AQA Maths Pure Core 1

Past Paper Pack

2006-2014

General Certificate of Education
January 2006
Advanced Subsidiary Examination

MATHEMATICS **MPC1**
Unit Pure Core 1

Tuesday 10 January 2006 1.30 pm to 3.00 pm

For this paper you must have: • an 8-page answer book • the **blue** AQA booklet of formulae and statistical tables You must **not** use a calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) Simplify $(\sqrt{5}+2)(\sqrt{5}-2)$. *(2 marks)*

(b) Express $\sqrt{8}+\sqrt{18}$ in the form $n\sqrt{2}$, where n is an integer. *(2 marks)*

2 The point A has coordinates $(1, 1)$ and the point B has coordinates $(5, k)$.

The line AB has equation $3x + 4y = 7$.

(a) (i) Show that $k = -2$. *(1 mark)*

(ii) Hence find the coordinates of the mid-point of AB. *(2 marks)*

(b) Find the gradient of AB. *(2 marks)*

(c) The line AC is perpendicular to the line AB.

(i) Find the gradient of AC. *(2 marks)*

(ii) Hence find an equation of the line AC. *(1 mark)*

(iii) Given that the point C lies on the x-axis, find its x-coordinate. *(2 marks)*

3 (a) (i) Express $x^2 - 4x + 9$ in the form $(x-p)^2 + q$, where p and q are integers. *(2 marks)*

(ii) Hence, or otherwise, state the coordinates of the minimum point of the curve with equation $y = x^2 - 4x + 9$. *(2 marks)*

(b) The line L has equation $y + 2x = 12$ and the curve C has equation $y = x^2 - 4x + 9$.

(i) Show that the x-coordinates of the points of intersection of L and C satisfy the equation

$$x^2 - 2x - 3 = 0$$ *(1 mark)*

(ii) Hence find the coordinates of the points of intersection of L and C. *(4 marks)*

4 The quadratic equation $x^2 + (m+4)x + (4m+1) = 0$, where m is a constant, has equal roots.

(a) Show that $m^2 - 8m + 12 = 0$. *(3 marks)*

(b) Hence find the possible values of m. *(2 marks)*

5 A circle with centre C has equation $x^2 + y^2 - 8x + 6y = 11$.

(a) By completing the square, express this equation in the form

$$(x-a)^2 + (y-b)^2 = r^2$$ *(3 marks)*

(b) Write down:

(i) the coordinates of C; *(1 mark)*

(ii) the radius of the circle. *(1 mark)*

(c) The point O has coordinates $(0, 0)$.

(i) Find the length of CO. *(2 marks)*

(ii) Hence determine whether the point O lies inside or outside the circle, giving a reason for your answer. *(2 marks)*

6 The polynomial $\mathrm{p}(x)$ is given by

$$\mathrm{p}(x) = x^3 + x^2 - 10x + 8$$

(a) (i) Using the factor theorem, show that $x - 2$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(ii) Hence express $\mathrm{p}(x)$ as the product of three linear factors. *(3 marks)*

(b) Sketch the curve with equation $y = x^3 + x^2 - 10x + 8$, showing the coordinates of the points where the curve cuts the axes.

(You are not required to calculate the coordinates of the stationary points.) *(4 marks)*

7 The volume, $V\ \text{m}^3$, of water in a tank at time t seconds is given by

$$V = \tfrac{1}{3}t^6 - 2t^4 + 3t^2\,, \quad \text{for } t \geqslant 0$$

(a) Find:

(i) $\dfrac{\mathrm{d}V}{\mathrm{d}t}$; *(3 marks)*

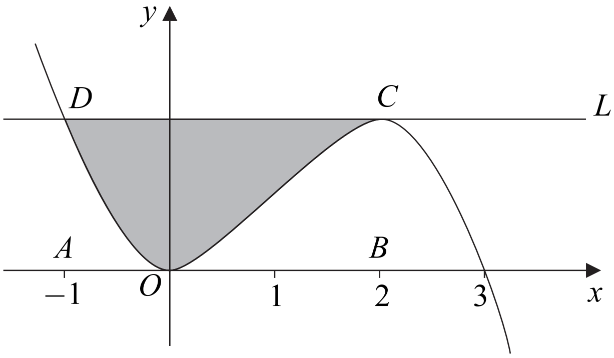
(ii) $\dfrac{\mathrm{d}^2V}{\mathrm{d}t^2}$. *(2 marks)*

(b) Find the rate of change of the volume of water in the tank, in $\text{m}^3\,\text{s}^{-1}$, when $t = 2$.
(2 marks)

(c) (i) Verify that V has a stationary value when $t = 1$. *(2 marks)*

(ii) Determine whether this is a maximum or minimum value. *(2 marks)*

8 The diagram shows the curve with equation $y = 3x^2 - x^3$ and the line L.



The points A and B have coordinates $(-1, 0)$ and $(2, 0)$ respectively. The curve touches the x-axis at the origin O and crosses the x-axis at the point $(3, 0)$. The line L cuts the curve at the point D where $x = -1$ and touches the curve at C where $x = 2$.

(a) Find the area of the rectangle $ABCD$. *(2 marks)*

(b) (i) Find $\int (3x^2 - x^3)\,\mathrm{d}x$. *(3 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the line L. *(4 marks)*

(c) For the curve above with equation $y = 3x^2 - x^3$:

(i) find $\frac{\mathrm{d}y}{\mathrm{d}x}$; *(2 marks)*

(ii) hence find an equation of the tangent at the point on the curve where $x = 1$; *(3 marks)*

(iii) show that y is decreasing when $x^2 - 2x > 0$. *(2 marks)*

(d) Solve the inequality $x^2 - 2x > 0$. *(2 marks)*

END OF QUESTIONS

General Certificate of Education
June 2006
Advanced Subsidiary Examination

MATHEMATICS **MPC1**
Unit Pure Core 1

Monday 22 May 2006 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You must **not** use a calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The point A has coordinates $(1, 7)$ and the point B has coordinates $(5, 1)$.

(a) (i) Find the gradient of the line AB. *(2 marks)*

(ii) Hence, or otherwise, show that the line AB has equation $3x + 2y = 17$. *(2 marks)*

(b) The line AB intersects the line with equation $x - 4y = 8$ at the point C. Find the coordinates of C. *(3 marks)*

(c) Find an equation of the line through A which is perpendicular to AB. *(3 marks)*

2 (a) Express $x^2 + 8x + 19$ in the form $(x + p)^2 + q$, where p and q are integers. *(2 marks)*

(b) Hence, or otherwise, show that the equation $x^2 + 8x + 19 = 0$ has no real solutions. *(2 marks)*

(c) Sketch the graph of $y = x^2 + 8x + 19$, stating the coordinates of the minimum point and the point where the graph crosses the y-axis. *(3 marks)*

(d) Describe geometrically the transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 8x + 19$. *(3 marks)*

3 A curve has equation $y = 7 - 2x^5$.

(a) Find $\dfrac{dy}{dx}$. *(2 marks)*

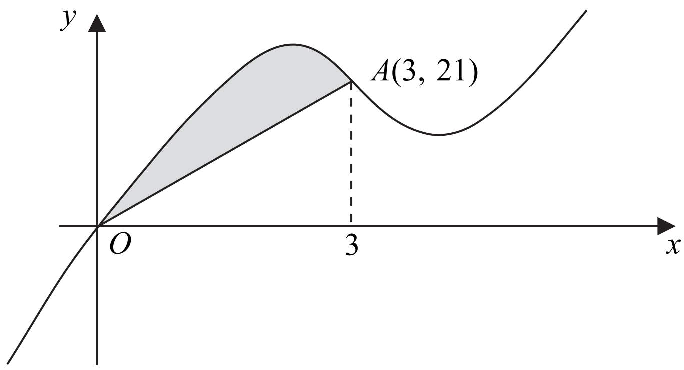
(b) Find an equation for the tangent to the curve at the point where $x = 1$. *(3 marks)*

(c) Determine whether y is increasing or decreasing when $x = -2$. *(2 marks)*

4 (a) Express $(4\sqrt{5} - 1)(\sqrt{5} + 3)$ in the form $p + q\sqrt{5}$, where p and q are integers. *(3 marks)*

(b) Show that $\dfrac{\sqrt{75} - \sqrt{27}}{\sqrt{3}}$ is an integer and find its value. *(3 marks)*

5 The curve with equation $y = x^3 - 10x^2 + 28x$ is sketched below.



The curve crosses the x-axis at the origin O and the point $A(3, 21)$ lies on the curve.

(a) (i) Find $\dfrac{dy}{dx}$. *(3 marks)*

(ii) Hence show that the curve has a stationary point when $x = 2$ and find the x-coordinate of the other stationary point. *(4 marks)*

(b) (i) Find $\int (x^3 - 10x^2 + 28x)\,dx$. *(3 marks)*

(ii) Hence show that $\int_0^3 (x^3 - 10x^2 + 28x)\,dx = 56\frac{1}{4}$. *(2 marks)*

(iii) Hence determine the area of the shaded region bounded by the curve and the line OA. *(3 marks)*

6 The polynomial $p(x)$ is given by $p(x) = x^3 - 4x^2 + 3x$.

(a) Use the Factor Theorem to show that $x - 3$ is a factor of $p(x)$. *(2 marks)*

(b) Express $p(x)$ as the product of three linear factors. *(2 marks)*

(c) (i) Use the Remainder Theorem to find the remainder, r, when $p(x)$ is divided by $x - 2$. *(2 marks)*

(ii) Using algebraic division, or otherwise, express $p(x)$ in the form

$$(x - 2)(x^2 + ax + b) + r$$

where a, b and r are constants. *(4 marks)*

Turn over for the next question

7 A circle has equation $x^2 + y^2 - 4x - 14 = 0$.

(a) Find:

(i) the coordinates of the centre of the circle; *(3 marks)*

(ii) the radius of the circle in the form $p\sqrt{2}$, where p is an integer. *(3 marks)*

(b) A chord of the circle has length 8. Find the perpendicular distance from the centre of the circle to this chord. *(3 marks)*

(c) A line has equation $y = 2k - x$, where k is a constant.

(i) Show that the x-coordinate of any point of intersection of the line and the circle satisfies the equation

$$x^2 - 2(k+1)x + 2k^2 - 7 = 0$$ *(3 marks)*

(ii) Find the values of k for which the equation

$$x^2 - 2(k+1)x + 2k^2 - 7 = 0$$

has equal roots. *(4 marks)*

(iii) Describe the geometrical relationship between the line and the circle when k takes either of the values found in part (c)(ii). *(1 mark)*

END OF QUESTIONS

General Certificate of Education
January 2007
Advanced Subsidiary Examination

MATHEMATICS **MPC1**
Unit Pure Core 1

Wednesday 10 January 2007 1.30 pm to 3.00 pm

For this paper you must have:
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Time allowed: 1 hour 30 minutes

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- Answer **all** questions.
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Information
- The maximum mark for this paper is 75.
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Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The polynomial $\mathrm{p}(x)$ is given by

$$\mathrm{p}(x) = x^3 - 4x^2 - 7x + k$$

where k is a constant.

