AQA Computer Science A-Level

4.5.2 Number bases

Advanced Notes

Specification:

3.5.2.1 Number base:

Be familiar with the concept of a number base, in particular:

- decimal (base 10)
- binary (base 2)
- hexadecimal (base 16)

Convert between decimal, binary and hexadecimal number bases.

Be familiar with, and able to use, hexadecimal as a shorthand for binary and to understand why it is used in this way.

Number bases

The same number can be represented in a variety of different ways. Humans use base 10, sometimes called denary or decimal, but we could use binary or hexadecimal.

Decimal (base 10)

Decimal is the number base that humans use to count, perhaps because we have ten fingers. Decimal uses the ten digits 0 through to 9 to represent numbers.

Decimal numbers can be denoted with a subscript 10, like so:

$$27_{10}$$

Binary (base 2)

Binary uses only two characters for each digit, either a 1 or a 0. These two values can easily be represented by computers with high or low current.

Binary numbers can be denoted with a subscript 2, like so:

$$10110010_{2}$$

Synoptic Link

Methods exist for representing **negative** and **non-integer** numbers with binary.

These methods are covered in **binary number system**.

Hexadecimal (base 16)

In contrast to decimal, hexadecimal uses the digits 0 through to 9 followed by the uppercase characters A to F to represent the decimal numbers 0 to 15.

Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Hexadecimal	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F

This means that hexadecimal can make use of 16 different characters for each digit.

Hexadecimal numbers can be denoted with a subscript 16, like so:

$$7E_{16}$$

Of all the number bases covered by this course, hexadecimal is the most compact. This means that it can represent the same number as binary or decimal while using far fewer digits.

For example, the number 733452 uses six digits in decimal, a whopping twenty in binary (10110011000100001100) but just five in hexadecimal (B310C).

As you'll see later in these notes, it's easy to convert between hexadecimal and binary. This, combined with its compact nature, makes hexadecimal useful as a shorthand representation for binary.

Conversions

Converting from binary to decimal

You can convert between binary and decimal by using place value headers. Starting with one and increasing in powers of two, placing larger values to the left of smaller values. For example, the binary number 10110010_2 could have place value headers added as follows:

128 (2^7)	64 (2^6)	32 (2^5)	16 (2^4)	8 (2^3)	4 (2^2)	2 (2^1)	1 (2^0)
1	0	1	1	0	0	1	0

The binary number could then be converted to decimal by adding together all of the place values with a binary one below them.

$$128 + 32 + 16 + 2 = 178$$

So the binary number 10110010_2 is equivalent to the decimal number 178_{10}.

Converting from decimal to binary

When converting from decimal to binary, you use the same place value headers. Starting from the left hand side, you place a one if the value is less than or equal to your number, and a zero otherwise.

Once you've placed a one, you must subtract the value of that position from your number and continue as before.

Let's say we're converting the number 53 to binary. First, write out your place value headers in powers of two. Keep on going until you've written a value which is larger than your number. For 53, we're going to go up to 64.

64	32	16	8	4	2	1

Now, starting from the left, compare the place value to your number. 64 is greater than 53 so we place a 0 under 64.

64	32	16	8	4	2	1
0						

Moving to the right, we see that 32 is lower than 53, so we place a 1 under 32.

64	32	16	8	4	2	1
0	1					

Because we've placed a one, we have to subtract 32 from 53 to find what's left to be represented. In this case, 53 - 32 = 21.

We move to the right again and find 16, which is lower than 21, so we place a 1 under 16.

64	32	16	8	4	2	1
0	1	1				

Again, because we've placed a one, we have to calculate a new value. 21 - 16 = 5.

Moving right, we find 8. This is larger than 5 so we place a 0.

64	32	16	8	4	2	1
0	1	1	0			

After moving right again, we find 4. As 4 is lower than 5, we place a 1.

64	32	16	8	4	2	1
0	1	1	0	1		

Having placed a 1, we must again calculate a new value. 5 - 4 = 1.

Moving right to find 2, we place a 0 as 2 is greater than 1.

64	32	16	8	4	2	1
0	1	1	0	1	0	

Moving right for the last time, we have 1. 1 = 1 so we place a 1.

64	32	16	8	4	2	1
0	1	1	0	1	0	1

Now that we've placed a 0 or a 1 under each place value, we have our answer. Although it's acceptable to remove any leading 0s, it may be preferable to add 0s to the start of your answer to make it a whole number of bytes (a multiple of 8 bits).

Note

If you're using signed binary, you'll need to take care with any leading 0s.

$$53_{10} = 0110101_2 = 110101_2 = 00110101_2$$

Converting from binary to hexadecimal

In order to convert from binary to hexadecimal, the binary number must first be split into nybbles. A nybble is four binary bits, or half a byte.

For example, the binary number 10110010_2 would be split into two nybbles:

$$10110010_2$$

$$1011_2 \qquad 0010_2$$

Each binary nybble is then converted to decimal as in the previous example:

8	4	2	1		8	4	2	1
1	0	1	1		0	0	1	0

$$8 + 2 + 1 = 11 \qquad\qquad 2 = 2$$

Once each nybble has been converted to decimal, the decimal value can be converted to its hexadecimal equivalent like so:

$$11_{10} = B_{16} \qquad\qquad 2_{10} = 2_{16}$$

Finally, the hexadecimal digits are concatenated to form a hexadecimal number:

$$10110010_2 = B2_{16}$$

Converting from hexadecimal to decimal (by converting from hexadecimal to binary)

When converting from hexadecimal to decimal, it's usually easiest to go via binary.

First convert your hexadecimal number to binary. Do this by converting each hexadecimal digit to a decimal digit and then to a binary nybble before combining the nybbles to form a single binary number.

$B2_{16}$

Split into hexadecimal digits

B_{16} 2_{16}

Convert hexadecimal to decimal

11_{10} 2_{10}

Convert decimal to binary nybbles

1011_2 0010_2

Combine binary nybbles

10110010_2

Note

When converting from hexadecimal to binary, use this method but stop here.

128	64	32	16	8	4	2	1
1	0	1	1	0	0	1	0

$$128 + 32 + 16 + 2 = 178$$

$$B2_{16} = 178_{10}$$

Converting from decimal to hexadecimal

Converting a number from decimal to hexadecimal is simply the reverse of converting from hexadecimal to decimal.

First convert your decimal number to binary, then split it into decimal nybbles. Each nybble can then be converted to decimal before converting decimal to hexadecimal.

Let's convert 178_{10} to hexadecimal.

First, convert to binary using the method described under "Converting from decimal to binary". If required, add leading 0s to your answer so that you have a whole number of nybbles.

$$10110010_2$$

Now split your binary number into nybbles.

$$1011_2 \qquad 0010_2$$

Then convert each nybble to decimal.

$$11_{10} \qquad 2_{10}$$

Lastly, convert decimal to hexadecimal and combine to form a single number.

$$B_{16} \qquad 2_{16}$$

$$B2_{16}$$

$$178_{10} = B2_{16}$$