Cambridge Assessment
Admissions Testing

TMUA/CTMUA **D513/01**

Model Solutions

PAPER 1

November 2020 **75 minutes**

Additional materials: Answer sheet

6019449018

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open the question paper until you are told that you may do so.

A separate answer sheet is provided for this paper. Please check you have one.
You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and full name.

This paper is the first of two papers.

There are 20 questions on this paper. For each question, choose the one answer you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

There are no penalties for incorrect responses, only marks for correct answers, so you should attempt **all** 20 questions. Each question is worth one mark.

You can use the question paper for rough working or notes, but **no extra paper** is allowed.

You **must** complete the answer sheet within the time limit.

Calculators and dictionaries are NOT permitted.

There is no formulae booklet for this test.

Please wait to be told you may begin before turning this page.

This question paper consists of 21 printed pages and 3 blank pages.

PV2

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1 Which of the following is an expression for the first derivative with respect to x of

$$\frac{x^3 - 5x^2}{2x\sqrt{x}}$$

A $-\frac{\sqrt{x}}{2}$

B $\frac{\sqrt{x}}{4}$

C $\frac{3x-5}{4\sqrt{x}}$

D $\frac{3\sqrt{x}-5}{4\sqrt{x}}$

E $\frac{3\sqrt{x}-10}{3\sqrt{x}}$

F $\frac{3x^2-10x}{3\sqrt{x}}$

$y = \frac{x^3 - 5x^2}{2x\sqrt{x}}$ and we want to find $\frac{dy}{dx}$ and we will do this using the quotient rule.

let $f(x) = x^3 - 5x^2 \Rightarrow f'(x) = 3x^2 - 10x$

and $g(x) = 2x\sqrt{x} = 2x x^{1/2} = 2x^{3/2}$

$\Rightarrow g'(x) = 3x^{1/2}$

$= 3\sqrt{x}$.

then from quotient rule:

$$\frac{dy}{dx} = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(3x^2 - 10x)2x^{3/2} - (x^3 - 5x^2)3x^{1/2}}{(2x^{3/2})^2}$$

$2x^{3/2} \cdot 2x^{3/2} = 4x^3$

$2x^{3/2}(3x^2 - 10x) = 6x^{7/2} - 20x^{5/2}$

$3x^{1/2}(x^3 - 5x^2) = 3x^{7/2} - 15x^{5/2}$

$$\Rightarrow \frac{dy}{dx} = \frac{6x^{7/2} - 20x^{5/2} - 3x^{7/2} + 15x^{5/2}}{4x^3}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x^{7/2} - 5x^{5/2}}{4x^3} = \frac{x^{5/2}(3x-5)}{4x^3}$$

then $3x^{7/2} - 5x^{5/2} = x^{5/2}(3x-5)$

then $\frac{x^{5/2}}{4x^3} = \frac{1}{4x^{1/2}} = \frac{1}{4\sqrt{x}}$

$$\Rightarrow \frac{dy}{dx} = \frac{3x-5}{4\sqrt{x}}$$

2 $(2x+1)$ and $(x-2)$ are factors of $2x^3 + px^2 + q$

What is the value of $2p+q$?

A -10

B $-\frac{38}{5}$

C $-\frac{22}{3}$

D $\frac{22}{3}$

E $\frac{38}{5}$

F 10

let $f(x) = 2x^3 + px^2 + q$

then for $(2x+1) \Rightarrow x = -\frac{1}{2} \Rightarrow f(-\frac{1}{2}) = 0$

and for $(x-2) \Rightarrow x = 2 \Rightarrow f(2) = 0$

$\Rightarrow f(-\frac{1}{2}) = -\frac{1}{4} + \frac{p}{4} + q = 0$

$\Rightarrow q = \frac{1}{4} - \frac{p}{4}$

and $f(2) = 16 + 4p + q = 0$

$\Rightarrow q = -4p - 16$

Then equating these:

$\frac{1}{4} - \frac{p}{4} = -4p - 16 \Rightarrow \frac{15}{4}p = -\frac{65}{4}$

$\Rightarrow p = -\frac{65}{15}$

$\Rightarrow q = -4 \cdot -\frac{65}{15} - 16 = \frac{4}{3}$

$\Rightarrow 2p + q = 2\left(-\frac{65}{15}\right) + \frac{4}{3} = \underline{-\frac{22}{3}}$

