Cambridge Assessment
Admissions Testing

Test of Mathematics for University Admission

Paper 1 2017 hand-written worked answers

TEST OF MATHEMATICS FOR UNIVERSITY ADMISSION

D513/11

PAPER 1

Model answers

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Wednesday 8 November 2017

Time: 75 minutes

Additional Materials: Answer sheet

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open the question paper until you are told that you may do so.

A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

This paper is the first of two papers.

There are 20 questions on this paper. For each question, choose the one answer you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

There are no penalties for incorrect responses, only marks for correct answers, so you should attempt all 20 questions. Each question is worth one mark.

Any rough work should be done on this question paper. No extra paper is allowed.

Please complete the answer sheet with your candidate number, centre number, date of birth, and full name.

Calculators and dictionaries must **NOT** be used.
There is no formulae booklet for this test.

Please wait to be told you may begin before turning this page.

This question paper consists of 21 printed pages and 3 blank pages.

PV2

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$$\frac{dy}{dx} = 3x^2 - \frac{2-3x}{x^3}$$

$$= 3x^2 - \frac{2}{x^3} + \frac{3}{x^2}$$

$$= 3x^2 - 2x^{-3} + 3x^{-2}$$

1 Given that

$$\frac{dy}{dx} = 3x^2 - \frac{2-3x}{x^3}, \quad x \neq 0$$

and $y = 5$ when $x = 1$, find y in terms of x.

A $y = \frac{1}{3}x^3 + x^{-2} - 3x^{-1} + 6\frac{2}{3}$

B $y = x^3 + \frac{1}{2}x^{-2} - 3x^{-1} + 6\frac{1}{2}$

C $y = x^3 + x^{-2} - 3x^{-1} + 6$

D $y = x^3 + x^{-2} - x^{-1} + 4$

E $y = x^3 + 2x^{-2} - x^{-1} + 3$

F $y = 3x^3 + x^{-2} - x^{-1} + 2$

Integrate:

$$\int 3x^2 - 2x^{-3} + 3x^{-2} \, dx$$

$$= \frac{3x^3}{3} + \frac{2x^{-2}}{2} - \frac{3x^{-1}}{1} + c$$

$$= x^3 + x^{-2} - 3x^{-1} + c$$

When $x = 1$, $y = 5$ so:

$y = x^3 + x^{-2} - 3x^{-1} + c$

$5 = 1^3 + 1^{-2} - 3 \times 1^{-1} + c$

$5 = 1 + 1 - 3 + c$

$5 = -1 + c$

$c = 6$

$\therefore y = x^3 + x^{-2} - 3x^{-1} + 6$

2 The function f is given by

$$f(x) = \left(\frac{2}{x} - \frac{1}{2x^2}\right)^2 \quad (x \neq 0)$$

What is the value of $f''(1)$?

$$f(x) = \left(\frac{2}{x} - \frac{1}{2x^2}\right)^2$$

$$= \frac{4}{x^2} - \frac{2}{2x^3} - \frac{2}{2x^3} + \frac{1}{4x^4}$$

$$= \frac{4}{x^2} - \frac{4}{2x^3} + \frac{1}{4x^4}$$

$$= \frac{4}{x^2} - \frac{2}{x^3} + \frac{1}{4x^4}$$

$$= 4x^{-2} - 2x^{-3} + \frac{1}{4}x^{-4}$$

A -3

B -1

C 5

D 17

E 29

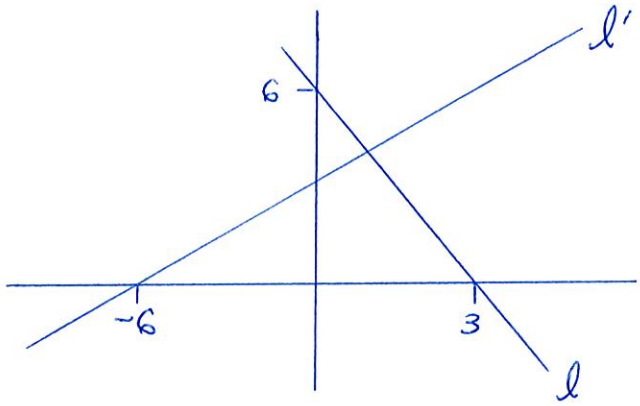
F 80

So $f'(x) = -8x^{-3} + 6x^{-4} - x^{-5}$

$f''(x) = 24x^{-4} - 24x^{-5} + 5x^{-6}$

Then $f''(1) = 24 - 24 + 5 = 5$

Call the second line l'



3 A line l has equation $y = 6 - 2x$

A second line is perpendicular to l and passes through the point $(-6, 0)$.

