OCR Computer Science A Level

2.3.1 Sorting Algorithms

Advanced Notes

Specification:

- Standard algorithms
  - Bubble sort
  - Insertion sort
  - Merge sort
  - Quick sort

Sorting Algorithms

Sorting algorithms are designed to take a number of elements in any order and output them in a logical order. This is usually numerical or lexicographic (phonebook style ordering).

Most sorting algorithms will output elements in ascending order, but the algorithms can typically be slightly altered (for example, by switching an inequality from less than to greater than) or their output simply reversed in order to produce an output in descending order.

Bubble Sort

Bubble sort makes comparisons and swaps between pairs of elements. The largest element in the unsorted part of the input is said to “bubble” to the top of the data with each iteration of the algorithm.

The algorithm starts at the first element in an array and compares it to the second. If they are in the wrong order, the algorithm swaps the pair. Otherwise, the algorithm moves on. The process is then repeated for every adjacent pair of elements in the array until the end of the array is reached (at which point the largest element is in the last position of the array).

This is referred to as one pass of the algorithm. For an array with n elements, the algorithm will perform n passes through the data, at which point the input is sorted and can be returned.

```
A = Array of data

    for i = 0 to A.length - 1:
        for j = 0 to A.length - 2:
            if A[j] > A[j+1]:
                temp = A[j]
                A[j] = A[j+1]
                A[j+1] = temp
            endif
    return A
```

The three lines highlighted could be written as “`swap A[j] and A[j+1]`” and indeed in most situations this would be acceptable. The example illustrates how a swap operation would work, making use of a temporary store.

Example

Use bubble sort to arrange the elements in the array into ascending order.

4	9	1	6	7

The two first elements are compared. They are in the correct order and so the algorithm moves on.

4	9	1	6	7
4	1	9	6	7

The second two elements are in the wrong order, and so they are swapped.

4	1	9	6	7
4	1	6	9	7

Again, the next two elements are in the wrong order, and so they are swapped.

4	1	6	9	7
4	1	6	7	9

The last two elements are also in the wrong order, so are swapped.

This marks the end of the first pass of the algorithm.

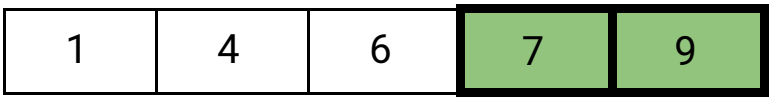
The second pass starts at the beginning of the array and compares the first two elements. They are in the wrong order, so are swapped.

4	1	6	7	9
1	4	6	7	9

The next two elements are in the correct order, so the algorithm moves on.

Again the comparison reveals that the elements are in the correct order.

The last pair are also in the correct order.



This is the end of the second pass, and we can see that the data is sorted. However, the algorithm wouldn't terminate here. It would carry out another two passes before terminating.

You might be thinking that this sounds like a waste of time, and you'd be right. If the algorithm goes on until it terminates, it could possibly make many pointless comparisons. The bubble sort algorithm can be improved to solve this issue, as the pseudocode below shows.

```
A = Array of data

    for i = 0 to A.length - 1:
        noSwap = True
        for j = 0 to A.length - (i + 1):
            if A[j] > A[j+1]:
                temp = A[j]
                A[j] = A[j+1]
                A[j+1] = temp
                noSwap = False
        if noSwap:
            break
    return A
```

The most important modification is the introduction of a flag recording whether a swap has occurred. If a full pass is made without any swaps, then the algorithm terminates earlier than it would have done originally.

The second, less significant, change is the inclusion of `j` in the second `for` loop. This means that as j increases, fewer comparisons are made. This works because the largest element is always in place at the end of the first pass, and the second largest element is in place at the end of the second pass and so on. There is no need to check these elements as we know they're in the right place.

Make sure you aren't caught out when using this version of the algorithm though. The algorithm terminates after a complete pass in which no swaps are completed, not at the end of the first pass at the end of which the elements are sorted!

Bubble sort is a fairly slow sorting algorithm, with a time complexity of $O(n^2)$.

Insertion Sort

Another sorting algorithm, insertion sort places elements into a sorted sequence. In the i^{th} iteration of the algorithm, the first `i` elements of the array are sorted. Be careful though, although `i` elements are sorted, they are not the `i` smallest elements in the input!