3 Find the complete set of values of x for which

$$(x+4)(x+3)(1-x) > 0 \quad \text{and} \quad (x+2)(x-2) < 0$$

A $1 < x < 2$

B $-2 < x < 1$

C $-2 < x < 2$

D $x < -2$ or $x > 1$

E $x < -4$ or $x > 2$

F $x < -4$ or $-3 < x < 1$

G $-4 < x < -2$ or $x > 1$

For $(x+4)(x+3)(1-x) > 0$

$\Rightarrow$ $x > -4$, $x > -3$ and $x < 1$

$\Rightarrow$ $-3 < x < 1$

For $(x+2)(x-2) < 0$

$\Rightarrow$ $x < -2$ and $x < 2$

$\Rightarrow$ Our final range must be

$-2 < x < 1$

4 The 1st, 2nd and 3rd terms of a geometric progression are also the 1st, 4th and 6th terms, respectively, of an arithmetic progression.

The sum to infinity of the geometric progression is 12.

Find the 1st term of the geometric progression.

A 1

B 2

C 3

D 4

E 5

F 6

Geometric : $S_\infty = \frac{a_1}{1-r} = 12 \Rightarrow a_1 = 12-12r$

and $a_n = a_1 r^{n-1}$

$\Rightarrow a_n = (12-12r)r^{n-1}$

Arithmetic : $a_n = a_1 + (n-1)d$

$\Rightarrow a_2 = (12-12r)r$ equals $a_4 = a_1 + 3d = 12-12r+3d$

$\Rightarrow a_3 = (12-12r)r^2$ equals $a_6 = a_1 + 5d = 12-12r+5d$

$\Rightarrow (12-12r)r = 12-12r+3d$ and $(12-12r)r^2 = 12-12r+5d$

$\Rightarrow 12r - 12r^2 - 12 + 12r = 3d$ $\quad 12r^2 - 12r^3 - 12 + 12r = 5d$

$\Rightarrow d = -4r^2 + 8r - 4$ and $d = \frac{-12r^3 + 12r^2 + 12r - 12}{5}$

$\Rightarrow$ Equate these $\Rightarrow$ $-20r^2 + 40r - 20 = -12r^3 + 12r^2 + 12r - 12$

$\Rightarrow 12r^3 - 32r^2 + 28r - 8 = 0$

$\Rightarrow 3r^3 - 8r^2 + 7r - 2 = 0$

$\Rightarrow (r-1)^2(3r-2) = 0$

$\Rightarrow r = 1$ or $r = \frac{2}{3}$

We know $r=1 \Rightarrow S_\infty$ doesn't exist, hence $r = \frac{2}{3}$. $\Rightarrow d = -4r^2 + 8r - 4 = -\frac{4}{9}$

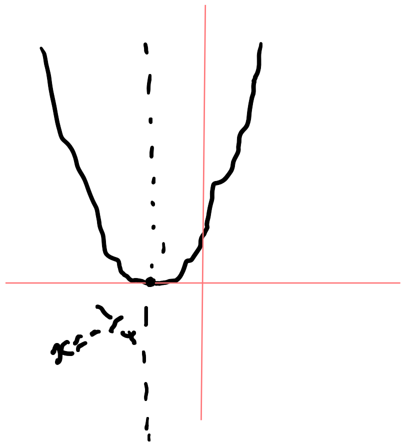
$\Rightarrow d = -\frac{4}{9}$

Geometric $a_1 = 12-12r$

$a_1 = 12 - 12 \times \frac{2}{3}$

$a_1 = 4$

5 The curve S has equation



$$y = px^2 + 6x - q$$

where p and q are constants.

S has a line of symmetry at $x = -\frac{1}{4}$ and touches the x-axis at exactly one point.

What is the value of $p + 8q$?

turning point

$b^2 - 4ac = 0$

A 6

B 18

C 21

D 25

E 38

We have a parabola with one real root and its turning point has x-coordinate $x = -\frac{1}{4}$.