Find the area of the region enclosed by the two lines and the x-axis.

A $16\frac{1}{5}$

B 18

C $21\frac{3}{5}$

D 27

E $40\frac{1}{2}$

l is $y = 6 - 2x$ Gradient is -2

$\therefore$ gradient of l' is $1/2$

For l' we know $y = mx + c$

$= \frac{1}{2}x + c$

Since $(-6, 0)$ lies on l', we get $0 = \frac{1}{2} \times {}^{-}6 + c$

$c = 3$

Hence the equation of l' is $y = \frac{1}{2}x + 3$

Our lines $y = 6 - 2x$ & $y = \frac{1}{2}x + 3$ intersect so $6 - 2x = \frac{1}{2}x + c$

$2.5x = 3$

$x = 6/5$

So $y = 6 - \frac{6}{5} \times 2 = \frac{18}{5}$

$\therefore$ Area $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times (3+6) \times \frac{18}{5} = 16\,1/5$

4 When $(3x^2 + 8x - 3)$ is multiplied by $(px - 1)$ and the resulting product is divided by $(x + 1)$, the remainder is 24.

What is the value of p?

$3x^2 + 8x - 3 = (3x-1)(x+3)$

A -4

B 2

C 4

D $\frac{8}{7}$

E $\frac{11}{4}$

If we let $x = -1$ then:

$$(3x-1)(x+3)(px-1) = (-3-1)(-1+3)(-p-1)$$
$$= -4 \times 2 \times (-p-1)$$
$$= -8(-p-1)$$
$$= 8p + 8$$

Remainder is 24 when divided by $(x+1)$ so $8p + 8 = 24$

$$8p = 16$$
$$p = 2$$

we have:

(1) $x^2-8x+12<0$

(2) $2x+1>9$

5 S is the complete set of values of x which satisfy **both** the inequalities

$$x^2 - 8x + 12 < 0 \text{ and } 2x + 1 > 9$$

The set S can also be represented as a single inequality.

Which one of the following single inequalities represents the set S ?

A $(x^2 - 8x + 12)(2x + 1) < 0$

B $(x^2 - 8x + 12)(2x + 1) > 0$

C $x^2 - 10x + 24 < 0$

D $x^2 - 10x + 24 > 0$

E $x^2 - 6x + 8 < 0$

F $x^2 - 6x + 8 > 0$

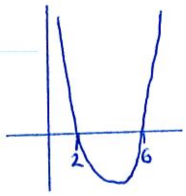
G $x < 2$

H $x > 6$

For (1):

$x^2-8x+12=(x-6)(x-2)$

If $y=(x-6)(x-2)$ then it looks like



So $(x-6)(x-2)<0$ when $2<x<6$

For (2):

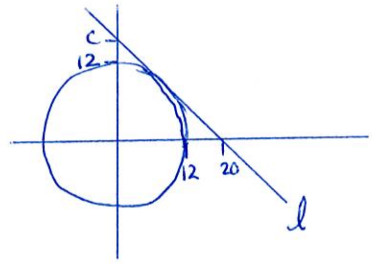
$2x+1>9$

$2x>8$

$x>4$

when both $x>4$ and $2<x<6$ are true, we have $4<x<6$

This is given by $(x-4)(x-6)<0$ ie $x^2-10x+24<0$



circle $x^2+y^2=144$

has centre $(0,0)$

radius $\sqrt{144}=12$

6 A tangent to the circle $x^2 + y^2 = 144$ passes through the point $(20, 0)$ and crosses the positive y-axis.

What is the value of y at the point where the tangent meets the y-axis?

