Cambridge Assessment
Admissions Testing

TEST OF MATHEMATICS FOR UNIVERSITY ADMISSION

D513/02

PAPER 2

November 2021 **75 minutes**

6767288541

Additional materials: Answer sheet

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open the question paper until you are told that you may do so.

A separate answer sheet is provided for this paper. Please check you have one.
You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and full name.

This paper is the second of two papers.

There are 20 questions on this paper. For each question, choose the one answer you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

There are no penalties for incorrect responses, only marks for correct answers, so you should attempt **all** 20 questions. Each question is worth one mark.

You can use the question paper for rough working or notes, but **no extra paper** is allowed.

You **must** complete the answer sheet within the time limit.

Calculators and dictionaries are NOT permitted.

There is no formulae booklet for this test.

Please wait to be told you may begin before turning this page.

PV2

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1 Find the value of

$$\int_1^4 \left(3\sqrt{x} + \frac{4}{x^2}\right) \mathrm{d}x$$

A -0.75

B 7.125

C 11

D 17

E 18

F 21.875

G 34.5

2 $A(0,2)$ and $C(4,0)$ are opposite vertices of the square $ABCD$.

What is the equation of the straight line through B and D?

A $y = -2x + 5$

B $y = -\frac{1}{2}x - 3$

C $y = -\frac{1}{2}x + 2$

D $y = x$

E $y = 2x - 3$

F $y = 2x + 2$

3 A student is chosen at random from a class. Each student is equally likely to be chosen.

Which of the following conditions is/are **necessary** for the probability that the student wears glasses to equal $\frac{4}{15}$?

I Exactly 11 students in the class do not wear glasses.

II The number of students in the class is divisible by 3.

III The class contains 30 students, and 8 of them wear glasses.

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

4 Consider the following claim about positive integers a, b and c:

if a is a factor of bc, **then** a is a factor of b **or** a is a factor of c

Which of the following provide(s) a **counterexample** to this claim?

I $a = 5$, $b = 10$, $c = 20$

II $a = 8$, $b = 4$, $c = 4$

III $a = 6$, $b = 7$, $c = 12$

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

5 On which line is the first error in the following argument?

A $\sin^2 x + \cos^2 x = 1$ for all values of x.

B Therefore $\cos x = \sqrt{1 - \sin^2 x}$ for all values of x.

C Hence $1 + \cos x = 1 + \sqrt{1 - \sin^2 x}$ for all values of x.

D Thus $(1 + \cos x)^2 = \left(1 + \sqrt{1 - \sin^2 x}\right)^2$ for all values of x.

E Substituting $x = \pi$ gives $0 = 4$.

6 Consider the following two statements about the polynomial $f(x)$:

P: $f(x) = 0$ for exactly three real values of x

Q: $f'(x) = 0$ for exactly two real values of x

Which one of the following is correct?

A P is **necessary** but **not sufficient** for Q.

B P is **sufficient** but **not necessary** for Q.

C P is **necessary and sufficient** for Q.

D P is **not necessary** and **not sufficient** for Q.

7 A circle has equation $(x-9)^2 + (y+2)^2 = 4$

A square has vertices at $(1, 0)$, $(1, 2)$, $(-1, 2)$ and $(-1, 0)$.

A straight line bisects both the area of the circle and the area of the square.

What is the x-coordinate of the point where this straight line meets the x-axis?

A 2

B 3

C 4

D 4.5

E 5

F 6

G The straight line is not uniquely determined by the information given, so there is more than one possible point of intersection.

H There is no straight line that bisects both the area of the circle and the area of the square.

8 Consider the following statement about the polynomial $\mathrm{p}(x)$, where a and b are real numbers with $a < b$:

(∗) There exists a number c with $a < c < b$ such that $\mathrm{p}'(c) = 0$.

Which one of the following is true?