$b^2 - 4ac = 6^2 - 4(p)(-q) = 36 + 4pq = 0$

$\Rightarrow pq = -9$

$\Rightarrow y = px^2 + 6x - q$

$\Rightarrow y = p\left(x^2 + \frac{6x}{p} - \frac{q}{p}\right)$

$y = p\left(x + \frac{3}{p}\right)^2 + \frac{9}{p^2} - \frac{q}{p}$

Remember we flip the sign to get our turning point!

Then $-\frac{3}{p} = -\frac{1}{4} \Rightarrow p = 12$

then using $pq = -9$

$\Rightarrow 12q = -9$

$\Rightarrow q = -\frac{9}{12} = -\frac{3}{4}$

$\Rightarrow p + 8q = 12 + 8 \cdot (-3/4)$

$= \underline{6}$

6 Find the maximum value of the function

$$f(x) = \frac{1}{5^{2x} - 4(5^x) + 7}$$

A $\frac{1}{7}$

B $\frac{1}{4}$

C $\frac{1}{3}$

D 3

E 4

F 7

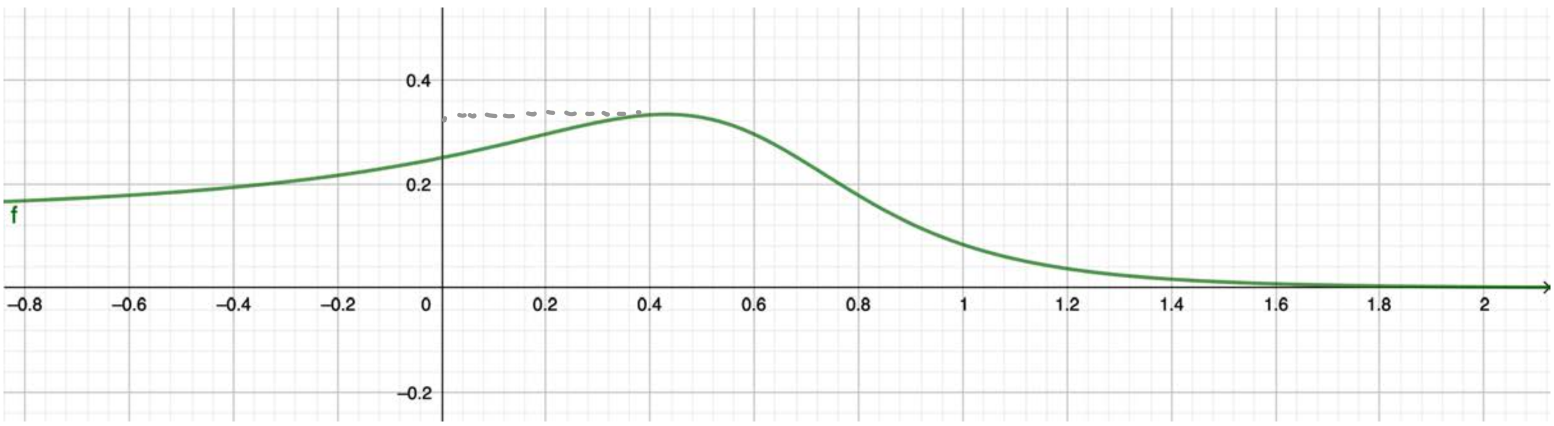
Let $u = 5^x$ so we have

$$\frac{1}{u^2 - 4u + 7} = \frac{1}{(u-2)^2 + 3}$$

maximum value will occur when denominator is at a minimum

$u = 2$

$5^x = 2 \Rightarrow x = \frac{1}{\ }$



7 Given that

$$2^{3x} = 8^{(y+3)}$$

and

$$4^{(x+1)} = \frac{16^{(y+1)}}{8^{(y+3)}}$$

what is the value of $x + y$?