(a) (i) Given that $x + 2$ is a factor of $\mathrm{p}(x)$, show that $k = 10$. *(2 marks)*

(ii) Express $\mathrm{p}(x)$ as the product of three linear factors. *(3 marks)*

(b) Use the Remainder Theorem to find the remainder when $\mathrm{p}(x)$ is divided by $x - 3$. *(2 marks)*

(c) Sketch the curve with equation $y = x^3 - 4x^2 - 7x + 10$, indicating the values where the curve crosses the x-axis and the y-axis. (You are **not** required to find the coordinates of the stationary points.) *(4 marks)*

2 The line AB has equation $3x + 5y = 8$ and the point A has coordinates $(6, -2)$.

(a) (i) Find the gradient of AB. *(2 marks)*

(ii) Hence find an equation of the straight line which is perpendicular to AB and which passes through A. *(3 marks)*

(b) The line AB intersects the line with equation $2x + 3y = 3$ at the point B. Find the coordinates of B. *(3 marks)*

(c) The point C has coordinates $(2, k)$ and the distance from A to C is 5. Find the **two** possible values of the constant k. *(3 marks)*

3 (a) Express $\dfrac{\sqrt{5}+3}{\sqrt{5}-2}$ in the form $p\sqrt{5} + q$, where p and q are integers. *(4 marks)*

(b) (i) Express $\sqrt{45}$ in the form $n\sqrt{5}$, where n is an integer. *(1 mark)*

(ii) Solve the equation

$$x\sqrt{20} = 7\sqrt{5} - \sqrt{45}$$

giving your answer in its simplest form. *(3 marks)*

4 A circle with centre C has equation $x^2 + y^2 + 2x - 12y + 12 = 0$.

(a) By completing the square, express this equation in the form

$$(x - a)^2 + (y - b)^2 = r^2$$ *(3 marks)*

(b) Write down:

(i) the coordinates of C; *(1 mark)*

(ii) the radius of the circle. *(1 mark)*

(c) Show that the circle does **not** intersect the x-axis. *(2 marks)*

(d) The line with equation $x + y = 4$ intersects the circle at the points P and Q.

(i) Show that the x-coordinates of P and Q satisfy the equation

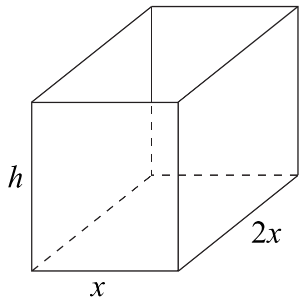
$$x^2 + 3x - 10 = 0$$ *(3 marks)*

(ii) Given that P has coordinates $(2, 2)$, find the coordinates of Q. *(2 marks)*

(iii) Hence find the coordinates of the midpoint of PQ. *(2 marks)*

Turn over for the next question

5 The diagram shows an **open-topped** water tank with a horizontal rectangular base and four vertical faces. The base has width x metres and length $2x$ metres, and the height of the tank is h metres.



The combined internal surface area of the base and four vertical faces is $54\,\text{m}^2$.

(a) (i) Show that $x^2 + 3xh = 27$. *(2 marks)*

(ii) Hence express h in terms of x. *(1 mark)*

(iii) Hence show that the volume of water, $V\ \text{m}^3$, that the tank can hold when full is given by

$$V = 18x - \frac{2x^3}{3}$$

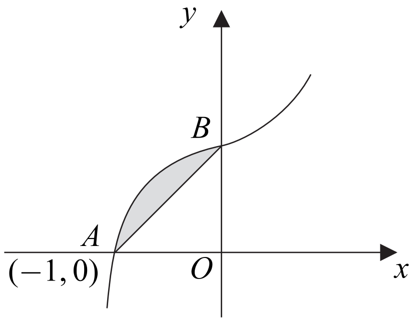
(1 mark)

(b) (i) Find $\dfrac{\mathrm{d}V}{\mathrm{d}x}$. *(2 marks)*

(ii) Verify that V has a stationary value when $x = 3$. *(2 marks)*

(c) Find $\dfrac{\mathrm{d}^2V}{\mathrm{d}x^2}$ and hence determine whether V has a maximum value or a minimum value when $x = 3$. *(2 marks)*

6 The curve with equation $y = 3x^5 + 2x + 5$ is sketched below.



The curve cuts the x-axis at the point $A(-1, 0)$ and cuts the y-axis at the point B.

(a) (i) State the coordinates of the point B and hence find the area of the triangle AOB, where O is the origin. *(3 marks)*

(ii) Find $\int (3x^5 + 2x + 5)\, dx$. *(3 marks)*

(iii) Hence find the area of the shaded region bounded by the curve and the line AB. *(4 marks)*

(b) (i) Find the gradient of the curve with equation $y = 3x^5 + 2x + 5$ at the point $A(-1, 0)$. *(3 marks)*

(ii) Hence find an equation of the tangent to the curve at the point A. *(1 mark)*

7 The quadratic equation $(k+1)x^2 + 12x + (k-4) = 0$ has real roots.

(a) Show that $k^2 - 3k - 40 \leqslant 0$. *(3 marks)*

(b) Hence find the possible values of k. *(4 marks)*

END OF QUESTIONS

General Certificate of Education
June 2007
Advanced Subsidiary Examination

MATHEMATICS **MPC1**
Unit Pure Core 1

Monday 21 May 2007 9.00 am to 10.30 am

For this paper you must have: • an 8-page answer book • the **blue** AQA booklet of formulae and statistical tables. You must **not** use a calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
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Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The points A and B have coordinates $(6, -1)$ and $(2, 5)$ respectively.

(a) (i) Show that the gradient of AB is $-\frac{3}{2}$. *(2 marks)*

(ii) Hence find an equation of the line AB, giving your answer in the form $ax + by = c$, where a, b and c are integers. *(2 marks)*

(b) (i) Find an equation of the line which passes through B and which is perpendicular to the line AB. *(2 marks)*

(ii) The point C has coordinates $(k, 7)$ and angle ABC is a right angle.

Find the value of the constant k. *(2 marks)*

2 (a) Express $\frac{\sqrt{63}}{3} + \frac{14}{\sqrt{7}}$ in the form $n\sqrt{7}$, where n is an integer. *(3 marks)*

(b) Express $\frac{\sqrt{7}+1}{\sqrt{7}-2}$ in the form $p\sqrt{7} + q$, where p and q are integers. *(4 marks)*

3 (a) (i) Express $x^2 + 10x + 19$ in the form $(x + p)^2 + q$, where p and q are integers. *(2 marks)*

(ii) Write down the coordinates of the vertex (minimum point) of the curve with equation $y = x^2 + 10x + 19$. *(2 marks)*

(iii) Write down the equation of the line of symmetry of the curve $y = x^2 + 10x + 19$. *(1 mark)*

(iv) Describe geometrically the transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 10x + 19$. *(3 marks)*

(b) Determine the coordinates of the points of intersection of the line $y = x + 11$ and the curve $y = x^2 + 10x + 19$. *(4 marks)*

4 A model helicopter takes off from a point O at time $t = 0$ and moves vertically so that its height, y cm, above O after time t seconds is given by

$$y = \frac{1}{4}t^4 - 26t^2 + 96t\ , \qquad 0 \leqslant t \leqslant 4$$

(a) Find:

(i) $\dfrac{\mathrm{d}y}{\mathrm{d}t}$; *(3 marks)*

(ii) $\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}$. *(2 marks)*

(b) Verify that y has a stationary value when $t = 2$ and determine whether this stationary value is a maximum value or a minimum value. *(4 marks)*

(c) Find the rate of change of y with respect to t when $t = 1$. *(2 marks)*

(d) Determine whether the height of the helicopter above O is increasing or decreasing at the instant when $t = 3$. *(2 marks)*

5 A circle with centre C has equation $(x + 3)^2 + (y - 2)^2 = 25$.

(a) Write down:

(i) the coordinates of C; *(2 marks)*

(ii) the radius of the circle. *(1 mark)*

(b) (i) Verify that the point $N(0, -2)$ lies on the circle. *(1 mark)*

(ii) Sketch the circle. *(2 marks)*

(iii) Find an equation of the normal to the circle at the point N. *(3 marks)*

(c) The point P has coordinates (2, 6).

(i) Find the distance PC, leaving your answer in surd form. *(2 marks)*

(ii) Find the length of a tangent drawn from P to the circle. *(3 marks)*

Turn over for the next question

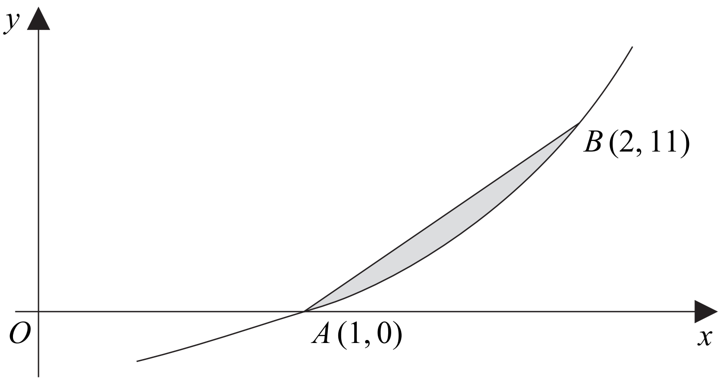
6 (a) The polynomial $f(x)$ is given by $f(x) = x^3 + 4x - 5$.

(i) Use the Factor Theorem to show that $x - 1$ is a factor of $f(x)$. *(2 marks)*

(ii) Express $f(x)$ in the form $(x - 1)(x^2 + px + q)$, where p and q are integers. *(2 marks)*

(iii) Hence show that the equation $f(x) = 0$ has exactly one real root and state its value. *(3 marks)*

(b) The curve with equation $y = x^3 + 4x - 5$ is sketched below.



The curve cuts the x-axis at the point $A(1, 0)$ and the point $B(2, 11)$ lies on the curve.

(i) Find $\int (x^3 + 4x - 5)\,dx$. *(3 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the line AB. *(4 marks)*

7 The quadratic equation

$$(2k - 3)x^2 + 2x + (k - 1) = 0$$

where k is a constant, has real roots.

(a) Show that $2k^2 - 5k + 2 \leqslant 0$. *(3 marks)*

(b) (i) Factorise $2k^2 - 5k + 2$. *(1 mark)*

(ii) Hence, or otherwise, solve the quadratic inequality

$$2k^2 - 5k + 2 \leqslant 0$$ *(3 marks)*

END OF QUESTIONS

General Certificate of Education
January 2008
Advanced Subsidiary Examination

MATHEMATICS **MPC1**
Unit Pure Core 1

Wednesday 9 January 2008 1.30 pm to 3.00 pm

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Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
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Answer **all** questions.

1 The triangle ABC has vertices $A(-2, 3)$, $B(4, 1)$ and $C(2, -5)$.

(a) Find the coordinates of the mid-point of BC. *(2 marks)*

(b) (i) Find the gradient of AB, in its simplest form. *(2 marks)*

(ii) Hence find an equation of the line AB, giving your answer in the form $x + qy = r$, where q and r are integers. *(2 marks)*

(iii) Find an equation of the line passing through C which is parallel to AB. *(2 marks)*

(c) Prove that angle ABC is a right angle. *(3 marks)*

2 The curve with equation $y = x^4 - 32x + 5$ has a single stationary point, M.

(a) Find $\dfrac{dy}{dx}$. *(3 marks)*

(b) Hence find the x-coordinate of M. *(3 marks)*

(c) (i) Find $\dfrac{d^2y}{dx^2}$. *(1 mark)*

(ii) Hence, or otherwise, determine whether M is a maximum or a minimum point. *(2 marks)*

(d) Determine whether the curve is increasing or decreasing at the point on the curve where $x = 0$. *(2 marks)*

3 (a) Express $5\sqrt{8} + \dfrac{6}{\sqrt{2}}$ in the form $n\sqrt{2}$, where n is an integer. *(3 marks)*

(b) Express $\dfrac{\sqrt{2} + 2}{3\sqrt{2} - 4}$ in the form $c\sqrt{2} + d$, where c and d are integers. *(4 marks)*

4 A circle with centre C has equation $x^2 + y^2 - 10y + 20 = 0$.

(a) By completing the square, express this equation in the form

$$x^2 + (y - b)^2 = k$$ *(2 marks)*

(b) Write down:

(i) the coordinates of C; *(1 mark)*

(ii) the radius of the circle, leaving your answer in surd form. *(1 mark)*

(c) A line has equation $y = 2x$.