A The condition $\mathrm{p}(a) = \mathrm{p}(b)$ is **necessary and sufficient** for (∗)

B The condition $\mathrm{p}(a) = \mathrm{p}(b)$ is **necessary** but **not sufficient** for (∗)

C The condition $\mathrm{p}(a) = \mathrm{p}(b)$ is **sufficient** but **not necessary** for (∗)

D The condition $\mathrm{p}(a) = \mathrm{p}(b)$ is **not necessary** and **not sufficient** for (∗)

9 Consider the following statements about a polynomial $\mathrm{f}(x)$:

I $\mathrm{f}(x) = px^3 + qx^2 + rx + s$, where $p \neq 0$.

II There is a real number t for which $\mathrm{f}'(t) = 0$.

III There are real numbers u and v for which $\mathrm{f}(u)\,\mathrm{f}(v) < 0$.

Which of these statements is/are **sufficient** for the equation $\mathrm{f}(x) = 0$ to have a real solution?

	Statement I is sufficient	**Statement II is sufficient**	**Statement III is sufficient**
A	Yes	Yes	Yes
B	Yes	Yes	No
C	Yes	No	Yes
D	Yes	No	No
E	No	Yes	Yes
F	No	Yes	No
G	No	No	Yes
H	No	No	No

10 The first seven terms of a sequence of positive integers are:

$$u_1 = 15$$
$$u_2 = 21$$
$$u_3 = 30$$
$$u_4 = 37$$
$$u_5 = 44$$
$$u_6 = 51$$
$$u_7 = 59$$

Consider the following statement about this sequence:

(∗) **If** n is a prime number, **then** u_n is a multiple of 3 **or** u_n is a multiple of 5.

What is the smallest value of n that provides a **counterexample** to (∗)?

A 1

B 2

C 3

D 4

E 5

F 6

G 7

11 A student attempts to solve the following problem, where a and b are non-zero real numbers:

> Show that **if** $a^2 - 4b^3 \geq 0$ **then** there exist real numbers x and y such that $a = xy(x+y)$ and $b = xy$.

Consider the following attempt:

$$(x-y)^2 \geq 0 \qquad \text{(I)}$$

$$\text{so} \quad x^2 + y^2 - 2xy \geq 0 \qquad \text{(II)}$$

$$\text{so} \quad (x+y)^2 - 4xy \geq 0 \qquad \text{(III)}$$

$$\text{so} \quad x^2y^2(x+y)^2 - 4x^3y^3 \geq 0 \qquad \text{(IV)}$$

$$\text{so} \quad a^2 - 4b^3 \geq 0 \qquad \text{(V)}$$

Which of the following best describes this attempt?

A It is completely correct.

B It is incorrect, but it would be correct if written in the reverse order.

C It is incorrect, but the student has correctly proved the converse.

D It is incorrect because there is an error in line (II).

E It is incorrect because there is an error in line (III).

F It is incorrect because there is an error in line (IV).

12 Which of the following statements about polynomials f and g is/are true?

I **If** $\mathrm{f}(x) \geq \mathrm{g}(x)$ for all $x \geq 0$, **then** $\int_0^x \mathrm{f}(t)\,\mathrm{d}t \geq \int_0^x \mathrm{g}(t)\,\mathrm{d}t$ for all $x \geq 0$.

II **If** $\mathrm{f}(x) \geq \mathrm{g}(x)$ for all $x \geq 0$, **then** $\mathrm{f}'(x) \geq \mathrm{g}'(x)$ for all $x \geq 0$.

III **If** $\mathrm{f}'(x) \geq \mathrm{g}'(x)$ for all $x \geq 0$, **then** $\mathrm{f}(x) \geq \mathrm{g}(x)$ for all $x \geq 0$.

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

13 A region R in the (x, y)-plane is defined by the simultaneous inequalities

$$y - x < 3$$

$$y - x^2 < 1$$

Which of the following statements is/are true for **every** point in R?