A -23

B -22

C -15

D -14

E -11

F -10

$2^{3x} = 8^{(y+3)}$

$\ln(2^{3x}) = \ln(8^{y+3})$

$3x \ln(2) = (y+3)\ln(8)$

$3x = \frac{(y+3)\ln(8)}{\ln(2)} \Rightarrow 3x = (y+3)\times 3$

$x = y+3$

$\Rightarrow y = x-3$

and

$4^{x+1} = \frac{16^{y+1}}{8^{y+3}}$

$\ln(16) = \ln 4^4$
$= 4\ln 2$

$\ln(8) = \ln 2^3$
$= 3\ln(2)$

$(x+1)\ln(4) = (y+1)\ln(16) - (y+3)\ln(8)$

$(x+1)\ln(4) = 4\ln(2)(y+1) - 3\ln(2)(y+3)$

$(x+1)\ln(4) = 4\ln(2)y + 4\ln(2) - 3\ln(2)y - 9\ln(2)$

$\ln(4)x = 4\ln(2)y - 3\ln(2)y - 7\ln(2)$

$2\ln(2)x = \ln(2)y - 7\ln(2)$

$\Rightarrow y = 2x+7$

$\Rightarrow x - 3 = 2x + 7$

$\Rightarrow -10 = x \Rightarrow y = -10 - 3 = -13$

$\Rightarrow x + y = -10 - 13 = \underline{\underline{-23}}$

8 The function f is defined for all real x as

$$f(x) = (p - x)(x + 2)$$

Find the complete set of values of p for which the maximum value of $f(x)$ is less than 4.

A $-2 - 4\sqrt{2} < p < -2 + 4\sqrt{2}$

B $-2 - 2\sqrt{2} < p < -2 + 2\sqrt{2}$

C $-2\sqrt{5} < p < 2\sqrt{5}$

D $-6 < p < 2$

E $-4 < p < 0$

F $-2 < p < 2$

$f(x) = (p-x)(x+2)$

$f(x) = px + 2p - x^2 - 2x$

$f'(x) = p - 2x - 2 = 0$

$\Rightarrow 2x = p - 2$

$\Rightarrow x = \frac{p-2}{2}$

$f(x) < 4$

$\Rightarrow px + 2p - x^2 - 2x < 4$

$\Rightarrow \frac{p(p-2)}{2} + 2p - \frac{(p-2)^2}{4} - 2\left(\frac{p-2}{2}\right)$

$\Rightarrow \frac{p^2+4p-4}{4} + 2 < 4$

$\Rightarrow p^2 + 4p - 4 < 8$

$\Rightarrow p^2 + 4p - 12 < 0$

$\Rightarrow (p+6)(p-2) < 0$

$\Rightarrow p < -6 \quad p < 2$

$\Rightarrow -6 < p < 2$

①

9 The quadratic expression $x^2 - 14x + 9$ factorises as $(x - \alpha)(x - \beta)$, where α and β are positive real numbers.

②

Which quadratic expression can be factorised as $(x - \sqrt{\alpha})(x - \sqrt{\beta})$?

A $x^2 - \sqrt{10}x + 3$

B $x^2 - \sqrt{14}x + 3$

C $x^2 - \sqrt{20}x + 3$

D $x^2 - 178x + 81$

E $x^2 - 176x + 81$

F $x^2 + 196x + 81$

Expanding the quadratics :

① : $x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - 14x + 9$

② : $x^2 - (\sqrt{\alpha} + \sqrt{\beta})x + \sqrt{\alpha}\sqrt{\beta}$

By ① we can see $\alpha + \beta = 14$ ③

$\alpha\beta = 9$ ④

So by ④ $\sqrt{\alpha\beta} = \sqrt{9}$

$\sqrt{\alpha}\sqrt{\beta} = 3$ (we reject -3 as α, β are both positive in the question)

By ③ $=14$

Now $(\sqrt{\alpha} + \sqrt{\beta})^2 = \alpha + \beta + 2\sqrt{\alpha}\sqrt{\beta}$

$= 14 + 2\sqrt{\alpha}\sqrt{\beta}$

$= 14 + 2 \times 3 = 20$

So ② becomes $x^2 - \sqrt{20}x + 3$

10 The following sequence of transformations is applied to the curve $y = 4x^2$

1. Translation by $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$

2. Reflection in the x-axis

3. Stretch parallel to the x-axis with scale factor 2

$4(x^2-6x+9)-5$

$4x^2-24x+36-5$

$4x^2-24x+31$

What is the equation of the resulting curve?