A 12

B 15

C $\frac{49}{3}$

D 20

E $\frac{64}{3}$

F $\frac{80}{3}$

Equation of l is of the form $y = mx + c$

It passes through $(20,0)$ so $0 = 20m + c$

$c = -20m$

ie. $y = mx - 20m$

$= m(x-20)$

If we sub this into $x^2+y^2=144$ we get:

$x^2 + (m(x-20))^2 = 144$

$x^2 + m^2(x^2 - 40x + 400) = 144$

$(m^2+1)x^2 - 40m^2x + (400m^2 - 144) = 0$

For this to have one root we need $b^2 - 4ac = 0$, ie

$(-40m^2)^2 - 4(m^2+1)(400m^2 - 144) = 0$

$1600m^4 - 1600m^4 - 1024m^2 + 576 = 0$

$-1024m^2 = -576$

$m^2 = \frac{9}{16}$

$m = \pm\frac{3}{4}$ but as we know the gradient is negative, ie $m<0$,

then $m = -3/4$

So we get $y = \frac{-3}{4}(x-20)$

Tangent meets the y axis at $x=0$, ie. $y = -3/4 \times -20 = 15$

call the common difference of the AP d
ratio GP r

AP is: (1) p
(2) $q = p + d$
(3) $p^2 = p + 2d$

GP is: (1) p
(2) $p^2 = pr$
(3) $q = pr^2$

7 The first three terms of an arithmetic progression are p, q and p^2 respectively, where $p < 0$

The first three terms of a geometric progression are p, p^2 and q respectively.

Find the sum of the first 10 terms of the arithmetic progression.

A $\frac{23}{8}$

B $\frac{95}{8}$

C $\frac{115}{8}$

D $\frac{185}{8}$

In the GP, from (2) we get $p = r$
AP (3) $q = p^3$

using $q = p^3$ in the AP we get: $q = p + d$
$p^3 = p + d$
$d = p^3 - p$
$2d = 2p^3 - 2p$

and $p^2 = p + 2d$
$2d = p^2 - p$

then we have:
$2p^3 - 2p = p^2 - p$
$2p^3 - p^2 - p = 0$
$p(2p^2 - p - 1) = 0$
$p(2p+1)(p-1) = 0$ so $p = 0, -1/2$ and 1

we were told $p < 0$ so $p = -1/2$

$d = p^3 - p = \frac{-1}{8} + \frac{1}{2} = \frac{3}{8}$ so $a = p = \frac{-1}{2}$

$S_n = \frac{n(2a + (n-1)d)}{2}$ $\therefore$ $S_{10} = \frac{10(-1 + 9 \times 3/8)}{2} = \frac{95}{8}$

Look at each of $(1-2\sin x)$ and $\cos x$ in turn.

8 Find the complete set of values of x, with $0 \le x \le \pi$, for which

$$(1 - 2\sin x)\cos x \ge 0$$

$\boxed{1-2\sin x = 0}$

A $0 \le x \le \frac{\pi}{6}$, $\frac{\pi}{2} \le x \le \frac{5\pi}{6}$

when $\sin x = \frac{1}{2}$ ie $x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

B $0 \le x \le \frac{\pi}{6}$, $\frac{5\pi}{6} \le x \le \pi$

when $x = 0$, $1 - 2\sin x = 1$

C $\frac{\pi}{6} \le x \le \frac{\pi}{2}$, $\frac{5\pi}{6} \le x \le \pi$

Hence $1-2\sin x \geqslant 0$ when $0 \le x \le \frac{\pi}{6}$ and $\frac{5\pi}{6} \le x \le \pi$

D $\frac{\pi}{6} \le x \le \frac{5\pi}{6}$

when $x = \pi$, $1 - 2\sin x = -1$

Hence $1 - 2\sin x \le 0$ when $\frac{\pi}{6} \le x \le \frac{5\pi}{6}$

$\boxed{\cos x = 0}$ when $x = \frac{\pi}{2}$

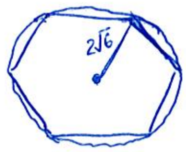
$\cos x \geqslant 0$ when $0 \le x \le \frac{\pi}{2}$

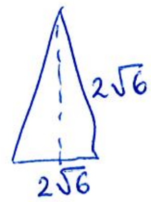
$\cos x \le 0$ when $\frac{\pi}{2} \le x \le \pi$

combining all of these expressions we get

$(1-2\sin x)\cos x \geqslant 0$ when $0 \le x \le \frac{\pi}{6}$

≤ 0 $\frac{\pi}{2} \le x \le \frac{5\pi}{6}$





9 A circle has equation $x^2 + y^2 - 18x - 22y + 178 = 0$

A regular hexagon is drawn inside this circle so that the vertices of the hexagon touch the circle.