(i) Show that the x-coordinate of any point of intersection of the line and the circle satisfies the equation $x^2 - 4x + 4 = 0$. *(2 marks)*

(ii) Hence show that the line is a tangent to the circle and find the coordinates of the point of contact, P. *(3 marks)*

(d) Prove that the point $Q(-1, 4)$ lies inside the circle. *(2 marks)*

5 (a) Factorise $9 - 8x - x^2$. *(2 marks)*

(b) Show that $25 - (x + 4)^2$ can be written as $9 - 8x - x^2$. *(1 mark)*

(c) A curve has equation $y = 9 - 8x - x^2$.

(i) Write down the equation of its line of symmetry. *(1 mark)*

(ii) Find the coordinates of its vertex. *(2 marks)*

(iii) Sketch the curve, indicating the values of the intercepts on the x-axis and the y-axis. *(3 marks)*

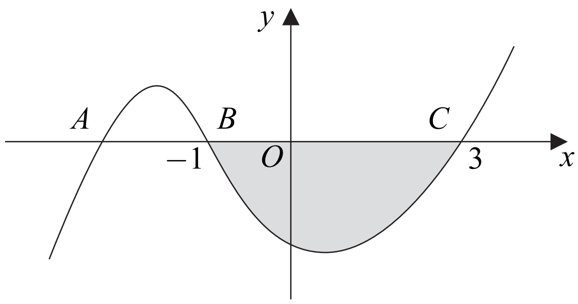
Turn over for the next question

6 (a) The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^3 - 7x - 6$.

(i) Use the Factor Theorem to show that $x + 1$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(ii) Express $\mathrm{p}(x) = x^3 - 7x - 6$ as the product of three linear factors. *(3 marks)*

(b) The curve with equation $y = x^3 - 7x - 6$ is sketched below.



The curve cuts the x-axis at the point A and the points $B(-1, 0)$ and $C(3, 0)$.

(i) State the coordinates of the point A. *(1 mark)*

(ii) Find $\int_{-1}^{3} (x^3 - 7x - 6)\, \mathrm{d}x$. *(5 marks)*

(iii) Hence find the area of the shaded region bounded by the curve $y = x^3 - 7x - 6$ and the x-axis between B and C. *(1 mark)*

(iv) Find the gradient of the curve $y = x^3 - 7x - 6$ at the point B. *(3 marks)*

(v) Hence find an equation of the normal to the curve at the point B. *(3 marks)*

7 The curve C has equation $y = x^2 + 7$. The line L has equation $y = k(3x + 1)$, where k is a constant.

(a) Show that the x-coordinates of any points of intersection of the line L with the curve C satisfy the equation

$$x^2 - 3kx + 7 - k = 0$$ *(1 mark)*

(b) The curve C and the line L intersect in two distinct points. Show that

$$9k^2 + 4k - 28 > 0$$ *(3 marks)*

(c) Solve the inequality $9k^2 + 4k - 28 > 0$. *(4 marks)*

END OF QUESTIONS

General Certificate of Education
June 2008
Advanced Subsidiary Examination

MATHEMATICS **MPC1**
Unit Pure Core 1

Thursday 15 May 2008 9.00 am to 10.30 am

For this paper you must have: • an 8-page answer book • the blue AQA booklet of formulae and statistical tables. You must **not** use a calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The straight line L has equation $y = 3x - 1$ and the curve C has equation

$$y = (x + 3)(x - 1)$$

(a) Sketch on the same axes the line L and the curve C, showing the values of the intercepts on the x-axis and the y-axis. *(5 marks)*

(b) Show that the x-coordinates of the points of intersection of L and C satisfy the equation $x^2 - x - 2 = 0$. *(2 marks)*

(c) Hence find the coordinates of the points of intersection of L and C. *(4 marks)*

2 It is given that $x = \sqrt{3}$ and $y = \sqrt{12}$.

Find, in the simplest form, the value of:

(a) xy ; *(1 mark)*

(b) $\dfrac{y}{x}$; *(2 marks)*

(c) $(x + y)^2$. *(3 marks)*

3 Two numbers, x and y, are such that $3x + y = 9$, where $x \geqslant 0$ and $y \geqslant 0$.

It is given that $V = xy^2$.

(a) Show that $V = 81x - 54x^2 + 9x^3$. *(2 marks)*

(b) (i) Show that $\dfrac{dV}{dx} = k(x^2 - 4x + 3)$, and state the value of the integer k. *(4 marks)*

(ii) Hence find the two values of x for which $\dfrac{dV}{dx} = 0$. *(2 marks)*

(c) Find $\dfrac{d^2V}{dx^2}$. *(2 marks)*

(d) (i) Find the value of $\dfrac{d^2V}{dx^2}$ for each of the two values of x found in part (b)(ii). *(1 mark)*

(ii) Hence determine the value of x for which V has a maximum value. *(1 mark)*

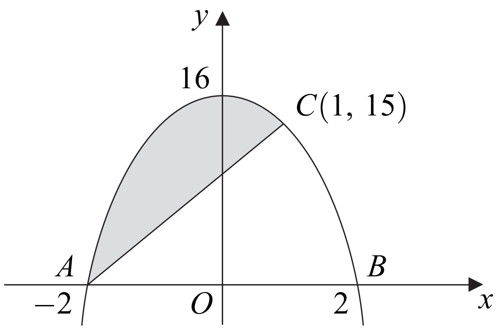
(iii) Find the maximum value of V. *(1 mark)*

4 (a) Express $x^2 - 3x + 4$ in the form $(x - p)^2 + q$, where p and q are rational numbers. *(2 marks)*

(b) Hence write down the minimum value of the expression $x^2 - 3x + 4$. *(1 mark)*

(c) Describe the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 - 3x + 4$. *(3 marks)*

5 The curve with equation $y = 16 - x^4$ is sketched below.



The points $A(-2, 0)$, $B(2, 0)$ and $C(1, 15)$ lie on the curve.

(a) Find an equation of the straight line AC. *(3 marks)*

(b) (i) Find $\int_{-2}^{1} (16 - x^4)\, dx$. *(5 marks)*

(ii) Hence calculate the area of the shaded region bounded by the curve and the line AC. *(3 marks)*

6 The polynomial $p(x)$ is given by $p(x) = x^3 + x^2 - 8x - 12$.

(a) Use the Remainder Theorem to find the remainder when $p(x)$ is divided by $x - 1$. *(2 marks)*

(b) (i) Use the Factor Theorem to show that $x + 2$ is a factor of $p(x)$. *(2 marks)*

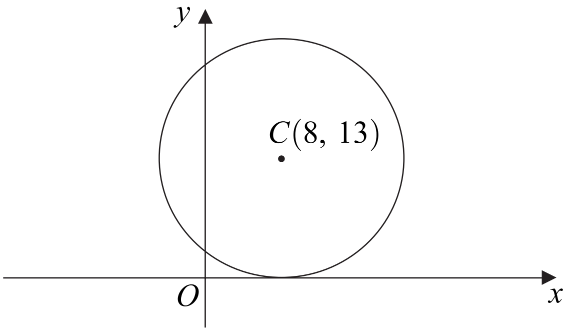
(ii) Express $p(x)$ as the product of linear factors. *(3 marks)*

(c) (i) The curve with equation $y = x^3 + x^2 - 8x - 12$ passes through the point $(0, k)$. State the value of k. *(1 mark)*

(ii) Sketch the graph of $y = x^3 + x^2 - 8x - 12$, indicating the values of x where the curve touches or crosses the x-axis. *(3 marks)*

Turn over for the next question

7 The circle S has centre $C(8, 13)$ and touches the x-axis, as shown in the diagram.



(a) Write down an equation for S, giving your answer in the form

$$(x - a)^2 + (y - b)^2 = r^2$$ *(2 marks)*

(b) The point P with coordinates $(3, 1)$ lies on the circle.

(i) Find the gradient of the straight line passing through P and C. *(1 mark)*

(ii) Hence find an equation of the tangent to the circle S at the point P, giving your answer in the form $ax + by = c$, where a, b and c are integers. *(4 marks)*

(iii) The point Q also lies on the circle S, and the length of PQ is 10. Calculate the shortest distance from C to the chord PQ. *(3 marks)*

8 The quadratic equation $(k + 1)x^2 + 4kx + 9 = 0$ has real roots.

(a) Show that $4k^2 - 9k - 9 \geqslant 0$. *(3 marks)*

(b) Hence find the possible values of k. *(4 marks)*

END OF QUESTIONS

General Certificate of Education
January 2009
Advanced Subsidiary Examination

MATHEMATICS **MPC1**
Unit Pure Core 1

Friday 9 January 2009 9.00 am to 10.30 am

For this paper you must have: • an 8-page answer book • the blue AQA booklet of formulae and statistical tables. You must **not** use a calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The points A and B have coordinates $(1, 6)$ and $(5, -2)$ respectively. The mid-point of AB is M.

(a) Find the coordinates of M. *(2 marks)*

(b) Find the gradient of AB, giving your answer in its simplest form. *(2 marks)*

(c) A straight line passes through M and is perpendicular to AB.

(i) Show that this line has equation $x - 2y + 1 = 0$. *(3 marks)*

(ii) Given that this line passes through the point $(k, k + 5)$, find the value of the constant k. *(2 marks)*

2 (a) Factorise $2x^2 - 5x + 3$. *(1 mark)*

(b) Hence, or otherwise, solve the inequality $2x^2 - 5x + 3 < 0$. *(3 marks)*

3 (a) Express $\dfrac{7 + \sqrt{5}}{3 + \sqrt{5}}$ in the form $m + n\sqrt{5}$, where m and n are integers. *(4 marks)*

(b) Express $\sqrt{45} + \dfrac{20}{\sqrt{5}}$ in the form $k\sqrt{5}$, where k is an integer. *(3 marks)*

4 (a) (i) Express $x^2 + 2x + 5$ in the form $(x + p)^2 + q$, where p and q are integers. *(2 marks)*

(ii) Hence show that $x^2 + 2x + 5$ is always positive. *(1 mark)*

(b) A curve has equation $y = x^2 + 2x + 5$.