$y = 4x^2$

A $y = -x^2 + 12x - 31$

B $y = -x^2 + 12x - 41$

C $y = x^2 + 12x + 31$

D $y = x^2 + 12x + 41$

E $y = -16x^2 + 48x - 31$

F $y = -16x^2 + 48x - 41$

G $y = 16x^2 - 48x + 31$

H $y = 16x^2 - 48x + 41$

1. $y = 4(x-3)^2 + 5$

$\Rightarrow y = 4x^2 - 24x + 31$

2. $y = -(4x^2 - 24x + 31)$

$\Rightarrow y = -4x^2 + 24x - 31$

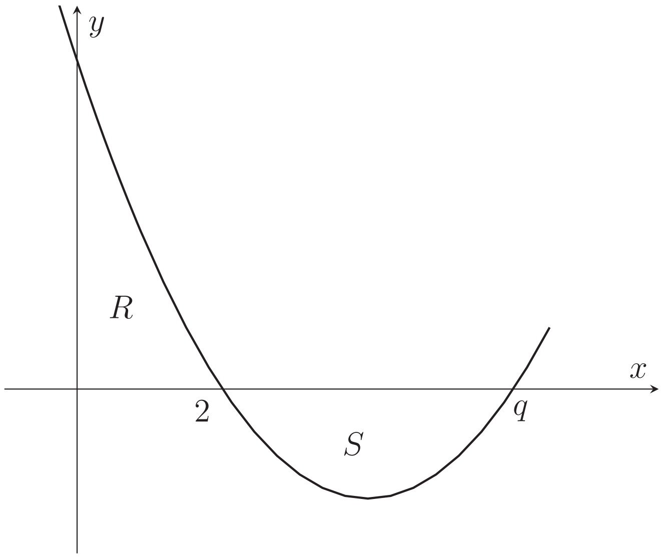
3. $y = -4(\frac{1}{2}x)^2 + 24(\frac{1}{2}x) - 31$

$\Rightarrow y = -4 \times \frac{1}{4}x^2 + 12x - 31$

$\Rightarrow y = -x^2 + 12x - 31$

$\Rightarrow$ option A

11 The quadratic function shown passes through $(2, 0)$ and $(q, 0)$, where $q > 2$.



What is the value of q such that the area of region R equals the area of region S?

A $\sqrt{6}$

B 3

C $\frac{18}{5}$

D 4

E 6

F $\frac{33}{5}$

Since root is 2, q equation can be written as $y = k(x-q)(x-2)$. Note that k will stretch the graph parallel to the y axis which will effect both area S and R the same so we can ignore it and take $k = 1$ for simplicity.

Since R and S need to be the same and S will be negative

$$\int_0^q (x-q)(x-2)\,dx = 0$$

(because the area between 0 and q will be 0)

$$\int_0^q x^2 - (q+2)x + 2q\,dx = \left[\frac{1}{3}x^3 - \frac{1}{2}(q+2)x^2 + 2qx\right]_0^q = 0$$

$$= -\frac{1}{6}q^3 + q^2 = 0$$

So $q = 6$

12 How many real solutions are there to the equation

$$3\cos x = \sqrt{x}$$

where x is in radians?

A 0

B 1

C 2

D 3

E 4

F 5

G infinitely many

$y = 3\cos x$ has maximum points of $y = 3$.

