AQA Computer Science A-Level

4.6.5 Boolean algebra

Past Paper Mark Schemes

January 2010 Comp 2

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ALGEBRAIC SOLUTION:

Method 1	Method 2
$\overline{A \cdot B} + A$ $= \overline{A} + \overline{B} + A$ $= 1 + \overline{B}$ $= 1$	$\overline{A \cdot B} + A$ $= \overline{A \cdot B \cdot \overline{A}}$ $= \overline{0 \cdot B}$ $= \overline{0}$ $= 1$

1 mark for an application of a DeMorgan's law
1 mark for realisation that $A + A + \overline{B} = 1 + \overline{B}$ or $\overline{0}.B = \overline{0}$ (must be written in method, not just inferred that student has done this if arrives at correct answer)
1 mark for correct answer

TRUTH TABLE SOLUTION:

		X	Y	Z
A	B	$A \cdot B$	$\overline{A \cdot B}$	$\overline{A \cdot B} + A$
0	0	0	1	1
0	1	0	1	1
1	0	0	1	1
1	1	1	0	1

1 mark for column Y correct
1 mark for column Z correct
1 mark for correct answer

ANY OTHER METHOD:
If student has used any other method to arrive at correct answer then award marks as follows:
1 mark for correct answer, no working out
2 marks for correct answer with working out, not all steps shown.
3 marks for correct answer with all steps of working out shown.

A True for 1, False for 0
A alternative notations :

- For $X \cdot Y$ allow X AND Y, $X \wedge Y$, $X \cap Y$, XY
- For X+Y allow X OR Y, $X \vee Y$, $X \cup Y$
- For $\overline{X}$ allow NOT X, $\neg X$

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3			$\overline{(\overline{A}.\overline{B})}$ becomes A + B ; **A** (A+B); **A** A OR B; $B+B.\overline{C}$ becomes B ; $A.B+A.\overline{B}$ becomes A ; $A.(B+1)$ becomes A ; 1 mark for each	4

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2	**d**		$(X+Y).(X+\bar{Y})$ [Fully expanding brackets – **1 mark**] $X.X + X.\bar{Y} + Y.X + Y.\bar{Y}$ [Recognising $X.X = X$ OR $Y.\bar{Y} = 0$ – **1 mark**] $X + X.\bar{Y} + Y.X + 0$ [Taking X outside brackets – **1 mark**] $X(1 + \bar{Y} + Y)$ OR $X + X(\bar{Y} + Y)$ X [Final Answer, **1 mark**] **Alternative Answer : (Distributive)** $(X+Y).(X+\bar{Y}) = X + (Y.\bar{Y})$ [Use of distributive law – 1 mark] $X + (Y.\bar{Y}) = X + 0$ [Recognising $Y.\bar{Y} = 0$ - 1 mark] X + 0 = X [1 mark] X [Final Answer, 1 mark] **Alternative Answer : (De Morgan's)** $\overline{\overline{X+Y} + \overline{X+\bar{Y}}} = Q$ [Use of De Morgan's – **1 mark**]		NOTE : Mark against one of the answers included. MAX 2 for working/method if mistakes occur. Any new solutions refer to team leader.

			$\overline{X+Y}+\overline{X+\bar{Y}}=\bar{Q}$ $\overline{\bar{X}.\bar{Y}}+\overline{\bar{X}.\bar{Y}}=\bar{Q}$ [Two further applications of De Morgan's] $\bar{X}.\bar{Y}+\bar{X}.Y=\bar{Q}$ $\bar{X}.(\bar{Y}+Y)=\bar{Q}$ [Taking X outside brackets – **1 mark**] [Recognising ~Y + Y = 1 – **1 mark**] $\bar{X}.1=\bar{Q}$ [Recognising X.1=X – **1 mark**] $\bar{X}=\bar{Q}$ X = Q [Final answer, **1 mark**] MAX 3 for working/method; 1 for final answer X on own with no working gains 1 mark.	**MAX 4**	

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4	**c**		De Morgan's (law);	**1**