(i) Write down the coordinates of the minimum point of the curve. *(2 marks)*

(ii) Sketch the curve, showing the value of the intercept on the y-axis. *(2 marks)*

(c) Describe the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 2x + 5$. *(3 marks)*

5 A model car moves so that its distance, x centimetres, from a fixed point O after time t seconds is given by

$$x = \frac{1}{2}t^4 - 20t^2 + 66t, \qquad 0 \leqslant t \leqslant 4$$

(a) Find:

(i) $\dfrac{dx}{dt}$; *(3 marks)*

(ii) $\dfrac{d^2x}{dt^2}$. *(2 marks)*

(b) Verify that x has a stationary value when $t = 3$, and determine whether this stationary value is a maximum value or a minimum value. *(4 marks)*

(c) Find the rate of change of x with respect to t when $t = 1$. *(2 marks)*

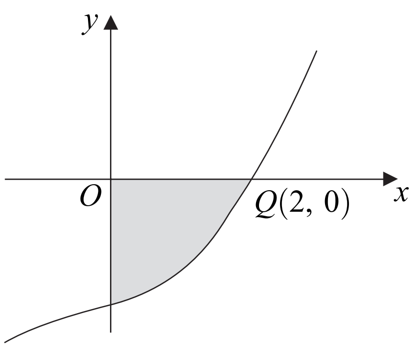
(d) Determine whether the distance of the car from O is increasing or decreasing at the instant when $t = 2$. *(2 marks)*

6 (a) The polynomial $p(x)$ is given by $p(x) = x^3 + x - 10$.

(i) Use the Factor Theorem to show that $x - 2$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x)$ in the form $(x - 2)(x^2 + ax + b)$, where a and b are constants. *(2 marks)*

(b) The curve C with equation $y = x^3 + x - 10$, sketched below, crosses the x-axis at the point $Q(2, 0)$.



(i) Find the gradient of the curve C at the point Q. *(4 marks)*

(ii) Hence find an equation of the tangent to the curve C at the point Q. *(2 marks)*

(iii) Find $\int (x^3 + x - 10)\, dx$. *(3 marks)*

(iv) Hence find the area of the shaded region bounded by the curve C and the coordinate axes. *(2 marks)*

Turn over for the next question

Turn over ▶

7 A circle with centre C has equation $x^2 + y^2 - 6x + 10y + 9 = 0$.

(a) Express this equation in the form

$$(x - a)^2 + (y - b)^2 = r^2$$ *(3 marks)*

(b) Write down:

(i) the coordinates of C;

(ii) the radius of the circle. *(2 marks)*

(c) The point D has coordinates $(7, \ -2)$.

(i) Verify that the point D lies on the circle. *(1 mark)*

(ii) Find an equation of the normal to the circle at the point D, giving your answer in the form $mx + ny = p$, where m, n and p are integers. *(3 marks)*

(d) (i) A line has equation $y = kx$. Show that the x-coordinates of any points of intersection of the line and the circle satisfy the equation

$$(k^2 + 1)x^2 + 2(5k - 3)x + 9 = 0$$ *(2 marks)*

(ii) Find the values of k for which the equation

$$(k^2 + 1)x^2 + 2(5k - 3)x + 9 = 0$$

has equal roots. *(5 marks)*

(iii) Describe the geometrical relationship between the line and the circle when k takes either of the values found in part (d)(ii). *(1 mark)*

END OF QUESTIONS

General Certificate of Education
June 2009
Advanced Subsidiary Examination

MATHEMATICS **MPC1**
Unit Pure Core 1

Wednesday 20 May 2009 1.30 pm to 3.00 pm

For this paper you must have: • an 8-page answer book • the blue AQA booklet of formulae and statistical tables. You must **not** use a calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The line AB has equation $3x + 5y = 11$.

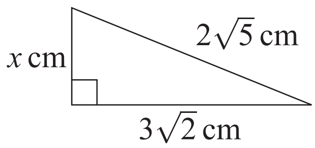
(a) (i) Find the gradient of AB. *(2 marks)*

(ii) The point A has coordinates $(2, 1)$. Find an equation of the line which passes through the point A and which is perpendicular to AB. *(3 marks)*

(b) The line AB intersects the line with equation $2x + 3y = 8$ at the point C. Find the coordinates of C. *(3 marks)*

2 (a) Express $\dfrac{5 + \sqrt{7}}{3 - \sqrt{7}}$ in the form $m + n\sqrt{7}$, where m and n are integers. *(4 marks)*

(b) The diagram shows a right-angled triangle.



The hypotenuse has length $2\sqrt{5}$ cm. The other two sides have lengths $3\sqrt{2}$ cm and x cm. Find the value of x. *(3 marks)*

3 The curve with equation $y = x^5 + 20x^2 - 8$ passes through the point P, where $x = -2$.

(a) Find $\dfrac{dy}{dx}$. *(3 marks)*

(b) Verify that the point P is a stationary point of the curve. *(2 marks)*

(c) (i) Find the value of $\dfrac{d^2y}{dx^2}$ at the point P. *(3 marks)*

(ii) Hence, or otherwise, determine whether P is a maximum point or a minimum point. *(1 mark)*

(d) Find an equation of the tangent to the curve at the point where $x = 1$. *(4 marks)*

4 (a) The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^3 - x + 6$.

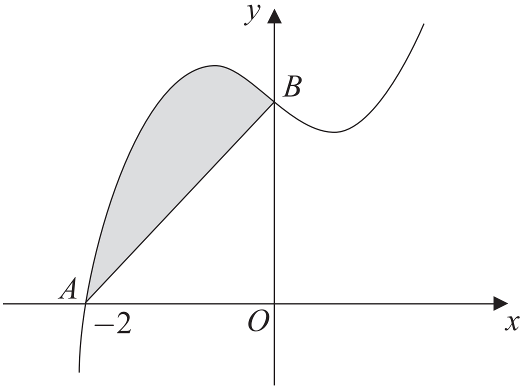
(i) Find the remainder when $\mathrm{p}(x)$ is divided by $x - 3$. *(2 marks)*

(ii) Use the Factor Theorem to show that $x + 2$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(iii) Express $\mathrm{p}(x) = x^3 - x + 6$ in the form $(x + 2)(x^2 + bx + c)$, where b and c are integers. *(2 marks)*

(iv) The equation $\mathrm{p}(x) = 0$ has one root equal to -2. Show that the equation has no other real roots. *(2 marks)*

(b) The curve with equation $y = x^3 - x + 6$ is sketched below.



The curve cuts the x-axis at the point $A\,(-2,\ 0)$ and the y-axis at the point B.

(i) State the y-coordinate of the point B. *(1 mark)*

(ii) Find $\int_{-2}^{0} (x^3 - x + 6)\,\mathrm{d}x$. *(5 marks)*

(iii) Hence find the area of the shaded region bounded by the curve $y = x^3 - x + 6$ and the line AB. *(3 marks)*

Turn over for the next question

5 A circle with centre C has equation

$$(x-5)^2+(y+12)^2=169$$

(a) Write down:

(i) the coordinates of C; *(1 mark)*

(ii) the radius of the circle. *(1 mark)*

(b) (i) Verify that the circle passes through the origin O. *(1 mark)*

(ii) Given that the circle also passes through the points $(10, 0)$ and $(0, p)$, sketch the circle and find the value of p. *(3 marks)*

(c) The point $A(-7, -7)$ lies on the circle.

(i) Find the gradient of AC. *(2 marks)*

(ii) Hence find an equation of the tangent to the circle at the point A, giving your answer in the form $ax+by+c=0$, where a, b and c are integers. *(3 marks)*

6 (a) (i) Express $x^2-8x+17$ in the form $(x-p)^2+q$, where p and q are integers. *(2 marks)*

(ii) Hence write down the minimum value of $x^2-8x+17$. *(1 mark)*

(iii) State the value of x for which the minimum value of $x^2-8x+17$ occurs. *(1 mark)*

(b) The point A has coordinates $(5, 4)$ and the point B has coordinates $(x, 7-x)$.

(i) Expand $(x-5)^2$. *(1 mark)*

(ii) Show that $AB^2=2(x^2-8x+17)$. *(3 marks)*

(iii) Use your results from part (a) to find the minimum value of the distance AB as x varies. *(2 marks)*

7 The curve C has equation $y = k(x^2 + 3)$, where k is a constant.

The line L has equation $y = 2x + 2$.

(a) Show that the x-coordinates of any points of intersection of the curve C with the line L satisfy the equation

$$kx^2 - 2x + 3k - 2 = 0$$ *(1 mark)*

(b) The curve C and the line L intersect in two distinct points.

(i) Show that

$$3k^2 - 2k - 1 < 0$$ *(4 marks)*

(ii) Hence find the possible values of k. *(4 marks)*

END OF QUESTIONS

General Certificate of Education
Advanced Subsidiary Examination
January 2010

Mathematics MPC1

Unit Pure Core 1

Monday 11 January 2010 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The polynomial $p(x)$ is given by $p(x) = x^3 - 13x - 12$.

(a) Use the Factor Theorem to show that $x + 3$ is a factor of $p(x)$. *(2 marks)*

(b) Express $p(x)$ as the product of three linear factors. *(3 marks)*

2 The triangle ABC has vertices $A(1, 3)$, $B(3, 7)$ and $C(-1, 9)$.

(a) (i) Find the gradient of AB. *(2 marks)*

(ii) Hence show that angle ABC is a right angle. *(2 marks)*

(b) (i) Find the coordinates of M, the mid-point of AC. *(2 marks)*

(ii) Show that the lengths of AB and BC are equal. *(3 marks)*

(iii) Hence find an equation of the line of symmetry of the triangle ABC. *(3 marks)*

3 The depth of water, y metres, in a tank after time t hours is given by

$$y = \frac{1}{8}t^4 - 2t^2 + 4t, \qquad 0 \leqslant t \leqslant 4$$

(a) Find:

(i) $\dfrac{dy}{dt}$; *(3 marks)*

(ii) $\dfrac{d^2y}{dt^2}$. *(2 marks)*

(b) Verify that y has a stationary value when $t = 2$ and determine whether it is a maximum value or a minimum value. *(4 marks)*

(c) (i) Find the rate of change of the depth of water, in metres per hour, when $t = 1$. *(2 marks)*

(ii) Hence determine, with a reason, whether the depth of water is increasing or decreasing when $t = 1$. *(1 mark)*

4 (a) Show that $\dfrac{\sqrt{50}+\sqrt{18}}{\sqrt{8}}$ is an integer and find its value. *(3 marks)*

(b) Express $\dfrac{2\sqrt{7}-1}{2\sqrt{7}+5}$ in the form $m+n\sqrt{7}$, where m and n are integers. *(4 marks)*

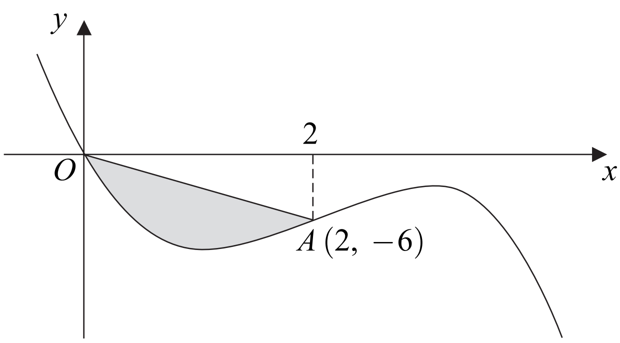
5 (a) Express $(x-5)(x-3)+2$ in the form $(x-p)^2+q$, where p and q are integers. *(3 marks)*

(b) (i) Sketch the graph of $y=(x-5)(x-3)+2$, stating the coordinates of the minimum point and the point where the graph crosses the y-axis. *(3 marks)*

(ii) Write down an equation of the tangent to the graph of $y=(x-5)(x-3)+2$ at its vertex. *(2 marks)*

(c) Describe the geometrical transformation that maps the graph of $y=x^2$ onto the graph of $y=(x-5)(x-3)+2$. *(3 marks)*

6 The curve with equation $y=12x^2-19x-2x^3$ is sketched below.



The curve crosses the x-axis at the origin O, and the point $A\,(2,\ -6)$ lies on the curve.