I $-1 < x < 2$

II $(y - x)(y - x^2) < 3$

III $y < 5$

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

14 Consider the following simultaneous equations, where p is a real number:

$$p2^x + \log_2 y = 2$$

$$2^x + \log_2 y = 1$$

What is the complete range of p for which these simultaneous equations have a real solution (x, y)?

A $p < 1$

B $p \neq 1$

C $p > 1$

D $p < 1$ or $p > 2$

E $p \neq 1$ and $p < 2$

F $p > 1$ and $p < 2$

G $p > 2$

H All real values of p

15 A circle has equation

$$x^2 + ax + y^2 + by + c = 0$$

where a, b and c are non-zero real constants.

Which one of the following is a **necessary and sufficient** condition for the circle to be tangent to the y-axis?

A $a^2 = 4c$

B $b^2 = 4c$

C $\frac{a}{2} = \sqrt{\frac{a^2 + b^2}{4} - c}$

D $\frac{b}{2} = \sqrt{\frac{a^2 + b^2}{4} - c}$

E $-\frac{a}{2} = \sqrt{\frac{a^2 + b^2}{4} - c}$

F $-\frac{b}{2} = \sqrt{\frac{a^2 + b^2}{4} - c}$

16 p and q are real numbers, and the equation

$$x\,|x| = px + q$$

has exactly k distinct real solutions for x.

Which one of the following is the complete list of possible values for k?

A 0, 1, 2

B 0, 1, 2, 3

C 0, 1, 2, 3, 4

D 0, 2, 4

E 1, 2, 3

F 1, 2, 3, 4

17 Consider the following functions defined for $x > 1$:

$$f(x) = \log_2\left(\log_2 \sqrt{x}\,\right)$$

$$g(x) = \log_2\left(\sqrt{\log_2 x}\,\right)$$

Which one of the following is true for **all** values of $x > 1$?

A $0 \leq f(x) \leq g(x)$ **or** $g(x) \leq f(x) \leq 0$

B $0 \leq g(x) \leq f(x)$ **or** $f(x) \leq g(x) \leq 0$

C $\frac{1}{2} \leq f(x) \leq g(x)$ **or** $g(x) \leq f(x) \leq \frac{1}{2}$

D $\frac{1}{2} \leq g(x) \leq f(x)$ **or** $f(x) \leq g(x) \leq \frac{1}{2}$

E $1 \leq f(x) \leq g(x)$ **or** $g(x) \leq f(x) \leq 1$

F $1 \leq g(x) \leq f(x)$ **or** $f(x) \leq g(x) \leq 1$

18 A student chooses two distinct real numbers x and y with $0 < x < y < 1$.

The student then attempts to draw a triangle ABC with:

$$AB = 1$$
$$\sin A = x$$
$$\sin B = y$$

Which of the following statements is/are correct?

I For some choice of x and y, there is exactly **one** triangle the student could draw.

II For some choice of x and y, there are exactly **two** different triangles the student could draw.

III For some choice of x and y, there are exactly **three** different triangles the student could draw.

(Note that congruent triangles are considered to be the same.)

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

19 The angle θ can take any of the values $1°$, $2°$, $3°$, $\ldots$, $359°$, $360°$.

For how many of these values of θ is it true that

$$\sin\theta\sqrt{1+\sin\theta}\sqrt{1-\sin\theta}+\cos\theta\sqrt{1+\cos\theta}\sqrt{1-\cos\theta}=0$$

A 0

B 1

C 2

D 4

E 93

F 182

G 271

H 360

20 A sequence of functions f_1, f_2, f_3, ... is defined by

$$f_1(x) = |x|$$

$$f_{n+1}(x) = |f_n(x) + x| \quad \text{for } n \geq 1$$

Find the value of

$$\int_{-1}^{1} f_{99}(x)\,dx$$

A 0

B 0.5

C 1

D 49.5

E 50

F 99

G 99.5

H 100

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