4	**d**		**Mark allocation:** One mark for taking either A, NOT C or A AND NOT C outside of brackets to produce a correct expression; One mark for eliminating B in a valid way; One mark for correct final answer; **Example One**: $A.B.\overline{C} + A.\overline{C}$ $A\,(B.\overline{C} + \overline{C})$ - taking A outside of brackets; $A\,(\overline{C}\,(B + 1))$ $(B + 1) = 1$ Simplifying to remove B using B + 1 = 1 ; $B.\overline{C} + \overline{C} = \overline{C}$ Simplifying to remove B using $B.\overline{C} + \overline{C} = \overline{C}$; **A.** $A\,(\overline{C}\,(B + 1)) \rightarrow A.\overline{C}$; Final answer $A.\overline{C}$	**3**

Example Two:

$A.B.\bar{C} + A.\bar{C}$

$A.\bar{C}(B + 1)$ – taking outside of brackets;

(B + 1) = 1; - simplifying to remove B

A. $A.\bar{C}(B + 1) \rightarrow A.\bar{C}$

Final answer $A.\bar{C}$

Truth Table Method

A	B	C		$A.\bar{C}$	$A.B.\bar{C} + A.\bar{C}$
0	0	0		0	0
0	0	1		0	0
0	1	0		0	0
0	1	1		0	0
1	0	0		1	1
1	0	1		0	0
1	1	0		1	1
1	1	1		0	0

(student answer may have more columns than this)

A mark for having correct column for $A.B.\bar{C} + A.\bar{C}$;
A mark for having correct column for $A.\bar{C}$;

Final answer $A.\bar{C}$

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9	(c)		**ALGEBRAIC SOLUTION:** $B \cdot (A + \overline{B})$ $B \cdot A + B \cdot \overline{B}$ [**1 mark** for expansion of brackets] $B \cdot A + 0$ [**1 mark** for identifying that $B \cdot \overline{B} = 0$] $B \cdot A$ [**1 mark** for correct answer] **TRUTH TABLE SOLUTION:**	3

		X	Y	Z
A	B	$\overline{B}$	$A + \overline{B}$	$B \cdot (A + \overline{B})$
0	0	1	1	0
0	1	0	0	0
1	0	1	1	0
1	1	0	1	1

1 mark for both columns X and Y correct
1 mark for column Z correct
1 mark for correct answer ($B \cdot A$)

ANY OTHER METHOD:
If student has used any other method to arrive at correct answer then award marks as follows:
1 mark for correct answer, no working out
2 marks for correct answer with working out, not all steps shown.
3 marks for correct answer with all steps of working out shown.

A True for 1, False for 0
A alternative notations :
- For $X \cdot Y$ allow X AND Y, $X \wedge Y$, $X \cap Y$, XY
- For X+Y allow X OR Y, $X \vee Y$, $X \cup Y$
- For $\overline{X}$ allow NOT X, $\neg X$

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3	c		$\overline{(A \cdot B)} + \overline{(A \cdot \overline{B})}$ $(\overline{A} + \overline{B}) + (\overline{A} + B)$; ; 2 marks – 1 each for De Morgans rule for each side of the central OR (award the mark for right hand expression, even if double NOT over B is not cancelled) $\overline{A} + \overline{B} + B$; Recognising NOT A OR NOT A is NOT A, and producing a correct expression $\overline{A} + 1$; Recognising B or NOT B is 1 Final answer 1 ; **Alternative answer** $\overline{(A \cdot B) \cdot (A \cdot \overline{B})}$; Application of De Morgan's to entire expression $\overline{(A \cdot B) \cdot (A \cdot \overline{B})}$; Cancellation of NOTs; 1 mark – De Morgans on entire expression $\overline{A \cdot B \cdot \overline{B}}$; Recognising A and A is A $\overline{A \cdot 0}$; Recognising B ANDed with its complement is 0 $\overline{0}$; Recognising 0 AND anything is 0 Final answer 1 ; **NOTE :** Marks can be awarded for the skills above if seen but MAX 3 (out of 4) for whole question if working has errors in it **A** T, True for 1 and F, False for 0 **A** alternative notations : • For $X \cdot Y$ allow X AND Y, $X \wedge Y$, $X \cap Y$, XY • For X+Y allow X OR Y, $X \vee Y$, $X \cup Y$ • For $\overline{X}$ allow NOT X, $\neg X$ Or by truth table M = marking point M M M M A B A.B $\overline{A.B}$ $\overline{B}$ $A.\overline{B}$ $\overline{A.\overline{B}}$ $\overline{A.B}$ + $\overline{A.B}$ 1 0 0 0 1 1 0 1 1 0 1 0 1 0 0 1 1 1 0 0 1 1 1 0 1 1 1 1 0 0 0 1 1	**MAX 3 for stages** **1 for final answer** **= 4**

