STEP MATHEMATICS 1

2019

Examiner's Report

SI (9465) 2019 Report

General Comments

In order to get the fullest picture, this document should be read in conjunction with the question paper, the marking scheme and (for comments on the underlying purpose and motivation for finding the right solution-approaches to questions) the *Hints and Solutions* document; all of which are available from the STEP and Cambridge Examinations Board websites.

The purpose of the STEPs is to learn what students are able to achieve mathematically when applying the knowledge, skills and techniques that they have learned within their standard A-level (or equivalent) courses … but seldom within the usual range of familiar settings. STEP questions require candidates to work at an extended piece of mathematics, often with the minimum of specific guidance, and to make the necessary connections. This requires a very different mind-set to that which is sufficient for success at A-level, and the requisite skills tend only to develop with prolonged and determined practice at such longer questions for several months beforehand.

One of the most crucial features of the STEPs is that the routine technical and manipulative skills are almost taken for granted; it is necessary for candidates to produce them with both speed and accuracy so that the maximum amount of time can be spent in thinking their way through the problem and the various hurdles and obstacles that have been set before them. Most STEP questions begin by asking the solver to do something relatively routine or familiar before letting them loose on the real problem. Almost always, such an opening has not been put there to allow one to pick up a few easy marks, but rather to point the solver in the right direction for what follows. Very often, the opening result or technique will need to be used, adapted or extended in the later parts of the question, with the demands increasing the further on that one goes. So it is that a candidate should never think that they are simply required to ‘go through the motions’ but must expect, sooner or later, to be required to show either genuine skill or real insight in order to make a reasonably complete effort. The more successful candidates are the ones who manage to figure out how to move on from the given starting-point.

Finally, reading through a finished solution is often misleading – even unhelpful – unless you have attempted the problem for yourself. This is because the thinking has been done for you. When you read through the report and look at the solutions (either in the mark-scheme or the *Hints & Solutions* booklet), try to figure out how you could have arrived at the solution, learn from your mistakes and pick up as many tips as you can whilst working through past paper questions.

This year’s paper produced the usual sorts of outcomes, with far too many candidates wasting valuable time by attempting more than six questions, and with many of these candidates picking up 0-4 marks on several ‘false starts’ which petered out the moment some understanding was required.

Around one candidate in eight failed to hit the 30 mark overall, though this is an improvement on last year. Most candidates were able to produce good attempts at two or more questions. At the top end of the scale, around a hundred candidates scored 100 or more out of 120, with four hitting the maximum of 120 and many others not far behind.

The paper is constructed so that question 1 is very approachable indeed, the intention being to get everyone started with some measure of success; unsurprisingly, Q1 was the most popular question of all, although under two-thirds of the entry attempted it this year, and it also turned out to be the most successful question on the paper with a mean score of about 12 out of 20.

In order of popularity, Q1 was followed by Qs.3, 4 and 2. Indeed, it was the pure maths questions in Section A that attracted the majority of attention from candidates, with the applied questions combined scoring fewer ‘hits’ than any one of the first four questions on its own. Though slightly more popular than the applied questions, the least successful question of all was Q5, on vectors. This question was attempted by almost 750 candidates, but 70% of these scored no more than 2 marks, leaving it with a mean score of just over 3 out of 20. Q9 (a statics question) was found only marginally more appetising, with a mean score of almost 3½ out of 20.

In general, it was found that explanations were poorly supplied, with many candidates happy to overlook completely any requests for such details.

Question 1

As intended, this was the most popular question on the paper and the one that elicited the highest average score. The set-up is a familiar one for A-level, though the "coefficients" involved are trigonometric throughout; it was this side of things that provided the only real degree of difficulty to the question.

Given that there was no requirement for candidates to justify the natures of the two extrema involved in the question, the issue was how accurately candidates could manage to deploy the necessary trig. identities at the appropriate points. Interestingly, it was clear that those who made tough going of the working were the ones who handled the three *reciprocal* trig. functions – cosec, sec and cot – and their derivatives less confidently. For instance, there are those who immediately convert any trig. situation exclusively into sines and cosines … this generally works perfectly well for A-level questions. Here, it simply turned out to be a hindrance as the resulting terms would contain rational functions which required heavier-duty methods for dealing with them; and many such candidates got into a muddle somewhere along the way, especially with the square of the distance XY in part (ii), which should have been turned into a perfect square reasonably swiftly.