What is the area of the hexagon?

A 6

B $6\sqrt{3}$

C 18

D $18\sqrt{3}$

E 36

F $36\sqrt{3}$

G 48

H $48\sqrt{3}$

Circle $x^2+y^2-18x-22y+178=0$

$(x^2-18x)+(y^2-22y)=-178$

$(x-9)^2-81+(y-11)^2-121=-178$

$(x-9)^2+(y-11)^2=24$

centre $(9, 11)$, radius $\sqrt{24}=2\sqrt{6}$

A hexagon is 6 triangles

Triangle height $h^2=(2\sqrt{6})^2-(\sqrt{6})^2$

$=4\times 6-6$

$=3\times 6$

$=18$

$h=\sqrt{18}$

$=3\sqrt{2}$

Triangle area $= \frac{1}{2}\times \text{base}\times\text{height} = \frac{1}{2}\times 2\sqrt{6}\times 3\sqrt{2} = 3\sqrt{2}\times\sqrt{6} = 6\sqrt{3}$

Hexagon area $= 6\times 6\sqrt{3} = 36\sqrt{3}$

[Turn over

10 A curve C has equation $y = f(x)$ where

$$f(x) = p^3 - 6p^2x + 3px^2 - x^3$$

and p is real. $= -x^3 + 3px^2 - 6p^2x + p^3$

The gradient of the normal to the curve C at the point where $x = -1$ is M.

What is the greatest possible value of M as p varies?

A $-\frac{3}{2}$

B $-\frac{2}{3}$

C $-\frac{1}{2}$

D $\frac{1}{4}$

E $\frac{2}{3}$

F $\frac{3}{2}$

$f'(x) = -3x^2 + 6px - 6p^2$

$f'(-1) = -3 - 6p - 6p^2$ ← gradient of C

Gradient of normal $= \dfrac{-1}{-6p^2 - 6p - 3} = \dfrac{1}{3(2p^2 + 2p + 1)}$

$\dfrac{d}{dp}(2p^2 + 2p + 1) = 4p + 2$

$4p + 2 = 0$
$p = -1/2$

so the denominator is minimised (& ∴ the gradient maximised) when $p = -1/2$ and denominator is $1/2$

so greatest gradient of normal $= \dfrac{1}{3 \times 1/2} = 2/3$

$x_1 = 7$

$x_2 = 3$

$x_3 = 1$

$x_4 = \frac{23 \times 1 - 53}{5 \times 1 + 1} = \frac{23 - 53}{6} = -5$

$x_5 = \frac{23 \times -5 - 53}{5 \times -5 + 1} = \frac{-115 - 53}{-25 + 1} = \frac{-168}{-24}$

$= 7$

etc.

11 The sequence x_n is defined by the rules

$$x_1 = 7$$

$$x_{n+1} = \frac{23x_n - 53}{5x_n + 1}$$

The first three terms in the sequence are 7, 3, 1

What is the value of x_{100}?

A −5

B 0

C 1

D 3

E 7

cycle length of 4 (7, 3, 1, −5)

$\frac{100}{4} = 25$ hence $x_{100} = -5$

12 The polynomial function $f(x)$ is such that $f(x) > 0$ for all values of x.

Given $\int_2^4 f(x)\,dx = A$, which one of the following statements **must** be correct?

A $\int_0^2 [f(x+2)+1]\,dx = A+1$

B $\int_0^2 [f(x+2)+1]\,dx = A+2$

C $\int_2^4 [f(x+2)+1]\,dx = A+1$

D $\int_2^4 [f(x+2)+1]\,dx = A+2$

E $\int_4^6 [f(x+2)+1]\,dx = A+1$

F $\int_4^6 [f(x+2)+1]\,dx = A+2$

$\int f(x)\,dx$ translated left by 2 gives:

$\int f(x+2)\,dx$

$\int_2^4 f(x)\,dx = \int_0^2 f(x+2)\,dx = A$

$\int_0^2 (f(x+2)+1)\,dx = \int_0^2 f(x+2)\,dx$

$= \int_0^2 1\,dx$

$= A+2$

13 In the expansion of $(a + bx)^5$ the coefficient of x^4 is 8 times the coefficient of x^2.

Given that a and b are non-zero **positive** integers, what is the smallest possible value of $a + b$?