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8	c		**Solution 1:** $Q = \overline{\bar{A} . (B . \bar{A})}$ [Application of De Morgan's Law –**1 mar** $Q = A.B.A$ [allow simplification of double nots at same time] [Simplification of A.A to A – **1 mark**] $Q = A.B$ [Correct solution – **1 mark**] **Solution 2:** $Q = \overline{\bar{A} + (\bar{B} + \bar{A})}$ [Application of De Morgan's Law – **1 mark**] $Q = \overline{A + B + A}$ [allow simplification of double nots at same time] $Q = \overline{A + B}$ [Simplification of NOT A OR NOT A to NOT A – **1 mark**] $Q = A.B$ [De Morgan's again to correct solution – **1 mark**] No working marks for truth table solution (asked to use De Morgan's in question)	3	1 mark for De Morgan; 1 mark for simplification; 1 mark for final answer; Other notations as for question 8b

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6	**(b)**	**(i)**	B;	**1**

6	**(b)**	**(ii)**	B;	**1**

6	**(b)**	**(iii)**	0;; Award 1 mark if De Morgan's has been applied once correctly but candidate does not end up simplifying to 0 Example: $\overline{B + (\bar{A} + \bar{B})}$ Example: $\bar{B} . (A . B)$	**2**

June 2016 AS Paper 2

03		**Marks are for AO2 (apply)** **Marking guidance for examiners** • Award marks for working out until an incorrect step has been made. • If, in any one step, a candidate is simplifying different parts of an expression simultaneously award all relevant marks for this multiple stage but don't award any further marks for working in any parts simplified incorrectly. Example, if the expression P.P.(P + Q) + P.P.1 was changed to P.(P+Q) + P.0 the candidate would get one mark for simplifying the first part to P.(P+Q) and could get further marks for correctly simplifying this part of the expression further but should not be awarded marks for simplifying the incorrectly changed part P.0 (ie to 0). **Mark as follows** **1 mark** for final answer: $\overline{A}.B$ **Max 3** for working; **Max 3** if answer is correct but any incorrect working or significant steps of working is missing: **1 mark** for a successful application of De Morgan's Law that would lead to a simpler expression. **Max 2** for applications of De Morgan's Law. **1 mark** for applying an identity other than cancelling nots that produces a simpler expression. **Max 2** for applying identities. **1 mark** for expanding brackets or putting an expression into brackets that would lead to a simpler expression. **Max 2** for expanding brackets or putting an expression into brackets. **Note:** a simpler expression is one that is logically equivalent to the original expression but uses fewer logical operators. **Example working (1)** $= (\overline{A} + B).(\overline{A}.(B + A))$ [application of De Morgan's law] $= (\overline{A} + B).(\overline{A}.B + \overline{A}.A)$ [expansion of brackets] $= (\overline{A} + B).(\overline{A}.B)$ [use of identities $X.\overline{X} = 0$ and $X+0 = X$] $= \overline{A}.\overline{A}.B + B.\overline{A}.B$ [expansion of brackets] $= \overline{A}.B + \overline{A}.B$ [use of identity $X.X = X$ twice] $= \overline{A}.B$ [use of identity $X + X = X$]	4

Alternative example working (2)

$= \overline{(\overline{A}+B)+(A+(\overline{B}+\overline{A}))}$ [application of De Morgan's Law]

$= \overline{A.\overline{B}+\overline{A}.(B+A)}$ [application of De Morgan's Law twice]

$= \overline{A.\overline{B}+(\overline{A}.B+\overline{A}.A)}$ [expansion of brackets]

$= \overline{A.\overline{B}+\overline{A}.B}$ [use of identities $\overline{A}.A=0$ and $X+0=X$]

$= \overline{\overline{A}+B}+\overline{A}.B$ [application of De Morgan's Law]

$= (\overline{A}+B).(\overline{A}.B)$ [application of De Morgan's Law]