(a) (i) Find the gradient of the curve with equation $y=12x^2-19x-2x^3$ at the point A. *(4 marks)*

(ii) Hence find the equation of the normal to the curve at the point A, giving your answer in the form $x+py+q=0$, where p and q are integers. *(3 marks)*

(b) (i) Find the value of $\displaystyle\int_0^2 (12x^2-19x-2x^3)\,\mathrm{d}x$. *(5 marks)*

(ii) Hence determine the area of the shaded region bounded by the curve and the line OA. *(3 marks)*

Turn over for the next question

7 A circle with centre C has equation $x^2 + y^2 - 4x + 12y + 15 = 0$.

(a) Find:

(i) the coordinates of C; *(2 marks)*

(ii) the radius of the circle. *(2 marks)*

(b) Explain why the circle lies entirely below the x-axis. *(2 marks)*

(c) The point P with coordinates $(5, k)$ lies outside the circle.

(i) Show that $PC^2 = k^2 + 12k + 45$. *(2 marks)*

(ii) Hence show that $k^2 + 12k + 20 > 0$. *(1 mark)*

(iii) Find the possible values of k. *(4 marks)*

END OF QUESTIONS

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

General Certificate of Education
Advanced Subsidiary Examination
June 2010

Mathematics MPC1

Unit Pure Core 1

Monday 24 May 2010 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

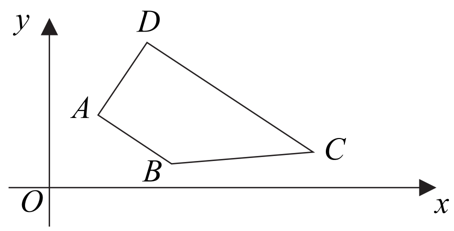
Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions in the spaces provided.

1 The trapezium $ABCD$ is shown below.



The line AB has equation $2x + 3y = 14$ and DC is parallel to AB.

(a) Find the gradient of AB. *(2 marks)*

(b) The point D has coordinates $(3, 7)$.

(i) Find an equation of the line DC. *(2 marks)*

(ii) The angle BAD is a right angle. Find an equation of the line AD, giving your answer in the form $mx + ny + p = 0$, where m, n and p are integers. *(4 marks)*

(c) The line BC has equation $5y - x = 6$. Find the coordinates of B. *(3 marks)*

QUESTION PART REFERENCE

Do not write outside the box

2 (a) Express $\left(3-\sqrt{5}\right)^2$ in the form $m+n\sqrt{5}$, where m and n are integers. *(2 marks)*

(b) Hence express $\dfrac{\left(3-\sqrt{5}\right)^2}{1+\sqrt{5}}$ in the form $p+q\sqrt{5}$, where p and q are integers. *(4 marks)*

QUESTION PART REFERENCE

Do not write outside the box

3 The polynomial $\mathrm{p}(x)$ is given by

$$\mathrm{p}(x) = x^3 + 7x^2 + 7x - 15$$

(a) (i) Use the Factor Theorem to show that $x + 3$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(ii) Express $\mathrm{p}(x)$ as the product of three linear factors. *(3 marks)*

(b) Use the Remainder Theorem to find the remainder when $\mathrm{p}(x)$ is divided by $x - 2$. *(2 marks)*

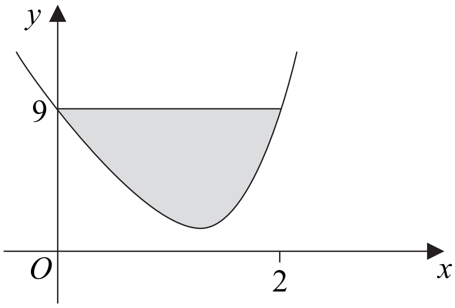
(c) (i) Verify that $\mathrm{p}(-1) < \mathrm{p}(0)$. *(1 mark)*

(ii) Sketch the curve with equation $y = x^3 + 7x^2 + 7x - 15$, indicating the values where the curve crosses the coordinate axes. *(4 marks)*

QUESTION PART REFERENCE

Do not write outside the box

4 The curve with equation $y = x^4 - 8x + 9$ is sketched below.



The point (2, 9) lies on the curve.

(a) (i) Find $\int_0^2 (x^4 - 8x + 9)\,\mathrm{d}x$. *(5 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the line $y = 9$. *(2 marks)*

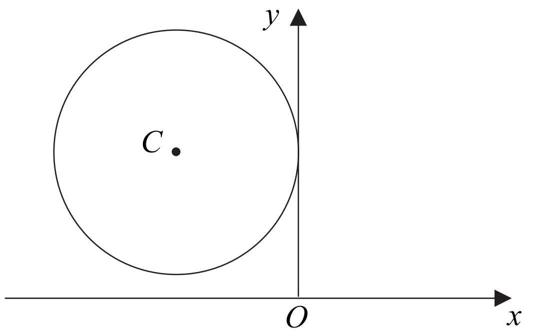
(b) The point $A(1, 2)$ lies on the curve with equation $y = x^4 - 8x + 9$.

(i) Find the gradient of the curve at the point A. *(4 marks)*

(ii) Hence find an equation of the tangent to the curve at the point A. *(1 mark)*

QUESTION PART REFERENCE

5 A circle with centre $C(-5, 6)$ touches the y-axis, as shown in the diagram.



(a) Find the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = r^2$$

(3 marks)

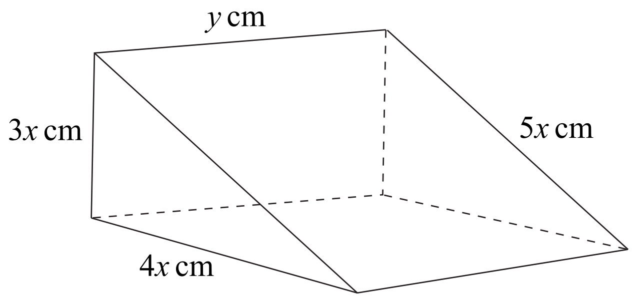
(b) (i) Verify that the point $P(-2, 2)$ lies on the circle. *(1 mark)*

(ii) Find an equation of the normal to the circle at the point P. *(3 marks)*

(iii) The mid-point of PC is M. Determine whether the point P is closer to the point M or to the origin O. *(4 marks)*

QUESTION PART REFERENCE

6 The diagram shows a block of wood in the shape of a prism with triangular cross-section. The end faces are right-angled triangles with sides of lengths $3x$ cm, $4x$ cm and $5x$ cm, and the length of the prism is y cm, as shown in the diagram.



The total surface area of the five faces is 144 cm^2.

(a) (i) Show that $xy + x^2 = 12$. *(3 marks)*

(ii) Hence show that the volume of the block, $V\text{ cm}^3$, is given by

$$V = 72x - 6x^3$$ *(2 marks)*

(b) (i) Find $\dfrac{dV}{dx}$. *(2 marks)*

(ii) Show that V has a stationary value when $x = 2$. *(2 marks)*

(c) Find $\dfrac{d^2V}{dx^2}$ and hence determine whether V has a maximum value or a minimum value when $x = 2$. *(2 marks)*

QUESTION PART REFERENCE

7 (a) (i) Express $2x^2 - 20x + 53$ in the form $2(x-p)^2 + q$, where p and q are integers. *(2 marks)*

(ii) Use your result from part **(a)(i)** to explain why the equation $2x^2 - 20x + 53 = 0$ has no real roots. *(2 marks)*

(b) The quadratic equation $(2k-1)x^2 + (k+1)x + k = 0$ has real roots.

(i) Show that $7k^2 - 6k - 1 \leqslant 0$. *(4 marks)*

(ii) Hence find the possible values of k. *(4 marks)*

QUESTION PART REFERENCE

General Certificate of Education
Advanced Subsidiary Examination
January 2011

Mathematics MPC1

Unit Pure Core 1

Monday 10 January 2011 9.00 am to 10.30 am

For this paper you must have: • the blue AQA booklet of formulae and statistical tables. You must **not** use a calculator.	

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The curve with equation $y = 13 + 18x + 3x^2 - 4x^3$ passes through the point P where $x = -1$.

(a) Find $\dfrac{dy}{dx}$. *(3 marks)*

(b) Show that the point P is a stationary point of the curve and find the other value of x where the curve has a stationary point. *(3 marks)*

(c) (i) Find the value of $\dfrac{d^2y}{dx^2}$ at the point P. *(3 marks)*

(ii) Hence, or otherwise, determine whether P is a maximum point or a minimum point. *(1 mark)*

---

2 (a) Simplify $\left(3\sqrt{3}\right)^2$. *(1 mark)*

(b) Express $\dfrac{4\sqrt{3} + 3\sqrt{7}}{3\sqrt{3} + \sqrt{7}}$ in the form $\dfrac{m + \sqrt{21}}{n}$, where m and n are integers. *(4 marks)*

---

3 The line AB has equation $3x + 2y = 7$. The point C has coordinates $(2, -7)$.

(a) (i) Find the gradient of AB. *(2 marks)*

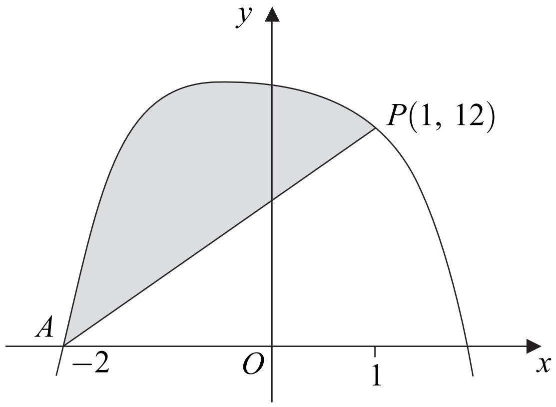
(ii) The line which passes through C and which is parallel to AB crosses the y-axis at the point D. Find the y-coordinate of D. *(3 marks)*

(b) The line with equation $y = 1 - 4x$ intersects the line AB at the point A. Find the coordinates of A. *(3 marks)*

(c) The point E has coordinates $(5, k)$. Given that CE has length 5, find the two possible values of the constant k. *(3 marks)*

---

4 The curve sketched below passes through the point $A(-2, 0)$.



The curve has equation $y = 14 - x - x^4$ and the point $P(1, 12)$ lies on the curve.

(a) (i) Find the gradient of the curve at the point P. *(3 marks)*

(ii) Hence find the equation of the tangent to the curve at the point P, giving your answer in the form $y = mx + c$. *(2 marks)*

(b) (i) Find $\int_{-2}^{1} (14 - x - x^4)\,dx$. *(5 marks)*

(ii) Hence find the area of the shaded region bounded by the curve $y = 14 - x - x^4$ and the line AP. *(2 marks)*

---

5 (a) (i) Sketch the curve with equation $y = x(x-2)^2$. *(3 marks)*

(ii) Show that the equation $x(x-2)^2 = 3$ can be expressed as

$$x^3 - 4x^2 + 4x - 3 = 0$$ *(1 mark)*

(b) The polynomial $p(x)$ is given by $p(x) = x^3 - 4x^2 + 4x - 3$.