A 3

B 4

C 5

D 9

E 13

F 17

$$(a+bx)^5 = a^5 + \binom{5}{1} a^4 (bx) + \binom{5}{2} a^3 (bx)^2 + \binom{5}{3} a^2 (bx)^3 + \binom{5}{4} a^1 (bx)^4 + \binom{5}{5} a^0 (bx)^5$$

$$= a^5 + 5a^4bx + 10a^3b^2x^2 + 100a^2b^3x^3 + 5ab^4x^4 + \ldots$$

Coefficient of $x^4 = 5ab^4$

$x^2 = 10a^3b^2$

we're told $8(10a^3b^2) = 5ab^4$

$80a^3b^2 = 5ab^4$

$80a^2 = 5b^2$

$16a^2 = b^2$

$4a = b$

$a+b = a+4a = 5a$

If both nonzero positive then the smallest a or b can be is 1,

i.e. max $(a+b = 5a) = 5$

14 The solution of the simultaneous equations

$$(1)\quad 2^x + 3 \times 2^y = 3$$

$$(2)\quad 2^{2x} - 9 \times 2^{2y} = 6$$

$$(2^x)^2 - 9 \times (2^y)^2 = 6$$

is $x = p,\ y = q$.

Find the value of $p - q$

Let $2^x = a$ then
$2^y = b$

A $\frac{5}{12}$

B $\frac{7}{3}$

C $\log_2 \frac{5}{12}$

D $\log_2 \frac{7}{3}$

E $\log_2 9$

F $\log_2 15$

$(1)\ a + 3b = 3 \Rightarrow a = 3 - 3b$ & sub into (2)

$(2)\ a^2 - 9b^2 = 6$

$(3-3b)^2 - 9b^2 = 6$

$(3-3b)(3-3b) - 9b^2 = 6$

$9 - 18b + 9b^2 - 9b^2 = 6$

$9 - 18b = 6$

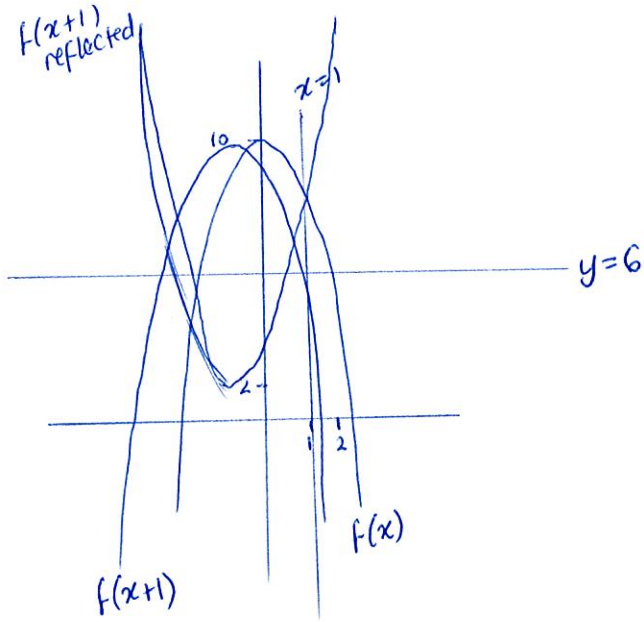
$3 = 18b$

$b = \frac{3}{18} = \frac{1}{6}$ so $a = 3 - 3(1/6) = 3 - 1/2 = 2\frac{1}{2} = 5/2$

So $a = \frac{5}{2} = 2^x$ then $x = \log_2 a = \log_2 5/2 = p$

$b = \frac{1}{6} = 2^y$ $y = \log_2 b = \log_2 1/6 = q$

So $p - q = \log_2 5/2 - \log_2 1/6 = \log_2 (5/2 \div 1/6) = \log_2 15$



15 It is given that $f(x) = -2x^2 + 10$

Consider the following three curves:

(**1**) $y = f(x)$ underestimate

(**2**) $y = f(x + 1)$ underestimate

(**3**) the curve $y = f(x + 1)$ reflected in the line $y = 6$ overestimate

The trapezium rule is used to estimate the area under each of these three curves between $x = 0$ and $x = 1$.