$= \overline{A}.\overline{A}.B+B.\overline{A}.B$ [expansion of brackets]

$= \overline{A}.B+\overline{A}.B$ [use of identity $X.X=X$ twice]

$= \overline{A}.B$ [use of identity $X+X=X$]

Alternative example working (3)

$= \overline{(\overline{A}+B)+(A+(\overline{B}+\overline{A}))}$ [application of De Morgan's Law]

$= \overline{A.\overline{B}+\overline{A}.(B+A)}$ [De Morgan's Law twice]

$= \overline{A.\overline{B}+(\overline{A}.B+\overline{A}.A)}$ [Expansion]

$= \overline{A.\overline{B}+\overline{A}.B}$ [Identity $\overline{A}.A=0$ and $A+0=A$]

$= \overline{\overline{A}+B}+\overline{A}.B$ [application of De Morgan's Law]

$= (\overline{A}+B).(\overline{A}.B)$ [application of De Morgan's Law]

$= \overline{A}.\overline{A}.B+B.\overline{A}.B$ [Expansion]

$= \overline{A}.B+\overline{A}.B$ [Identity $\overline{A}.\overline{A}=\overline{A}$ and $B.B=B$]

$= \overline{A}.B$ [Final answer via identity $A+A=A$]

June 2017 AS Paper 2

05	3	**Marks are for AO2 (apply)** **Marking guidance for examiners** • Award marks for working out until an incorrect step has been made. • If, in any one step, a candidate is simplifying different parts of an expression simultaneously award all relevant marks for this multiple stage but don't award any further marks for working in any parts simplified incorrectly. Example, if the expression P.P.(P+Q) + P.P.1 was changed to P.(P+Q)+P.0, the candidate would get one mark for simplifying the first part to P.(P+Q) and could get further marks for correctly simplifying this part of the expression further but should not be awarded marks for simplifying the incorrectly changed part P.0 (ie to 0) **Mark as follows** **1 mark** for final answer X **Max 3 marks for working**: • **1 mark** for each application of an identity other than cancelling NOTs that produces a simpler expression. • **1 mark** for expanding brackets • **1 mark** for putting an expression into brackets that would lead to a simpler expression. **Note:** a simpler expression is one that is logically equivalent to the original expression but uses fewer logical operators. **Max 3** if answer is correct but any incorrect working or significant steps of working is missing. **Example working (1)** $X.X + X.\overline{Y} + Y.X + Y.\overline{Y}$ [expansion of brackets] $X + X.\overline{Y} + Y.X + 0$ [use of $X \cdot X = X$ and $Y.\overline{Y} = 0$] $X(1 + \overline{Y} + Y)$ or $X + X(\overline{Y} + Y)$ [taking X outside of brackets] **Alternative example working (2)** $X + (Y.\overline{Y})$ [Use of distributive law] $X + 0$ [$Y.\overline{Y} = 0$] X [Recognising X+0 = X]	**4**

June 2017 Paper 2

04	3	**All marks AO2 (apply)** **Marking guidance for examiners** • Award marks for working out until an incorrect step has been made. • If, in any one step, a candidate is simplifying different parts of an expression simultaneously award all relevant marks for this multiple stage but don't award any further marks for working in any parts simplified incorrectly. For example, if the expression P.P.(P+Q) + P.P.1 was changed to P.(P+Q)+P.0, the candidate would get one mark for simplifying the first part to P.(P+Q) and could get further marks for correctly simplifying this part of the expression further but should not be awarded marks for simplifying the incorrectly changed part P.0 (ie to 0) **1 mark** for final answer: $B + C$ **MAX 3** for working. Award up to two marks for applying each one of the three techniques (one mark per application): • a successful application of De Morgan's Law (and any associated cancellation of NOTs) that produces a simpler expression. • applying an identity other than cancelling NOTs that produces a simpler expression. • successfully expanding brackets. **Note:** A simpler expression is one that is logically equivalent to the original expression but uses fewer logical operators. **Example Working (1)** $\overline{\left(\overline{A} + A \cdot (A + B)\right) + (\overline{B} \cdot \overline{C})}$ $= \left(\overline{A} + A \cdot (A + B)\right) \cdot \overline{(\overline{B} \cdot \overline{C})}$ Application of DeMorgan $= \left(\overline{A} + A \cdot (A + B)\right) \cdot (B + C)$ Application of DeMorgan $= (\overline{A} + A) \cdot (B + C)$ By identity $A = A \cdot (A + B)$ $= 1 \cdot (B + C)$ By identity $\overline{A} + A = 1$ $= B + C$ By identity $1 \cdot X = X$ **Example Working (2)**	4