(i) Find the remainder when $p(x)$ is divided by $x + 1$. *(2 marks)*

(ii) Use the Factor Theorem to show that $x - 3$ is a factor of $p(x)$. *(2 marks)*

(iii) Express $p(x)$ in the form $(x-3)(x^2 + bx + c)$, where b and c are integers. *(2 marks)*

(c) Hence show that the equation $x(x-2)^2 = 3$ has only one real root and state the value of this root. *(3 marks)*

Turn over ▶

6 A circle has centre $C(-3,\ 1)$ and radius $\sqrt{13}$.

(a) (i) Express the equation of the circle in the form

$$(x-a)^2+(y-b)^2=k$$ *(2 marks)*

(ii) Hence find the equation of the circle in the form

$$x^2+y^2+mx+ny+p=0$$

where m, n and p are integers. *(3 marks)*

(b) The circle cuts the y-axis at the points A and B. Find the distance AB. *(3 marks)*

(c) (i) Verify that the point $D(-5,\ -2)$ lies on the circle. *(1 mark)*

(ii) Find the gradient of CD. *(2 marks)*

(iii) Hence find an equation of the tangent to the circle at the point D. *(2 marks)*

---

7 (a) (i) Express $4-10x-x^2$ in the form $p-(x+q)^2$. *(2 marks)*

(ii) Hence write down the equation of the line of symmetry of the curve with equation $y=4-10x-x^2$. *(1 mark)*

(b) The curve C has equation $y=4-10x-x^2$ and the line L has equation $y=k(4x-13)$, where k is a constant.

(i) Show that the x-coordinates of any points of intersection of the curve C with the line L satisfy the equation

$$x^2+2(2k+5)x-(13k+4)=0$$ *(1 mark)*

(ii) Given that the curve C and the line L intersect in two distinct points, show that

$$4k^2+33k+29>0$$ *(3 marks)*

(iii) Solve the inequality $4k^2+33k+29>0$. *(4 marks)*

General Certificate of Education
Advanced Subsidiary Examination
June 2011

Mathematics MPC1

Unit Pure Core 1

Wednesday 18 May 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The line AB has equation $7x + 3y = 13$.

(a) Find the gradient of AB. *(2 marks)*

(b) The point C has coordinates $(-1, 3)$.

(i) Find an equation of the line which passes through the point C and which is parallel to AB. *(2 marks)*

(ii) The point $(1\frac{1}{2}, -1)$ is the mid-point of AC. Find the coordinates of the point A. *(2 marks)*

(c) The line AB intersects the line with equation $3x + 2y = 12$ at the point B. Find the coordinates of B. *(3 marks)*

---

2 (a) (i) Express $\sqrt{48}$ in the form $k\sqrt{3}$, where k is an integer. *(1 mark)*

(ii) Simplify $\dfrac{\sqrt{48} + 2\sqrt{27}}{\sqrt{12}}$, giving your answer as an integer. *(3 marks)*

(b) Express $\dfrac{1 - 5\sqrt{5}}{3 + \sqrt{5}}$ in the form $m + n\sqrt{5}$, where m and n are integers. *(4 marks)*

---

3 The volume, $V\,\text{m}^3$, of water in a tank after time t seconds is given by

$$V = \frac{t^3}{4} - 3t + 5$$

(a) Find $\dfrac{dV}{dt}$. *(2 marks)*

(b) (i) Find the rate of change of volume, in $\text{m}^3\,\text{s}^{-1}$, when $t = 1$. *(2 marks)*

(ii) Hence determine, with a reason, whether the volume is increasing or decreasing when $t = 1$. *(1 mark)*

(c) (i) Find the positive value of t for which V has a stationary value. *(3 marks)*

(ii) Find $\dfrac{d^2V}{dt^2}$, and hence determine whether this stationary value is a maximum value or a minimum value. *(3 marks)*

4 (a) Express $x^2 + 5x + 7$ in the form $(x + p)^2 + q$, where p and q are rational numbers. *(3 marks)*

(b) A curve has equation $y = x^2 + 5x + 7$.

(i) Find the coordinates of the vertex of the curve. *(2 marks)*

(ii) State the equation of the line of symmetry of the curve. *(1 mark)*

(iii) Sketch the curve, stating the value of the intercept on the y-axis. *(3 marks)*

(c) Describe the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 5x + 7$. *(3 marks)*

---

5 The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^3 - 2x^2 + 3$.

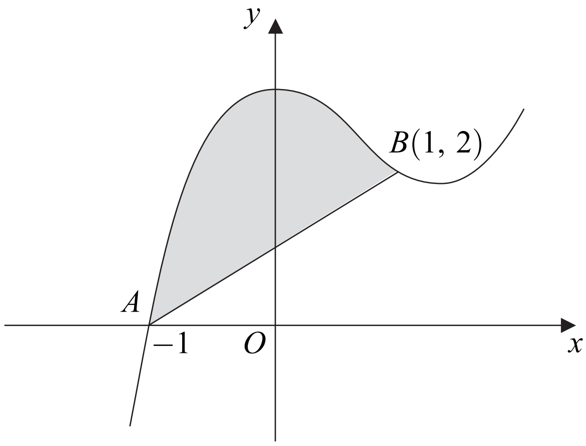
(a) Use the Remainder Theorem to find the remainder when $\mathrm{p}(x)$ is divided by $x - 3$. *(2 marks)*

(b) Use the Factor Theorem to show that $x + 1$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(c) (i) Express $\mathrm{p}(x) = x^3 - 2x^2 + 3$ in the form $(x + 1)(x^2 + bx + c)$, where b and c are integers. *(2 marks)*

(ii) Hence show that the equation $\mathrm{p}(x) = 0$ has exactly one real root. *(2 marks)*

6 The curve with equation $y = x^3 - 2x^2 + 3$ is sketched below.



The curve cuts the x-axis at the point $A(-1, 0)$ and passes through the point $B(1, 2)$.

(a) Find $\int_{-1}^{1} (x^3 - 2x^2 + 3)\,\mathrm{d}x$. *(5 marks)*

(b) Hence find the area of the shaded region bounded by the curve $y = x^3 - 2x^2 + 3$ and the line AB. *(3 marks)*

---

7 Solve each of the following inequalities:

(a) $2(4 - 3x) > 5 - 4(x + 2)$; *(2 marks)*

(b) $2x^2 + 5x \geqslant 12$. *(4 marks)*

8 A circle has centre $C(3, -8)$ and radius 10.

(a) Express the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = k$$

(2 marks)

(b) Find the x-coordinates of the points where the circle crosses the x-axis. *(3 marks)*

(c) The tangent to the circle at the point A has gradient $\frac{5}{2}$. Find an equation of the line CA, giving your answer in the form $rx + sy + t = 0$, where r, s and t are integers. *(3 marks)*

(d) The line with equation $y = 2x + 1$ intersects the circle.

(i) Show that the x-coordinates of the points of intersection satisfy the equation

$$x^2 + 6x - 2 = 0$$

(3 marks)

(ii) Hence show that the x-coordinates of the points of intersection are of the form $m \pm \sqrt{n}$, where m and n are integers. *(2 marks)*

END OF QUESTIONS

General Certificate of Education
Advanced Subsidiary Examination
January 2012

Mathematics MPC1

Unit Pure Core 1

Friday 13 January 2012 9.00 am to 10.30 am

For this paper you must have: • the blue AQA booklet of formulae and statistical tables. You must **not** use a calculator.	

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The point A has coordinates $(6, -4)$ and the point B has coordinates $(-2, 7)$.

(a) Given that the point O has coordinates $(0, 0)$, show that the length of OA is less than the length of OB. *(3 marks)*

(b) (i) Find the gradient of AB. *(2 marks)*

(ii) Find an equation of the line AB in the form $px + qy = r$, where p, q and r are integers. *(3 marks)*

(c) The point C has coordinates $(k, 0)$. The line AC is perpendicular to the line AB. Find the value of the constant k. *(3 marks)*

---

2 (a) Factorise $x^2 - 4x - 12$. *(1 mark)*

(b) Sketch the graph with equation $y = x^2 - 4x - 12$, stating the values where the curve crosses the coordinate axes. *(4 marks)*

(c) (i) Express $x^2 - 4x - 12$ in the form $(x - p)^2 - q$, where p and q are positive integers. *(2 marks)*

(ii) Hence find the minimum value of $x^2 - 4x - 12$. *(1 mark)*

(d) The curve with equation $y = x^2 - 4x - 12$ is translated by the vector $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$.
Find an equation of the new curve. You need not simplify your answer. *(2 marks)*

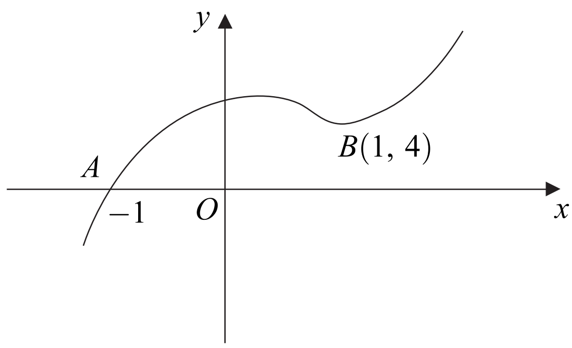
---

3 (a) (i) Simplify $(3\sqrt{2})^2$. *(1 mark)*

(ii) Show that $(3\sqrt{2} - 1)^2 + (3 + \sqrt{2})^2$ is an integer and find its value. *(4 marks)*

(b) Express $\dfrac{4\sqrt{5} - 7\sqrt{2}}{2\sqrt{5} + \sqrt{2}}$ in the form $m - \sqrt{n}$, where m and n are integers. *(4 marks)*

4 The curve with equation $y = x^5 - 3x^2 + x + 5$ is sketched below. The point O is at the origin and the curve passes through the points $A(-1, 0)$ and $B(1, 4)$.



(a) Given that $y = x^5 - 3x^2 + x + 5$, find:

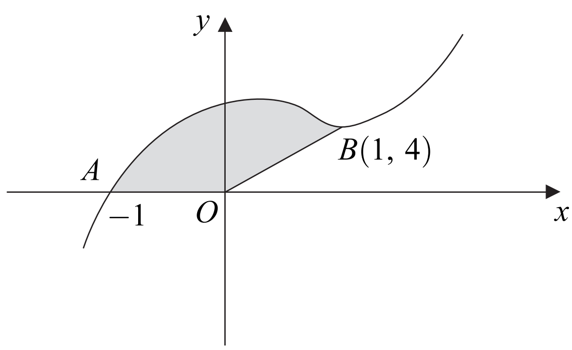
(i) $\dfrac{dy}{dx}$; *(3 marks)*

(ii) $\dfrac{d^2y}{dx^2}$. *(1 mark)*

(b) Find an equation of the tangent to the curve at the point $A(-1, 0)$. *(2 marks)*

(c) Verify that the point B, where $x = 1$, is a minimum point of the curve. *(3 marks)*

(d) The curve with equation $y = x^5 - 3x^2 + x + 5$ is sketched below. The point O is at the origin and the curve passes through the points $A(-1, 0)$ and $B(1, 4)$.



(i) Find $\displaystyle\int_{-1}^{1} (x^5 - 3x^2 + x + 5)\,dx$. *(5 marks)*

(ii) Hence find the area of the shaded region bounded by the curve between A and B and the line segments AO and OB. *(2 marks)*

Turn over ▶

5 The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^3 + cx^2 + dx - 12$, where c and d are constants.

(a) When $\mathrm{p}(x)$ is divided by $x + 2$, the remainder is -150.