For example, the first three elements in the sequence 3, 7, 9, 4, 8 are sorted, but are not the three smallest elements in the input.

Insertion sort starts at the second element in the input, and compares it to the element to its left. If the two elements are in the wrong order, the smaller element is placed in the lowest position.

The third element in the input is then selected. It is inserted into the correct position in the sorted portion of the input to its left. This continues until the last element is inserted into the correct position, resulting in a fully sorted array.

```
A = Array of data

    for i = 1 to A.length - 1:
        elem = A[i]
        j = i - 1
        while j >= 0 and A[j] > elem:
                A[j+1] = A[j]
                  j = j - 1
                A[j+1] = elem
```

The pseudocode for insertion sort, shown above, shows how the algorithm starts at the second item and places it into a sorted sequence by performing consecutive swaps (within

the while loop) until every item has been inserted, leaving behind a sorted array. Insertion sort has the same time complexity as bubble sort, $O(n^2)$.

Example

The algorithm can ignore the first element, as a single element is always a sorted sequence. Starting at the second element, we see it is greater than the element to its left and so it remains in place.

4	9	1	6	7

Moving to the third element, which is smaller than the element to its left, the algorithm has to move it into the right place in the sorted sequence that is forming at the start of the data. It moves the element one place to the left, but it is still smaller than the new element to its left, so is moved left again at which point it is in the correct place in the sorted sequence.

4	9	1	6	7
4	1	9	6	7
1	4	9	6	7

The next element to be sorted is also smaller than the element to its left, so is moved left.

1	4	9	6	7
1	4	6	9	7

The same occurs with the next element, which is also moved left into the correct place.

1	4	6	9	7
1	4	6	7	9

Finally, the last element is checked, and is already in the right place.

1	4	6	7	9

1	4	6	7	9

Merge Sort

Merge sort is an example of a class of algorithms known as "divide and conquer" for reasons which will soon become clear.

Merge sort is formed from two functions. One called `MergeSort` and another called `Merge`. `MergeSort` divides its input into two parts and recursively calls `MergeSort` on each of those two parts until they are of length 1 at which point `Merge` is called. `Merge` puts groups of elements back together in a special way, ensuring that the final group produced is sorted.

`A` = Array of data

```
MergeSort(A)
     if A.length <= 1:
          return A
     else:
          mid = A.length / 2
          left = A[0...mid]
          right = A[mid+1...A.length-1]
          leftSort = MergeSort(left)
          rightSort = MergeSort(right)
          return Merge(leftSort, rightSort)
```

The exact implementation of merge isn't required, but knowledge of how it works is. The following example shows how the function works.

MergeSort is a more efficient algorithm than bubble sort and insertion sort, with a worst case time complexity of $O(n \log n)$.

Example

Firstly, the input is halved until individual elements are isolated. At this point there are four sorted lists (remember from insertion sort that a single element counts as being sorted).

7	4	2	6

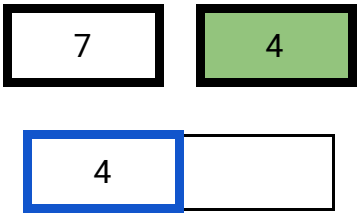
7	4

2	6

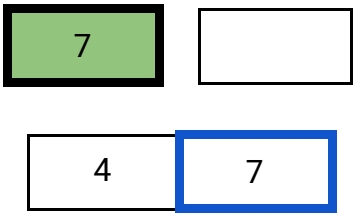
7	4	2	6

Each of the four lists must now be merged back together. This is done by inspecting the first element of two lists and placing the lowest in a new list. This is repeated until both lists are empty.

The first two lists are 7 and 4. The first elements of each (the whole list, as these only have length 1) are compared and the smallest, 4, inserted into a new list.

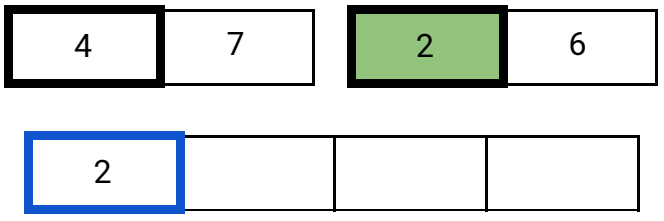


Now only 7 remains, and so it is placed behind the 4 in the new list.