This means that it will intersect with $\sqrt{x}$ until $\sqrt{x} > 3$

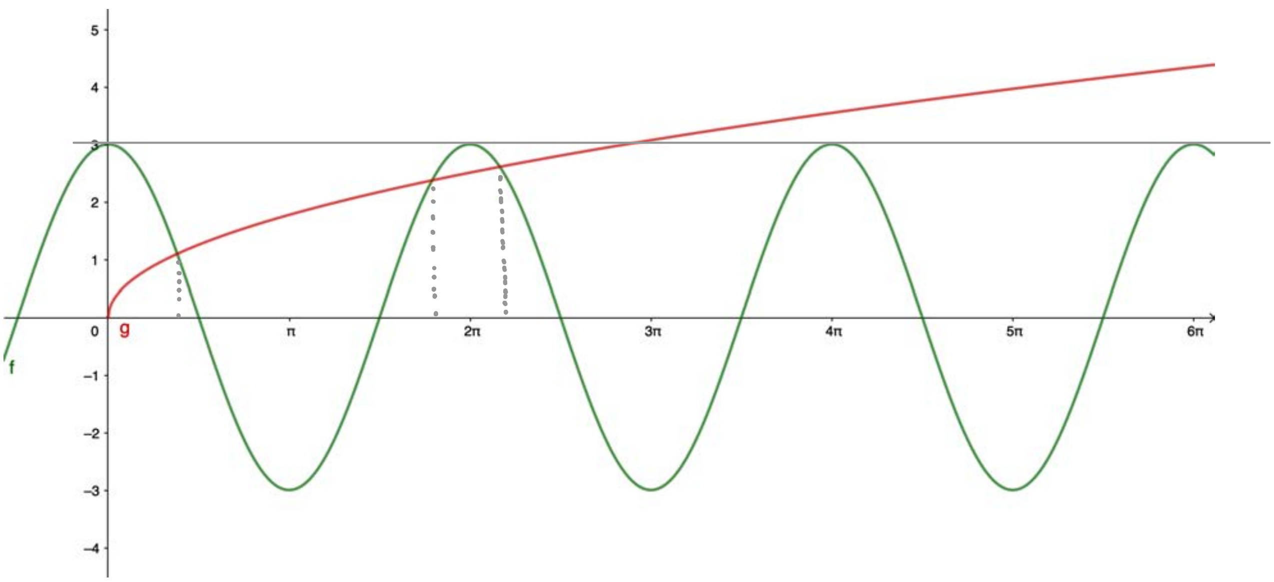
$\Rightarrow$ $x > 9$.

One period of $3\cos x$ goes from 0 to 2π.

and we have that: $2\pi > 9$ is false but $3\pi > 9$ is true

$\Rightarrow$ There will be 2 points of intersection between $3\cos x$ and $\sqrt{x}$ in $x: 0 \to 2\pi$ and then one further intersection between 2π and 3π. $\Rightarrow$ $\underline{\underline{3}}$

$\Rightarrow$ We can see this in the plot.



13 Find the coefficient of x^2y^4 in the expansion of $(1+x+y^2)^7$

A 6

B 10

C 21

D 35

E 105

F 210

let $u = 1+x$ and then we have $(u+y^2)^7$

$$\Rightarrow \binom{7}{0}u^7(y^2)^0 + \binom{7}{1}u^6(y^2)^1 + \binom{7}{2}u^5(y^2)^2 + \binom{7}{3}u^4(y^2)^3 + \dots$$

$$\dots \binom{7}{4}u^3(y^2)^4 + \binom{7}{5}u^2(y^2)^5 + \binom{7}{6}u(y^2)^6 + \binom{7}{7}(y^2)^{12}$$

$\Rightarrow$ we see x^2y^4 comes from :

$$\binom{7}{2}u^5(y^2)^2 = \binom{7}{2}(1+x)^5y^4 = 21(1+x)^5y^4$$

$\Rightarrow$ Now we do a seperate binomial expansion for $(1+x)^5$
and x^2 will occur for : $\binom{5}{2}x^2 1^3 = 10x^2$

$$\Rightarrow 21 \times 10x^2y^4 \Rightarrow 210x^2y^4$$

$\Rightarrow$ 210 is the coefficient.

14 The area enclosed between the line $y = mx$ and the curve $y = x^3$ is 6.

What is the value of m?

A 2

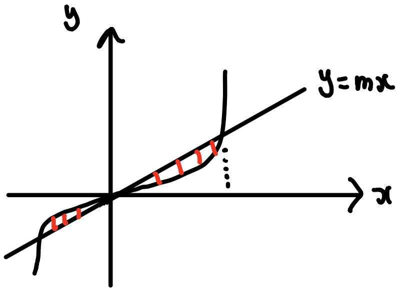
B 4

C $\sqrt{3}$

D $\sqrt{6}$

E $2\sqrt{3}$

F $2\sqrt{6}$



$mx = x^3$
$m = x^2$
$\Rightarrow x = \sqrt{m}$
and $x = 0$.
So, our limits are 0 and $\sqrt{m}$.