Show that $2c - d + 65 = 0$. *(3 marks)*

(b) Given that $x - 3$ is a factor of $\mathrm{p}(x)$, find another equation involving c and d. *(2 marks)*

(c) By solving these two equations, find the value of c and the value of d. *(3 marks)*

---

6 A rectangular garden is to have width x metres and length $(x + 4)$ metres.

(a) The perimeter of the garden needs to be greater than 30 metres.

Show that $2x > 11$. *(1 mark)*

(b) The area of the garden needs to be less than 96 square metres.

Show that $x^2 + 4x - 96 < 0$. *(1 mark)*

(c) Solve the inequality $x^2 + 4x - 96 < 0$. *(4 marks)*

(d) Hence determine the possible values of the width of the garden. *(1 mark)*

7 A circle with centre C has equation $x^2 + y^2 + 14x - 10y + 49 = 0$.

(a) Express this equation in the form

$$(x - a)^2 + (y - b)^2 = r^2$$ *(3 marks)*

(b) Write down:

(i) the coordinates of C;

(ii) the radius of the circle. *(2 marks)*

(c) Sketch the circle. *(2 marks)*

(d) A line has equation $y = kx + 6$, where k is a constant.

(i) Show that the x-coordinates of any points of intersection of the line and the circle satisfy the equation $(k^2 + 1)x^2 + 2(k + 7)x + 25 = 0$. *(2 marks)*

(ii) The equation $(k^2 + 1)x^2 + 2(k + 7)x + 25 = 0$ has equal roots. Show that

$$12k^2 - 7k - 12 = 0$$ *(3 marks)*

(iii) Hence find the values of k for which the line is a tangent to the circle. *(2 marks)*

General Certificate of Education
Advanced Subsidiary Examination
June 2012

Mathematics MPC1

Unit Pure Core 1

Wednesday 16 May 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 Express $\dfrac{5\sqrt{3}-6}{2\sqrt{3}+3}$ in the form $m+n\sqrt{3}$, where m and n are integers. *(4 marks)*

---

2 The line AB has equation $4x-3y=7$.

(a) (i) Find the gradient of AB. *(2 marks)*

(ii) Find an equation of the straight line that is parallel to AB and which passes through the point $C(3, -5)$, giving your answer in the form $px+qy=r$, where p, q and r are integers. *(3 marks)*

(b) The line AB intersects the line with equation $3x-2y=4$ at the point D. Find the coordinates of D. *(3 marks)*

(c) The point E with coordinates $(k-2, 2k-3)$ lies on the line AB. Find the value of the constant k. *(2 marks)*

---

3 The polynomial $\mathrm{p}(x)$ is given by

$$\mathrm{p}(x)=x^3+2x^2-5x-6$$

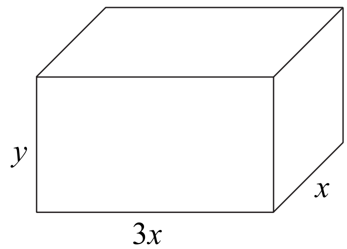
(a) (i) Use the Factor Theorem to show that $x+1$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(ii) Express $\mathrm{p}(x)$ as the product of three linear factors. *(3 marks)*

(b) Verify that $\mathrm{p}(0)>\mathrm{p}(1)$. *(2 marks)*

(c) Sketch the curve with equation $y=x^3+2x^2-5x-6$, indicating the values where the curve crosses the x-axis. *(3 marks)*

4 The diagram shows a solid cuboid with sides of lengths x cm, $3x$ cm and y cm.



The total surface area of the cuboid is 32 cm^2.

(a) (i) Show that $3x^2 + 4xy = 16$. *(2 marks)*

(ii) Hence show that the volume, $V\text{ cm}^3$, of the cuboid is given by

$$V = 12x - \frac{9x^3}{4}$$ *(2 marks)*

(b) Find $\frac{dV}{dx}$. *(2 marks)*

(c) (i) Verify that a stationary value of V occurs when $x = \frac{4}{3}$. *(2 marks)*

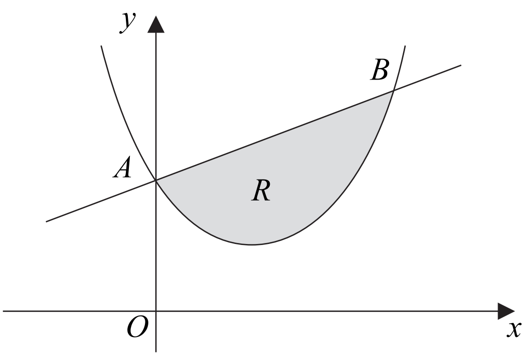
(ii) Find $\frac{d^2V}{dx^2}$ and hence determine whether V has a maximum value or a minimum value when $x = \frac{4}{3}$. *(2 marks)*

Turn over ▶

5 (a) (i) Express $x^2 - 3x + 5$ in the form $(x - p)^2 + q$. *(2 marks)*

(ii) Hence write down the equation of the line of symmetry of the curve with equation $y = x^2 - 3x + 5$. *(1 mark)*

(b) The curve C with equation $y = x^2 - 3x + 5$ and the straight line $y = x + 5$ intersect at the point $A(0, 5)$ and at the point B, as shown in the diagram below.

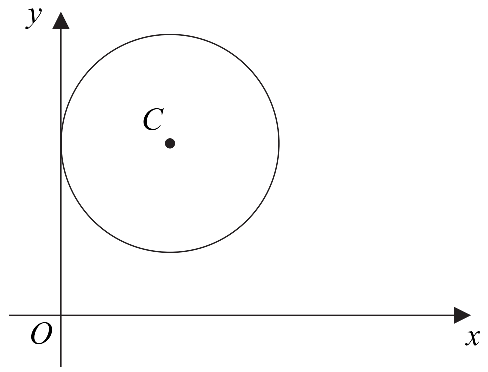


(i) Find the coordinates of the point B. *(3 marks)*

(ii) Find $\int (x^2 - 3x + 5)\,dx$. *(3 marks)*

(iii) Find the area of the shaded region R bounded by the curve C and the line segment AB. *(4 marks)*

6 The circle with centre $C(5,\ 8)$ touches the y-axis, as shown in the diagram.



(a) Express the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = k$$ *(2 marks)*

(b) (i) Verify that the point $A(2,\ 12)$ lies on the circle. *(1 mark)*

(ii) Find an equation of the tangent to the circle at the point A, giving your answer in the form $sx + ty + u = 0$, where s, t and u are integers. *(5 marks)*

(c) The points P and Q lie on the circle, and the mid-point of PQ is $M(7,\ 12)$.

(i) Show that the length of CM is $n\sqrt{5}$, where n is an integer. *(2 marks)*

(ii) Hence find the area of triangle PCQ. *(3 marks)*

---

7 The gradient, $\dfrac{dy}{dx}$, of a curve C at the point $(x,\ y)$ is given by

$$\frac{dy}{dx} = 20x - 6x^2 - 16$$

(a) (i) Show that y is increasing when $3x^2 - 10x + 8 < 0$. *(2 marks)*

(ii) Solve the inequality $3x^2 - 10x + 8 < 0$. *(4 marks)*

(b) The curve C passes through the point $P(2,\ 3)$.

(i) Verify that the tangent to the curve at P is parallel to the x-axis. *(2 marks)*

(ii) The point $Q(3,\ -1)$ also lies on the curve. The normal to the curve at Q and the tangent to the curve at P intersect at the point R. Find the coordinates of R. *(7 marks)*

General Certificate of Education
Advanced Subsidiary Examination
January 2013

Mathematics MPC1

Unit Pure Core 1

Monday 14 January 2013 9.00 am to 10.30 am

For this paper you must have: • the blue AQA booklet of formulae and statistical tables. You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The point A has coordinates $(-3, 2)$ and the point B has coordinates $(7, k)$.

The line AB has equation $3x + 5y = 1$.

(a) (i) Show that $k = -4$. *(1 mark)*

(ii) Hence find the coordinates of the midpoint of AB. *(2 marks)*

(b) Find the gradient of AB. *(2 marks)*

(c) A line which passes through the point A is perpendicular to the line AB. Find an equation of this line, giving your answer in the form $px + qy + r = 0$, where p, q and r are integers. *(3 marks)*

(d) The line AB, with equation $3x + 5y = 1$, intersects the line $5x + 8y = 4$ at the point C. Find the coordinates of C. *(3 marks)*

---

2 A bird flies from a tree. At time t seconds, the bird's height, y metres, above the horizontal ground is given by

$$y = \frac{1}{8}t^4 - t^2 + 5, \quad 0 \leqslant t \leqslant 4$$

(a) Find $\frac{dy}{dt}$. *(2 marks)*

(b) (i) Find the rate of change of height of the bird in metres per second when $t = 1$. *(2 marks)*

(ii) Determine, with a reason, whether the bird's height above the horizontal ground is increasing or decreasing when $t = 1$. *(1 mark)*

(c) (i) Find the value of $\frac{d^2y}{dt^2}$ when $t = 2$. *(2 marks)*

(ii) Given that y has a stationary value when $t = 2$, state whether this is a maximum value or a minimum value. *(1 mark)*

---

3 (a) (i) Express $\sqrt{18}$ in the form $k\sqrt{2}$, where k is an integer. *(1 mark)*

(ii) Simplify $\frac{\sqrt{8}}{\sqrt{18} + \sqrt{32}}$. *(3 marks)*

(b) Express $\frac{7\sqrt{2} - \sqrt{3}}{2\sqrt{2} - \sqrt{3}}$ in the form $m + \sqrt{n}$, where m and n are integers. *(4 marks)*

4 (a) (i) Express $x^2 - 6x + 11$ in the form $(x - p)^2 + q$. *(2 marks)*

(ii) **Use the result from part (a)(i)** to show that the equation $x^2 - 6x + 11 = 0$ has no real solutions. *(2 marks)*

(b) A curve has equation $y = x^2 - 6x + 11$.

(i) Find the coordinates of the vertex of the curve. *(2 marks)*

(ii) Sketch the curve, indicating the value of y where the curve crosses the y-axis. *(3 marks)*

(iii) Describe the geometrical transformation that maps the curve with equation $y = x^2 - 6x + 11$ onto the curve with equation $y = x^2$. *(3 marks)*

---

5 The polynomial $\mathrm{p}(x)$ is given by

$$\mathrm{p}(x) = x^3 - 4x^2 - 3x + 18$$

(a) Use the Remainder Theorem to find the remainder when $\mathrm{p}(x)$ is divided by $x + 1$. *(2 marks)*

(b) (i) Use the Factor Theorem to show that $x - 3$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(ii) Express $\mathrm{p}(x)$ as a product of linear factors. *(3 marks)*

(c) Sketch the curve with equation $y = x^3 - 4x^2 - 3x + 18$, stating the values of x where the curve meets the x-axis. *(3 marks)*

---

6 The gradient, $\frac{\mathrm{d}y}{\mathrm{d}x}$, of a curve at the point (x, y) is given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 10x^4 - 6x^2 + 5$$

The curve passes through the point $P(1, 4)$.

(a) Find the equation of the tangent to the curve at the point P, giving your answer in the form $y = mx + c$. *(3 marks)*

(b) Find the equation of the curve. *(5 marks)*

7 A circle with centre $C(-3, 2)$ has equation

$$x^2 + y^2 + 6x - 4y = 12$$

(a) Find the y-coordinates of the points where the circle crosses the y-axis. *(3 marks)*

(b) Find the radius of the circle. *(3 marks)*

(c) The point $P(2, 5)$ lies outside the circle.