State whether the trapezium rule gives an overestimate or underestimate for each of these areas.

	(**1**)	(**2**)	(**3**)
A	underestimate	underestimate	underestimate
B	underestimate	underestimate	overestimate
C	underestimate	overestimate	underestimate
D	underestimate	overestimate	overestimate
E	overestimate	underestimate	underestimate
F	overestimate	underestimate	overestimate
G	overestimate	overestimate	underestimate
H	overestimate	overestimate	overestimate

$f(x) = 3x^2 + 12x + 4$

$f'(x) = 6x + 12$

$g(x) = x^3 + 6x^2 + 9x - 8$

$g'(x) = 3x^2 + 12x + 9$
$= 3(x^2 + 4x + 3)$
$= 3(x+1)(x+3)$

A function $f(x)$ is increasing when $f'(x) \geq 0$
decreasing $f'(x) \leq 0$

16 The functions f and g are given by $f(x) = 3x^2 + 12x + 4$ and $g(x) = x^3 + 6x^2 + 9x - 8$.

What is the complete set of values of x for which one of the functions is increasing and the other decreasing?

A $x \geq -1$

B $x \leq -1$

C $-3 \leq x \leq -2,\ x \geq -1$

D $x \leq -2,\ x \geq -1$

E $x \leq -3,\ -2 \leq x \leq -1$

F $x \leq -3,\ x \geq -2$

G $-2 \leq x \leq -1$

(1) If $f(x)$ is increasing & $g(x)$ decreasing then we have:

$6x + 12 \geq 0$
$6x \geq -12$
$x \geq -2$

$3(x+1)(x+3) \leq 0$
$-3 \leq x \leq -1$

Combine those & get $-2 \leq x \leq -1$

(2) If $f(x)$ is decreasing & $g(x)$ is increasing then we have:

$6x + 12 \leq 0$
$6x \leq -12$
$x \leq -2$

$3(x+1)(x+3) \geq 0$
$x \leq -3$ or $x \geq -1$

Combine those & get $x \leq -3$

So the complete set is $x \leq -3$ or $-2 \leq x \leq -1$

$F(n) = \frac{1}{n}\int_0^n (n-x)\,dx = \frac{1}{n}\left(nx - \frac{1}{2}x^2\right)\Big|_0^n = \frac{1}{n}\left(n^2 - \frac{1}{2}n^2\right) - \frac{1}{n}(0-0)$

$= \frac{\frac{1}{2}n^2}{n} = \frac{1}{2}n$

17 The two functions $F(n)$ and $G(n)$ are defined as follows for positive integers n:

$$F(n) = \frac{1}{n}\int_0^n (n-x)\,dx$$

$$G(n) = \sum_{r=1}^{n} F(r)$$

What is the smallest positive integer n such that $G(n) > 150$?

A 22

B 23

C 24

D 25

E 26

$G(n) = \sum_{r=1}^{n} F(r) = \sum_{r=1}^{n} \frac{1}{2}r = \frac{1}{2}\sum_{r=1}^{n} r$

$= \frac{1}{2}(1+2+3+\dots+n)$

(arithmetic series) $= \frac{1}{2} \times \frac{1}{2}n(n+1)$

$= \frac{1}{4}n(n+1)$

For $G(n) > 150$ we need $\frac{1}{4}n(n+1) > 150$

$n(n+1) > 600$

Trying some of the options, we get:

(1) For 24 we would have $24 \times 25 = 600$ which isn't > 600 so doesn't work

(2) For 25 we would have $25 \times 26 = 650 > 600$

18 The graph of $y = \log_{10}x$ is translated in the positive y-direction by 2 units.

This translation is equivalent to a stretch of factor k parallel to the x-axis.