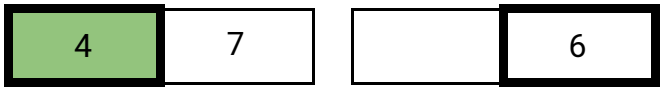


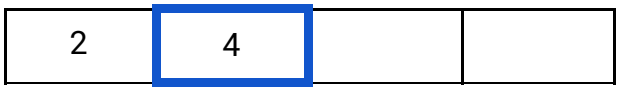
The same is applied to the two lists with single elements 2 and 6, resulting in a single list containing the elements 2 and 6 in that order.

Now the two two-element lists are merged. The first elements in each list are compared and the lowest, 2, placed into a new four-element list before being removed from the shorter list.

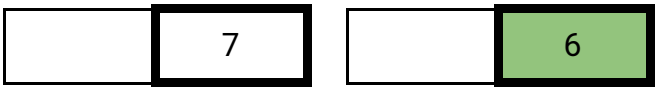


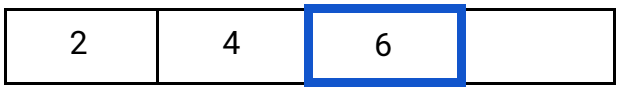
The process is now repeated, and the first two remaining elements of each list (now 4 and 6) are compared. The lowest, 4, is then added to the new list and removed from the shorter list.



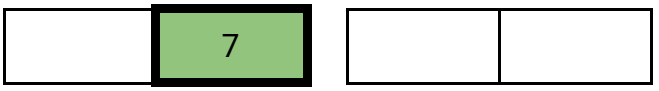


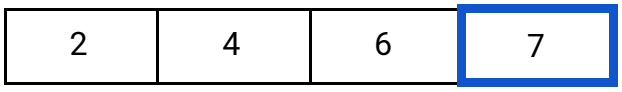
The process continues, with 7 and 6 being compared.



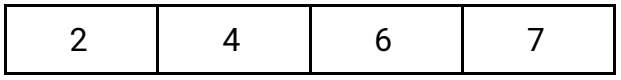


Now there's only one element remaining, 7, which is added to the back of the new list.





The sorted list can now be returned.



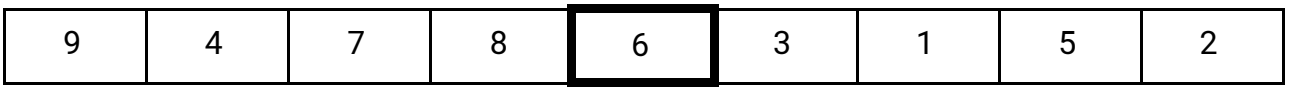
Quick Sort

Quick sort works by selecting an element, often the central element (called a pivot), and dividing the input around it. Elements smaller than the pivot are placed in a list to the left of the pivot and others are placed in a list to the right.

This process is then repeated recursively on each new list until all elements in the input are old pivots themselves or form a list of length 1. Despite the name, quick sort isn't particularly fast, in fact it's time complexity is $O(n^2)$.

Example

In this example, the pivot is chosen to be the central element. The first pivot is 6. Comparing elements from left to right, elements larger than the pivot are placed in a list to the right and elements smaller are placed in a list to the left.



6 is now an old pivot, and is in the correct position, so is shaded. There are now two lists - one of length 5 and another of length 3. A new pivot is selected for each of these lists and the elements partitioned around the pivot as before.

4	3	1	5	2	6	9	7	8

1, 6 and 7 are now old pivots, so are shaded. No elements were smaller than 1, and so there is no list to the left of 1. In a similar way, no elements were greater than 9, meaning that there is no list to the right of 9. From the two new lists, two pivots are chosen. When there is no central element, the pivot is rounded up (hence 5 is chosen over 3).

1	4	3	5	2	6	7	8	9

Again the elements are partitioned around the pivots and the old pivots are shaded. Note that once a pivot has been shaded, it never moves. This is because an old pivot is always in the correct position, even if the data isn't yet sorted.

Because 8 is a list of length 1, it can be ignored. Only one new pivot is chosen.

1	4	3	2	5	6	7	8	9

Elements are partitioned around the pivot and the old pivot is shaded. We can now see that the stopping condition (every element is an old pivot or a list of length 1) has been met, meaning that the algorithm can terminate and return the sorted data.

1	2	3	4	5	6	7	8	9