The area is equal below and above x-axis so we can integrate from 0 to $\sqrt{m}$ but we half the area.

$$3 = \int_0^{\sqrt{m}} mx - x^3 \, dx$$

$$3 = \left[\frac{mx^2}{2} - \frac{x^4}{4}\right]_0^{\sqrt{m}}$$

$$\Rightarrow \frac{m^2}{2} - \frac{m^2}{4} = 3$$

$$\Rightarrow \frac{1}{4}m^2 = 3$$

$$\Rightarrow m^2 = 12$$

$$\Rightarrow m = \underline{2\sqrt{3}}$$

15 Find the positive difference between the two real values of x for which

$$(\log_2 x)^4 + 12\left(\log_2\left(\frac{1}{x}\right)\right)^2 - 2^6 = 0$$

A 4

B 16

C $\frac{15}{4}$

D $\frac{17}{4}$

E $\frac{255}{16}$

F $\frac{257}{16}$

$\Rightarrow \log_2(x)^4 + 12\log_2(x)^2 - 64 = 0$

Then let $u = \log_2(x)$

$\Rightarrow u^4 + 12u^2 - 64 = 0$ and let $v = u^2$

$\Rightarrow v^2 + 12v - 64 = 0 \Rightarrow v = 4$ and $v = -16$

u won't be real $\Rightarrow$ ignore.

$\Rightarrow u = \sqrt{4}$

$\Rightarrow u = \pm 2$

$\Rightarrow \log_2(x) = 2$ and $\log_2(x) = -2$

$x = 2^2$ $\quad$ $x = 2^{-2}$

$\Rightarrow x = 4$ $\quad$ $x = \frac{1}{4}$

$\Rightarrow 4 - \frac{1}{4} = \underline{\underline{\frac{15}{4}}}$

16 The circle C_1 has equation $(x + 2)^2 + (y - 1)^2 = 3$

The circle C_2 has equation $(x - 4)^2 + (y - 1)^2 = 3$

The straight line l is a tangent to both C_1 and C_2 and has positive gradient.

The acute angle between l and the x-axis is θ

Find the value of $\tan\theta$

A $\frac{1}{2}$

B 2

C $\frac{\sqrt{2}}{2}$

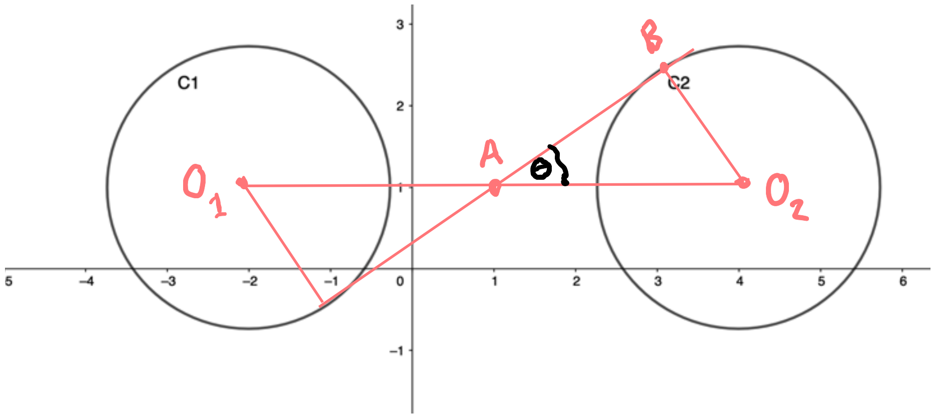
D $\sqrt{2}$

E $\frac{\sqrt{6}}{2}$

F $\frac{\sqrt{6}}{3}$

G $\frac{\sqrt{3}}{3}$

H $\sqrt{3}$

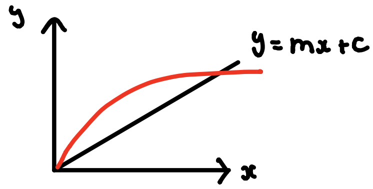


To find $\tan\theta$ we find lengths AB and BO_2 as ABO_2 is a right angled triangle. Length BO_2 is $\sqrt{3}$ from equation of circle. Length $O_1O_2 = 6$ so length $AO_2 = 3$. Using pythagoras on triangle ABO_2

$$\sqrt{3}^2 + AB^2 = 3$$

$$3 + AB^2 = 9 \Rightarrow AB = \sqrt{6}$$

so $\tan\theta = \frac{\sqrt{3}}{\sqrt{6}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ as required



17 Find the complete set of values of m in terms of c such that the graphs of $y = mx + c$ and $y = \sqrt{x}$ have two points of intersection.