What is the value of k?

$$y = \log_{10} x \xrightarrow{\text{translate } +2} y = \log_{10} x + 2$$

$$\xrightarrow{\text{stretch factor } k} y = f\left(\tfrac{1}{k}x\right) = f\left(\tfrac{x}{k}\right)$$

A 0.01

B $\log_{10} 2$

C 0.5

D 2

E $\log_2 10$

F 100

$$\log_{10} x + 2 = \log_{10}\left(\frac{x}{k}\right)$$

$$\log_{10} x + 2 = \log_{10} x - \log_{10} k$$

$$2 = -\log_{10} k$$

so $k = 10^{-2} = \frac{1}{100} = 0.01$

19 The set of solutions to the inequality $x^2 + bx + c < 0$ is the interval $p < x < q$

where b, c, p and q are real constants with $c < 0$.

In terms of p, q and c, what is the set of solutions to the inequality $x^2 + bcx + c^3 < 0$?

A $\frac{p}{c} < x < \frac{q}{c}$

B $\frac{q}{c} < x < \frac{p}{c}$

C $pc < x < qc$

D $qc < x < pc$

E $pc^2 < x < qc^2$

F $qc^2 < x < pc^2$

For $x^2+bcx+c^3=0$ we would have:

$$x = \frac{-bc \pm \sqrt{b^2c^2-4c^3}}{2} = \frac{-bc \pm \sqrt{c^2(b^2-4c)}}{2}$$

$$= \frac{-bc \pm c\sqrt{b^2-4c}}{2}$$

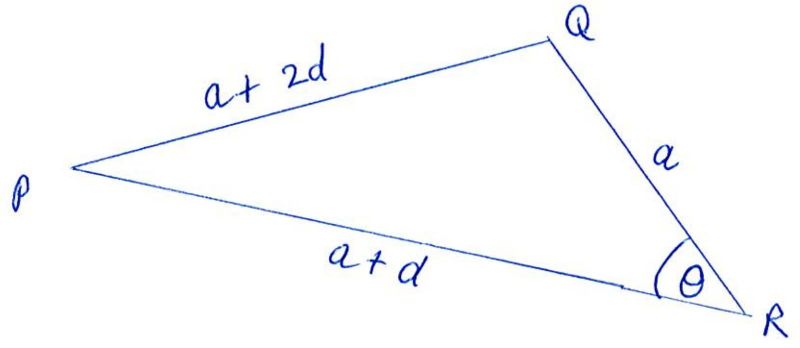
$$= \frac{c(-b \pm \sqrt{b^2-4c})}{2}$$

so then p & q are one of $\frac{1}{2}c(-b \pm \sqrt{b^2-4c})$ each

For x^2+bx+c the roots are obviousley $-\frac{1}{2}b \pm \sqrt{b^2-4ac}$

Hence the roots of $x^2+bcx+c^3$ are pc and qc so the answer must be $\boxed{C}$ or $\boxed{D}$

we need to find if $pc > qc$ or $qc > pc$. we know $c<0$ and $p<q$ so $\therefore$ $pc > qc$



$3d > 2a$

20 The lengths of the sides QR, RP and PQ in triangle PQR are a, $a + d$ and $a + 2d$ respectively, where a and d are positive and such that $3d > 2a$.

What is the full range, in degrees, of possible values for angle PRQ?

A $0 <$ angle $PRQ < 60$

B $0 <$ angle $PRQ < 120$

C $60 <$ angle $PRQ < 120$

D $60 <$ angle $PRQ < 180$

E $120 <$ angle $PRQ < 180$

By the cosine rule which says:

$$(a+2d)^2 = a^2 + (a+d)^2 - 2a(a+d)\cos\theta$$

we have

$$(a+2d)(a+2d) = a^2 + (a+d)(a+d) - 2a(a+d)\cos\theta$$

$$a^2 + 4ad + 4d^2 = a^2 + a^2 + 2ad + d^2 - 2a(a+d)\cos\theta$$

$$-a^2 + 2ad + 3d^2 = -2a(a+d)\cos\theta$$

$$\cos\theta = \frac{-a^2 + 2ad + 3d^2}{-2a(a+d)}$$

$$= \frac{a^2 - 2ad - 3d^2}{2a(a+d)}$$

$$= \frac{(a+d)(a-3d)}{2a(a+d)}$$

$$= \frac{a-3d}{2a}$$

Since $3d > 2a$, $-a > a - 3d$ so $\cos\theta < \frac{-a}{2a} = \frac{-1}{2}$

so $\theta > 120°$

END OF TEST

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