-10(x+1)$ *or* $y=-10x-8$ etc | A1 | 3 | any correct tangent eqn from correct $\frac{dy}{dx}$ |
| (c) | $\left(\frac{dy}{dx}=\right)$ $10(-2)^4+20(-2)^3$ | M1 | | correctly sub $x=-2$ into their $\frac{dy}{dx}$ |
| | $=160-160=0 \Rightarrow$ **stationary** point (when $x=-2$) | A1 | | correctly shown that $\frac{dy}{dx}=0$ **plus** correct statement |
| | $\left(\frac{d^2y}{dx^2}=\right)$ $40(-2)^3+60(-2)^2$ | M1 | | correctly sub $x=-2$ into their $\frac{d^2y}{dx^2}$ or other suitable test for max/min |
| | $=-320+240=-80<0$<br>(Therefore) **maximum** (point at $Q$) | A1 | 4 | **either** $\frac{d^2y}{dx^2}=-320+240<0$<br>**or** $\frac{d^2y}{dx^2}=-80<0$<br>**plus** conclusion |
| | **Total** | | **12** | |
| **(b) (i)** | Accept "gradient is negative so decreasing" for **E1**<br>Do **not** accept "because **it** is negative" or "$\frac{dy}{dx}=-10$" as reasons for **E1** | | | |
| **(ii)** | May earn **M1** for attempt to find $c$ using $y=mx+c$ if clearly finding tangent and not normal.<br>Must simplify $x--1$ to $x+1$ for **A1** | | | |
| **(c)** | May write "their" $10x^4+20x^3=0$ and attempt to find $x$ for first **M1** leading to "$x=-2$ …stationary pt" for **A1** | | | |

| Q | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| 4 (a)(i) | $k-(x+3)^2$ | M1 | | or $x^2+6x-16=(x+3)^2-25$ <br> or $q$ = 3 stated |
| | $25-(x+3)^2$ | A1 | 2 | |
| (ii) | (Max value =) 25 | B1√ | 1 | ft their $p$ |
| (b)(i) | $(8+x)(2-x)$ | B1 | 1 | |
| (ii) |  | M1 | | ∩ shape |
| | | A1 | | curve roughly symmetrical with max to left of $y$-axis, <br> curve in all 4 quadrants <br> **and** $y$-intercept 16 stated or marked on $y$-axis |
| | crosses $x$-axis at –8 and 2 | B1 | 3 | correct - stated or marked on $x$-axis |
| | **Total** | | **7** | |

| | |
|---|---|
| (a)(i) | **Example** $16-(x+3)^2-9$ earns **M1** |
| (ii) | (–3, 25) scores **B0** since maximum value not identified <br> Allow maximum given as "$y=25$" |
| (b)(i) | Condone $-(x-2)(x+8)$, $(x-2)(-x-8)$ etc |
| (ii) | Withhold **B1** if more than 2 intercepts |

| Q | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| 5 (a) | $(-3)^3 + c(-3)^2 + d(-3) + 3$ | M1 | | p(–3) attempted |
| | $-27 + 9c - 3d + 3 = 0$ | | | must see this line or equivalent, **and** must have = 0 on right or left before final result |
| | $\Rightarrow 3c - d = 8$ | A1 | 2 | **AG** be convinced |
| (b) | $2^3 + c\times 2^2 + d\times 2 + 3 = 65$ | M1 | | p(2) attempted & … = 65 |
| | $8 + 4c + 2d + 3 = 65$ | A1 | 2 | correct equation in any form simplifying powers of 2 eg $4c + 2d = 54$ |
| (c) | $5c = 35$ or $10d = 130$ OE | M1 | | correct elimination of $c$ or $d$ using both $3c - d = 8$ and their equation from (b) |
| | $c = 7$ | A1 | | |
| | $d = 13$ | A1 | 3 | |
| | **Total** | | **7** | |

**(a)** May use long division by $x+3$ but must reach remainder term for **M1**
Condone missing brackets in p(–3) expression if recovered later as –27 +9*c* +… to earn **A1**