A $0 < m < \frac{1}{4c}$

B $0 < m < 4c^2$

C $m > \frac{1}{4c}$

D $m < \frac{1}{4c}$

E $m > 4c^2$

F $m < 4c^2$

We first see that the lower bound will always be $m > 0$, as anything with a negative gradient will obviously never have two points of intersection.

$mx + c = \sqrt{x}$

$(mx+c)^2 = x$

$m^2x^2 + 2cmx + c^2 - x = 0$

$m^2x^2 + x(2cm - 1) + c^2 = 0$

$b^2 - 4ac = (2cm-1)^2 - 4(m^2)(c^2) = 1 - 4cm > 0$

$\Rightarrow \quad -4cm > -1$

$\Rightarrow \quad 4cm < 1$

$\Rightarrow \quad m < \frac{1}{4c}$

hence $\quad 0 < m < \frac{1}{4c}$

18 Find the number of solutions and the sum of the solutions of the equation

$$1 - 2\cos^2 x = |\cos x|$$

where $0 \leq x \leq 180°$

A Number of solutions $= 2$ Sum of solutions $= 180°$

B Number of solutions $= 2$ Sum of solutions $= 240°$

C Number of solutions $= 3$ Sum of solutions $= 180°$

D Number of solutions $= 3$ Sum of solutions $= 360°$

E Number of solutions $= 4$ Sum of solutions $= 240°$

F Number of solutions $= 4$ Sum of solutions $= 360°$

$1 - 2\cos^2 x = |\cos x|$

$\Rightarrow 2\cos^2 x + |\cos x| - 1 = 0$ let $\cos x = x$

$\Rightarrow 2x^2 + |x| - 1 = 0$

$\Rightarrow |x| = 1 - 2x^2$

+ve x : $x = 1 - 2x^2 \Rightarrow 2x^2 + x - 1 = 0 \Rightarrow x = \frac{1}{2}$ or $x = -1$

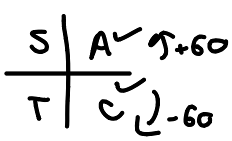
-ve x : $-x = 1 - 2x^2 \Rightarrow 2x^2 - x - 1 = 0 \Rightarrow x = 1$ or $x = -\frac{1}{2}$

We check our answers in original equation $2x^2 + |x| - 1 = 0$ and we see that for $x = \frac{1}{2}$ and $-\frac{1}{2}$ work (they give 0) but -1 and 1 do not, so we discard these.

$\Rightarrow \cos x = \frac{1}{2}$

$\Rightarrow x = \cos^{-1}\left(\frac{1}{2}\right)$

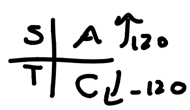
$x = 60°$



$\cos x = -\frac{1}{2}$

$x = \cos^{-1}\left(-\frac{1}{2}\right)$

$x = 120°$



$\Rightarrow x = 60°$ and $x = 300°$

$\Rightarrow x = 120°$ and $x = 240°$

But we only have $0 \leq x \leq 180°$ $\Rightarrow$ x values are $x = 60°$ and $x = 120°$

$\Rightarrow$ We have two solutions and their sum is $\underline{180°}$.

19 Find the lowest positive integer for which $x^2 - 52x - 52$ is positive.

A 26

B 27

C 51

D 52

E 53

F 54

$x^2 - 52x - 52 > 0$

$(x - 26)^2 - 26^2 - 52 > 0$

$(x - 26)^2 - 728 > 0$

$\Rightarrow \quad (x - 26)^2 > 728$

$\Rightarrow \quad x - 26 > \sqrt{728}$

$\Rightarrow \quad x > 26 + \sqrt{728}$

$x > 52.981\ldots$

$\Rightarrow$ Positive for $x = \underline{53}$

20 For how many values of a is the equation

$$(x-a)(x^2-x+a)=0$$

satisfied by exactly two distinct values of x?

A 0

B 1

C 2

D 3

E 4

F more than 4

$(x-a)(x^2-x+a)=0$

$x=a$ and $x^2-x+a=0$

One distinct Value of x.

$b^2-4ac=0$

other distinct x will occur when $b^2-4ac=0$

$\Rightarrow 1-4(a)=1-4a=0$

$\Rightarrow a=\frac{1}{4}$

$\Rightarrow a=x$ and $a=\frac{1}{4}$

$\Rightarrow$ 2 values of a.

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