In order to get the given result for k towards the end of (ii), a number of candidates resorted to verification, though this worked reasonably well provided they didn't somehow confuse α s with θ s. Those who wrapped up the required final answer correctly were those who spotted the *difference-of-two-squares* factorisation embedded in there *and* spotted that $(c + s)^2 = 1 + 2sc$ at the right moment (using the usual abbreviations for cos and sin).

Question 2

Given that there has been a question of a similar nature to this on several of the STEP I's of recent years, the demands of these sorts of coordinate geometry questions have become relatively routine. Attracting the interest of 60% of the candidature, this question drew the second highest mean score overall, just over half-marks.

The first major stumbling block in this question was a lack of technical precision when taking square roots of equations. If done using parametric differentiation this was not needed, but too many students rearranged $x = 3t^2$ to form $t = \sqrt{\frac{x}{3}}$. Many candidates also assumed that if $3p^2 = 3t^2$ then p must equal t. Candidates should also be aware that not all letters are equivalent. In the first part of this question, t was a variable but p was a fixed value. Many candidates wrote $\frac{dx}{dp}$ showing a fundamental misunderstanding, although it led algebraically to the correct result in this instance.

Lots of candidates did not read the question carefully. In the second paragraph it was required that the point of intersection of the tangents be found in general. Many students who clearly could have done this conflated it with the constraint that the tangents had to be perpendicular, which was a separate question.

Finding the parameterised form of the locus of intersections of perpendicular tangents required some judicious algebra and use of $pq = -1$. This was generally done quite well, although many candidates were not aware of the difference of two cubes which made the algebra much nastier. It is often worth neatening up algebraic expressions before continuing to work with them. When this curve was intersected with the original curve it became clear that many candidates were not clear which x coordinate was related to which point. Correct simultaneous equations usually led to a disguised cubic. Candidates seemed quite good at spotting one solution and factorising to find the remaining ones. However, not all candidates explained why one solution was not possible.

The final sketch was rarely done well. The semi-cubical parabola was meant to be unfamiliar, but most students could not piece together the information to realise that it must have a cusp. Overall, most candidates were able to engage and make progress with this question, albeit with several technical errors.

Question 3

This question was clearly found an attractive proposition, drawing almost 2000 'hits', partly due to the clearly directed beginning and the immediate likelihood that the second integral could also be attacked with an exactly similar tactic: replacing the factor of $(1 - \sin x)$ given for the first integral by $(1 - \sec x)$ for the second … which, as it happens, works as well as any other method. As a result, the question was answered relatively well by those who could take advantage of the starting prompt. However, many candidates failed to make much of an impression, to the extent that a fifth of all takers scored only 0, 1 or 2 marks on it.

The main issue is that the question requires a considerable degree of dexterity in one's approach, switching between the various integration techniques without any further signposting: "recognition" (a.k.a. "reverse chain rule") integration was the most direct but required the clearest grasp of the various trigonometric relationships that could be used, but "substitution" and "by parts" were also necessary at times, depending upon how one split up the integrand within one's working. Those who were most successful used the method introduced by the first part to tackle the second, but many candidates succeeded with other, sometimes considerably longer methods. Most candidates could remember integrals of simple trigonometric functions, e.g. $\sec^2 x$. Trigonometric identities were generally applied accurately, though sometimes over-zealously; for example simplifying $1 - \sin^2 x$ by a double angle formula rather than to $\cos^2 x$. The most successful candidates made judicious use of these known formulae to produce an integrand which they could integrate directly, while those who appeared less discerning in their choice of identities cycled through many expressions which were difficult to integrate. The integral of $\sec^4 x$ was the most difficult in this respect, though plenty of candidates were successful in their handling of it.

Question 4

This question was also attempted by many candidates, being the last of the "big four" early questions, and – along with questions 1 and 2 – one of the only questions for which scores exceeded 10/20 on average.

Most students did part (i) well, although it was quite common to see negative or non-integer values of m and n included in the answer, despite the question's clear wording. It was also unfortunate that many candidates went about it in the longest way imaginable, squaring $m + n\sqrt{2}$ and then comparing the result with the intended answer … *then* solving for m and n from a pair of simultaneous equations (one linear and one quadratic) when the small numbers involved, and the fact that the question stated that they would be integers, required a careful evaluation of the situation.