Treat parts **(b)** and **(c)** holistically

**(b)** May use long division by $x-2$ as far as remainder and equate their remainder to 65 for **M1**

**(c)** **Example** $4c + 2(3c - 8) = 54$ earns **M1** for eliminating *d* if equation in part **(b)** is correct

| Q | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| 6 (a)(i) | $x^3 - x^2 - 5x + 7 = x + 7$ | M1 | | |
| | $\Rightarrow x^3 - x^2 - 5x = x$ | | | must see this line OE eg $x^3 - x^2 - 6x = 0$ |
| | $(x \neq 0) \Rightarrow x^2 - x - 6 = 0$ | A1 | 2 | AG |
| (ii) | $(x-3)(x+2)$ | M1 | | correct |
| | $x = 3, \quad x = -2$ | A1 | | both $x$ values correct |
| | $A(-2, 5)$ **and** $C(3, 10)$ | A1 | 3 | both pairs of coordinates correct |
| (b) | $\frac{x^4}{4} - \frac{x^3}{3} - \frac{5x^2}{2} + 7x \quad (+c)$ | M1<br>A1<br>A1 | 3 | 2 terms correct<br>another term correct<br>all correct |
| (c) | $\mathrm{F}(-2) = \left[\frac{(-2)^4}{4} - \frac{(-2)^3}{3} - \frac{5(-2)^2}{2} + 7(-2)\right]$ | M1 | | F('their'–2) correctly substituting into their answer to (b) , but must have scored M1 in part (b) |
| | F(0) – F(–2) = $0 - \left(\frac{16}{4} + \frac{8}{3} - \frac{20}{2} - 14\right) = \frac{52}{3}$ | A1 | | correct value using limits correctly |
| | Area of trapezium = $\left(\frac{1}{2}(5+7) \times 2\right) = 12$ | B1 | | or rectangle plus triangle |
| | Area of $R = \frac{52}{3} - 12 = \frac{16}{3}$ | A1 | 4 | $5\frac{1}{3}$ or $5.\dot{3}$ |
| | **Total** | | 12 | |

**(a)(ii)** **NMS either** $(-2, 5)$ **or** $(3, 10)$ scores **SC1** and **both correct** scores **SC3**
Allow "when $x = 3, \quad y = 10$ **and** when $x = -2, \quad y = 5$" instead of coordinates for final **A1**

**(c)** Condone missing brackets around "their" –2 for **M1** and if recovered and correct on next line for **A1**
Area of trapezium found by integration $\int_{-2}^{0} (x+7)\,\mathrm{d}x = \left[\frac{x^2}{2} + 7x\right]_{-2}^{0} = 12$ earns **B1**
Accept rounded answer of 5.3 etc after correct exact answer seen.

| Q | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| 7 (a) | $(x-5)^2+(y--6)^2$ | M1 | | one term correct |
| | | A1 | | LHS correct with perhaps extra constant terms |
| | $(x-5)^2+(y+6)^2=20$ | A1 | 3 | equation completely correct |
| (b) (i) | $C(5,-6)$ | B1√ | 1 | correct or ft their **(a)** |
| (ii) | (radius =) $\sqrt{20}$ | M1 | | correct or ft 'their' $\sqrt{k}$ provided RHS > 0 |
| | $= 2\sqrt{5}$ | A1 | 2 | must see $\sqrt{20}$ **first** |
| (c) | Grad $AC = \dfrac{-6--2}{5-3}$ $(=-2)$ | M1 | | correct unsimplified, ft their coords of $C$ |
| | Grad of tangent $= \dfrac{1}{2}$ | B1√ | | ft their $-1/$ grad $AC$ |
| | Equation of tangent is $(y--2)="their\dfrac{1}{2}"(x-3)$ | M1 | | clear attempt at **tangent** not normal through $(3,-2)$ |
| | $y+2=\dfrac{1}{2}(x-3)$ | A1 | | correct equation in any form but $y--2$ must be simplified to $y+2$ |
| | $x-2y=7$ | A1 cso | 5 | |
| (d) | $AB^2+(their\ \ r)^2 = 6^2$ | M1 | | Pythagoras used with 6 as hypotenuse |
| | $d^2+20=36$ or $(AB^2)=36-20$ | A1 | | values correct with $(2\sqrt{5})^2=20$ PI |
| | $AB^2=16$ Hence $AB=4$ | A1cso | 3 | notation all correct |
| | **Total** | | 14 | |