(i) Find the length of CP, giving your answer in the form $\sqrt{n}$, where n is an integer. *(2 marks)*

(ii) The point Q lies on the circle so that PQ is a tangent to the circle. Find the length of PQ. *(2 marks)*

---

8 A curve has equation $y = 2x^2 - x - 1$ and a line has equation $y = k(2x - 3)$, where k is a constant.

(a) Show that the x-coordinate of any point of intersection of the curve and the line satisfies the equation

$$2x^2 - (2k + 1)x + 3k - 1 = 0$$ *(1 mark)*

(b) The curve and the line intersect at two distinct points.

(i) Show that $4k^2 - 20k + 9 > 0$. *(3 marks)*

(ii) Find the possible values of k. *(4 marks)*

General Certificate of Education
Advanced Subsidiary Examination
June 2013

Mathematics MPC1

Unit Pure Core 1

Monday 13 May 2013 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The line AB has equation $3x - 4y + 5 = 0$.

(a) The point with coordinates $(p, p + 2)$ lies on the line AB. Find the value of the constant p. *(2 marks)*

(b) Find the gradient of AB. *(2 marks)*

(c) The point A has coordinates $(1, 2)$. The point $C(-5, k)$ is such that AC is perpendicular to AB. Find the value of k. *(3 marks)*

(d) The line AB intersects the line with equation $2x - 5y = 6$ at the point D. Find the coordinates of D. *(3 marks)*

---

2 (a) (i) Express $\sqrt{48}$ in the form $n\sqrt{3}$, where n is an integer. *(1 mark)*

(ii) Solve the equation

$$x\sqrt{12} = 7\sqrt{3} - \sqrt{48}$$

giving your answer in its simplest form. *(3 marks)*

(b) Express $\dfrac{11\sqrt{3} + 2\sqrt{5}}{2\sqrt{3} + \sqrt{5}}$ in the form $m - \sqrt{15}$, where m is an integer. *(4 marks)*

---

3 A circle C has equation

$$x^2 + y^2 - 10x + 14y + 25 = 0$$

(a) Write the equation of C in the form

$$(x - a)^2 + (y - b)^2 = k$$

where a, b and k are integers. *(3 marks)*

(b) Hence, for the circle C, write down:

(i) the coordinates of its centre; *(1 mark)*

(ii) its radius. *(1 mark)*

(c) (i) Sketch the circle C. *(2 marks)*

(ii) Write down the coordinates of the point on C that is furthest away from the x-axis. *(2 marks)*

(d) Given that k has the same value as in part **(a)**, describe geometrically the transformation which maps the circle with equation $(x + 1)^2 + y^2 = k$ onto the circle C. *(3 marks)*

4 (a) The polynomial $\mathrm{f}(x)$ is given by $\mathrm{f}(x) = x^3 - 4x + 15$.

(i) Use the Factor Theorem to show that $x + 3$ is a factor of $\mathrm{f}(x)$. *(2 marks)*

(ii) Express $\mathrm{f}(x)$ in the form $(x + 3)(x^2 + px + q)$, where p and q are integers. *(2 marks)*

(b) A curve has equation $y = x^4 - 8x^2 + 60x + 7$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. *(3 marks)*

(ii) Show that the x-coordinates of any stationary points of the curve satisfy the equation

$$x^3 - 4x + 15 = 0$$ *(1 mark)*

(iii) Use the results above to show that the only stationary point of the curve occurs when $x = -3$. *(2 marks)*

(iv) Find the value of $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ when $x = -3$. *(3 marks)*

(v) Hence determine, with a reason, whether the curve has a maximum point or a minimum point when $x = -3$. *(1 mark)*

---

5 (a) (i) Express $2x^2 + 6x + 5$ in the form $2(x + p)^2 + q$, where p and q are rational numbers. *(2 marks)*

(ii) Hence write down the minimum value of $2x^2 + 6x + 5$. *(1 mark)*

(b) The point A has coordinates $(-3, 5)$ and the point B has coordinates $(x, 3x + 9)$.

(i) Show that $AB^2 = 5(2x^2 + 6x + 5)$. *(3 marks)*

(ii) Use your result from part **(a)(ii)** to find the minimum value of the length AB as x varies, giving your answer in the form $\frac{1}{2}\sqrt{n}$, where n is an integer. *(2 marks)*

Turn over ▶

6 A curve has equation $y = x^5 - 2x^2 + 9$. The point P with coordinates $(-1, 6)$ lies on the curve.

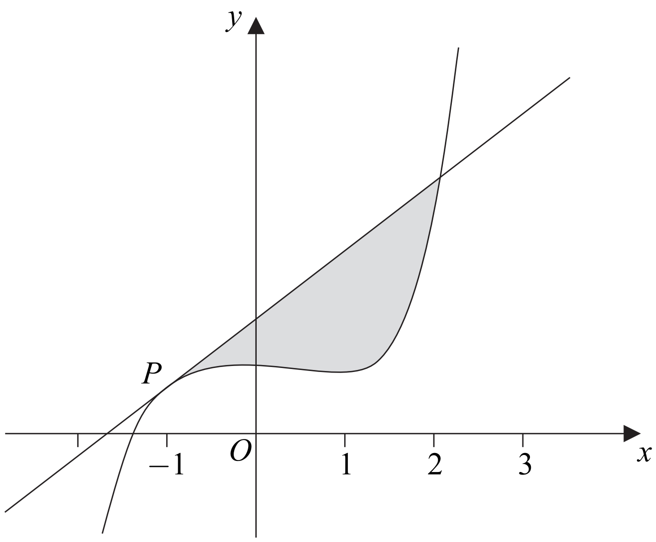
(a) Find the equation of the tangent to the curve at the point P, giving your answer in the form $y = mx + c$. *(5 marks)*

(b) The point Q with coordinates $(2, k)$ lies on the curve.

(i) Find the value of k. *(1 mark)*

(ii) Verify that Q also lies on the tangent to the curve at the point P. *(1 mark)*

(c) The curve and the tangent to the curve at P are sketched below.



(i) Find $\int_{-1}^{2} (x^5 - 2x^2 + 9)\,dx$. *(5 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the tangent to the curve at P. *(3 marks)*

---

7 The quadratic equation

$$(2k - 7)x^2 - (k - 2)x + (k - 3) = 0$$

has real roots.

(a) Show that $7k^2 - 48k + 80 \leqslant 0$. *(4 marks)*

(b) Find the possible values of k. *(4 marks)*

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
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7	
8	
TOTAL	

AQA

General Certificate of Education
Advanced Subsidiary Examination
June 2014

Mathematics MPC1

Unit Pure Core 1

Monday 19 May 2014 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
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- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 The point A has coordinates $(-1, 2)$ and the point B has coordinates $(3, -5)$.

(a) (i) Find the gradient of AB.

[2 marks]

(ii) Hence find an equation of the line AB, giving your answer in the form $px + qy = r$, where p, q and r are integers.

[3 marks]

(b) The midpoint of AB is M.

(i) Find the coordinates of M.

[1 mark]

(ii) Find an equation of the line which passes through M and which is perpendicular to AB.

[3 marks]

(c) The point C has coordinates $(k, 2k + 3)$. Given that the distance from A to C is $\sqrt{13}$, find the two possible values of the constant k.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 1

2 A rectangle has length $(9 + 5\sqrt{3})\,\text{cm}$ and area $(15 + 7\sqrt{3})\,\text{cm}^2$.

Find the width of the rectangle, giving your answer in the form $(m + n\sqrt{3})\,\text{cm}$, where m and n are integers.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 2

3 A curve has equation $y = 2x^5 + 5x^4 - 1$.

(a) Find:

(i) $\dfrac{dy}{dx}$

[2 marks]

(ii) $\dfrac{d^2y}{dx^2}$

[1 mark]

(b) The point on the curve where $x = -1$ is P.

(i) Determine whether y is increasing or decreasing at P, giving a reason for your answer.
[2 marks]

(ii) Find an equation of the tangent to the curve at P.

[3 marks]

(c) The point $Q(-2, 15)$ also lies on the curve. Verify that Q is a maximum point of the curve.

[4 marks]

QUESTION PART REFERENCE	Answer space for question 3

Do not write outside the box

4 (a) (i) Express $16 - 6x - x^2$ in the form $p - (x + q)^2$ where p and q are integers.

[2 marks]

(ii) Hence write down the maximum value of $16 - 6x - x^2$.

[1 mark]

(b) (i) Factorise $16 - 6x - x^2$.

[1 mark]

(ii) Sketch the curve with equation $y = 16 - 6x - x^2$, stating the values of x where the curve crosses the x-axis and the value of the y-intercept.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 4

5 The polynomial $p(x)$ is given by

$$p(x) = x^3 + cx^2 + dx + 3$$

where c and d are integers.

(a) Given that $x + 3$ is a factor of $p(x)$, show that

$$3c - d = 8$$

[2 marks]

(b) The remainder when $p(x)$ is divided by $x - 2$ is 65.

Obtain a further equation in c and d.

[2 marks]

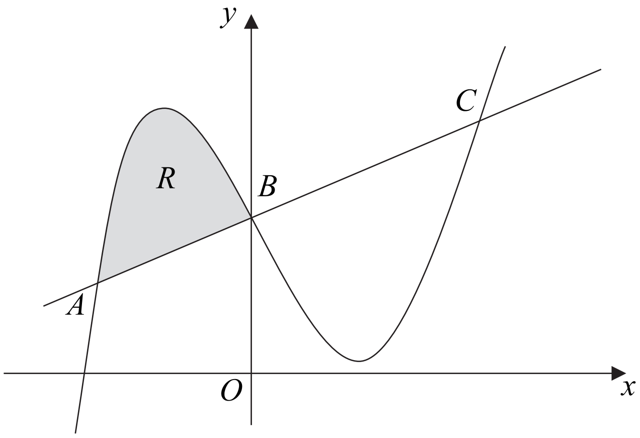
(c) Use the equations from parts **(a)** and **(b)** to find the value of c and the value of d.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 5

6 The diagram shows a curve and a line which intersect at the points A, B and C.



The curve has equation $y = x^3 - x^2 - 5x + 7$ and the straight line has equation $y = x + 7$. The point B has coordinates $(0, 7)$.

(a) (i) Show that the x-coordinates of the points A and C satisfy the equation

$$x^2 - x - 6 = 0$$

[2 marks]

(ii) Find the coordinates of the points A and C.

[3 marks]

(b) Find $\int (x^3 - x^2 - 5x + 7)\, dx$.

[3 marks]

(c) Find the area of the shaded region R bounded by the curve and the line segment AB.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 6

Do not write outside the box

7 A circle with centre C has equation $x^2 + y^2 - 10x + 12y + 41 = 0$. The point $A(3, -2)$ lies on the circle.

(a) Express the equation of the circle in the form

$$(x-a)^2 + (y-b)^2 = k$$

[3 marks]

(b) (i) Write down the coordinates of C.

[1 mark]

(ii) Show that the circle has radius $n\sqrt{5}$, where n is an integer.

[2 marks]

(c) Find the equation of the tangent to the circle at the point A, giving your answer in the form $x + py = q$, where p and q are integers.

[5 marks]

(d) The point B lies on the tangent to the circle at A and the length of BC is 6. Find the length of AB.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 7

8 Solve the following inequalities:

(a) $3(1-2x)-5(3x+2)>0$

[2 marks]

(b) $6x^2 \leqslant x+12$

[4 marks]

QUESTION PART REFERENCE

Answer space for question 8