		$\overline{\left(\overline{A} + A \cdot (A + B)\right) + (\overline{B} \cdot \overline{C})}$ $= \overline{(\overline{A} + A \cdot A + A \cdot B) + (\overline{B} \cdot \overline{C})}$ Expansion of brackets $= \overline{(\overline{A} + A + A \cdot B) + (\overline{B} \cdot \overline{C})}$ By identity $A \cdot A = A$ $= \overline{(1 + A \cdot B) + (\overline{B} \cdot \overline{C})}$ By identity $\overline{A} + A = 1$ $= \overline{(1) + (\overline{B} \cdot \overline{C})}$ By identity $1 + X = 1$ $= \overline{0 + (\overline{B} \cdot \overline{C})}$ By identity $\overline{0} = 1$ $= \overline{(\overline{B} \cdot \overline{C})}$ By identity $0 + X = X$ $= B + C$ Application of DeMorgan	

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(c) **ALGEBRAIC SOLUTION:**

$\overline{\overline{A} + \overline{B}} + B \cdot \overline{A}$ [Application of DeMorgan's Law **1 mark**]

$A \cdot B + B \cdot \overline{A}$

$B \cdot (A + \overline{A}) \,//\, B \cdot 1$ [Common term B taken out **1 mark**]

B [Correct answer **1 mark**]

A alternative notations :

- For $X \cdot Y$ allow X AND Y, $X \wedge Y$, $X \cap Y$, XY
- For X+Y allow X OR Y, $X \vee Y$, $X \cup Y$
- For $\overline{X}$ allow NOT X, $\neg X$

TRUTH TABLE SOLUTION:

				X	**Y**	**Z**
A	B	$\overline{A}$	$\overline{B}$	$\overline{A} + \overline{B}$	$B \cdot \overline{A}$	$\overline{\overline{A} + \overline{B}} + B \cdot \overline{A}$
0	0	1	1	0	0	0
0	1	1	0	0	1	1
1	0	0	1	0	0	0
1	1	0	0	1	0	1

1 mark for both columns X and Y correct
1 mark for column Z correct
1 mark for correct answer (B)
A Rightmost column labelled as L or Q

3

Specimen AS Paper 2

09	**2**		**Marks are for AO2 (apply)** A.B.(A + B) A.B.A + A.B.B ; [expansion of brackets] B.A + A.B ; [use of A.A = A] A.B ; [use of A + A = A] **1 mark:** Final answer: A.B; **Max 2 for working**	3

09	**3**		**Marks are for AO2 (apply)** (X + Y).(X + NOT Y) XX + X(NOT Y) + XY + Y(NOT Y) ; [expansion of brackets] X + X(NOT Y) + XY ; [use of X.X = X or use of Y(NOT Y) = 0] X (1 + NOT Y + Y) ; [use of 1 + X = 1] **1 mark:** Final answer - X; **Max 2** for working	3

Specimen Paper 2

11	**1**	**All marks AO1 (understanding)** | **Equation**	**Correct? (Shade three)**
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$A \cdot \overline{A} = 1$	
$A + B = \overline{\overline{A} \cdot \overline{B}}$	✓
$A + 1 = 1$	✓
$A \cdot (A + B) = A$	✓
$A + (A \cdot B) = B$	
$A \cdot 1 = 1$		 **If more than three lozenges shaded then take the number of incorrect answers from the number of correct answers to arrive at the total mark**	3

11	**2**	**All marks AO2 (apply)** Example solution: $\overline{\overline{A}} + \overline{\overline{B}} + B \cdot \overline{A}$ $= A \cdot B + B \cdot \overline{A}$ $= B \cdot (A + \overline{A})$ $= B \cdot 1$ $= B$ **In any attempted solution award:** **1 mark** for an application of DeMorgan's law **1 mark** for an application of a Boolean identity or expanding the brackets **1 mark** for correct answer	3