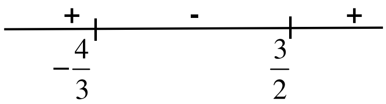
**(a)** $(x-5)^2+(y--6)^2=\left(\sqrt{20}\right)^2$ scores full marks

If final equation is correct then award 3 marks, treating earlier lines with extra terms etc as rough working.
If final equation has sign errors then check to see if M1 is earned.
**Example** $(x-5)^2+(y+6)^2-25+36+41=0$ earns **M1 A1** but if this is part of preliminary working and final equation is offered as $(x-5)^2+(y+6)^2=20$ then award **M1 A1 A1.**
**Example** $(x-5)^2+(y-6)^2=20$ earns **M1 A0 ; Example** $(x+5)^2+(y-6)^2=20$ earns **M0**

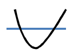
**(b)(ii)** Candidates may still earn A1 here provided RHS of circle equation is 20.
**Example** $(x+5)^2+(y-6)^2=20$ earns **M0** in **(a)** but can then earn **M1 A1** for radius = $\sqrt{20}=2\sqrt{5}$
**NMS** or no $\sqrt{20}$ seen; " radius $=2\sqrt{5}$ " scores **SC1** since question says "show that"

**(c)** May earn second **M1** for attempt to find $c$ using $y=mx+c$ if clearly finding tangent and not normal.
If their gradient of $AC$ is $m$, then use of $-m$ or $1/m$ with correct coordinates can earn second **M1**

**(d)** **Example** $AB = 36-(2\sqrt{5})^2 = 16 = 4$ scores **M1 A1 A0** for poor notation
**NMS** $AB = 4$ scores **SC1** since no evidence that exact value of radius has been used.

| Q | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| 8 | | | | |
| (a) | $3-6x-15x-10>0$ | M1 | | Correctly multiplied out with $>0$ |
| | $-21x>7$ | | | |
| | $\Rightarrow x<-\frac{1}{3}$ | A1cso | 2 | all working correct |
| (b) | $6x^2-x-12\leqslant 0$ | | | |
| | $(3x+4)(2x-3)$ | M1 | | correct factors or correct use of formula as far as $\frac{1\pm\sqrt{289}}{12}$ |
| | CVs are $-\frac{4}{3}, \frac{3}{2}$ | A1 | | |
| |  | M1 | | use of sign diagram or graph with CVs clearly shown |
| | $-\frac{4}{3}\leqslant x\leqslant\frac{3}{2}$ | A1 | 4 | *or* $\frac{3}{2}\geqslant x\geqslant-\frac{4}{3}$ |
| | **Total** | | **6** | |
| | **TOTAL** | | **75** | |

**(a)** Allow final answer in form $-\frac{1}{3}>x$ .

**(b)** For second **M1**, if critical values are correct then sign diagram or sketch  must be correct ***with correct CVs marked***.
However, if CVs are not correct then second **M1** can be earned for attempt at sketch or sign diagram but ***their CVs*** MUST be marked on the diagram or sketch.
Final **A1,** inequality must have $x$ and no other letter.

***Final answer of*** $x\leqslant\frac{3}{2}$ AND $x\geqslant-\frac{4}{3}$ (with or without working) scores 4 marks .

(A) $-\frac{4}{3}<x<\frac{3}{2}$ (B) $x\leqslant\frac{3}{2}$ OR $x\geqslant-\frac{4}{3}$ (C) $x\leqslant\frac{3}{2}$ , $x\geqslant-\frac{4}{3}$ **(D)** $-\frac{4}{3}\leqslant k\leqslant\frac{3}{2}$
with or without working each score 3 marks (**SC3**)

**Example NMS** $\frac{4}{3}\leqslant x\leqslant\frac{3}{2}$ scores **M0** (since one CV is incorrect)

**Example NMS** $x<\frac{3}{2}$ , $x<-\frac{4}{3}$ scores **M1 A1 M0** (since both CVs are correct)