In part (ii), most students correctly got three equations for p, q and s by expanding the factorisation given and comparing coefficients. However, relatively few gave a clear justification that such a factorisation must exist; there were some wordy but vague attempts at this, with the logic commonly being reversed. Some candidates found it difficult to manipulate the simultaneous non-linear equations to obtain useful expressions, but there were also many very good derivations of the desired equation, following several different routes.

Most students, including those who had been unable to derive it, reduced the given equation to a cubic equation for s^2 and solved this with no problems. A good number simply spotted the roots with no working shown. In using the value $s = \sqrt{2}$ to obtain two quadratic equations, students often obtained $p, q = -4 \pm 3\sqrt{2}$ but confused which variable took which sign, or even used both possibilities in each case, leading either to incorrect signs on all four roots or to eight roots of which four were spurious. We also saw a significant number incorrectly taking $s = 2$. Most people who got to the end were very good at getting rid of the nested square roots using the method of part (i).

Question 5

It is clear that either vectors questions are not very popular, or that the topic itself is found difficult. Almost a quarter of the overall entry of 2000+ made a start at this question, most of which candidates (70% of them, in fact) attempted the opening explanation (usually very poorly) and then gave up. As a result, Q5 drew the lowest mean score of any of the paper's 11 questions, dipping down to a miserable 3/20 on average. Indeed, of the very few candidates who scored high marks on this question, most attacked it with more advanced (further maths) methods than are currently within the scope of the 9465 syllabus. It is, therefore, very difficult to say what it is that candidates did well, or did poorly, at.

To begin with, very few had a sufficient grasp of the properties of quadrilaterals and were unable to identify the parallelogram and rhombus of part (i). The only upside to this poor start for many was that it did help re-direct their attentions towards other questions. The beginning of (ii) then deterred further progress for most of the remaining candidates who had little idea as to what to do: the most "on-spec" approach being to check that the two diagonal lines intersect.

In (ii) (b), most good efforts used the scalar product, though the equivalent approach using the *Cosine rule* was the one intended (given that the SP is not actually on this part of the syllabus).

Question 6

As with several of the questions on the paper, the "start" supplied in the question gave many candidates the prospect of a grip on the content of the question, although – as was frequently the case – a significant proportion of starts petered out relatively quickly. Almost half of all candidates attempted this question, but then around a quarter of them fell by the wayside before making any substantial progress.

Many candidates preferred to find extrema by differentiation. Such efforts were rewarded for the quartic, but not for the quadratic since that part of the question required candidates to complete the square. Common mistakes followed from incorrectly dealing with the coefficient 9, and often candidates obtained different extrema for the two polynomials and were unable to make further progress. In many cases, candidates were unable to use the completed square form to find a minimum, while those who used differentiation often neglected to check the nature of the stationary points.

When sketching the graph, candidates often worked backwards from the inequalities given in the question to find turning points and did not receive many marks. The argument showing $\frac{\sin^2\theta\cos^2 x}{1+\cos^2\theta\sin^2 x} \leq 1$ was overcomplicated by many, though many approaches were successful. While some candidates could justify $\sin^2\theta = 1$ and $\cos^2 x = 1$ with clarity, many struggled for a cogent argument.

Question 7

Once again, the prospect of gaining an easy few marks at the question's opening drew in a lot of interest; but many of these attempts (a third of them, in fact) were of negligible success.

For serious takers, there was a good spread of marks, with some sensible comments and explanations being offered in several places. In part (i), steps 1, 3 and 4 were generally done well, although a common mistake in step 4 was to obtain an expression for b^2 that contained a "factor" of 3 but in which the other factor was not obviously an integer. Having shown step 4, however, a great many candidates thought that the contradiction obtained if a is a multiple of 3 was sufficient to conclude that $\sqrt{2}+\sqrt{3}$ was irrational, and didn't consider the case where a is not a multiple of 3, thus losing most of the marks for step 5. Some of those who avoided this pitfall forgot to appeal to symmetry to rule out the case of b being a multiple of 3. It was sometimes difficult to interpret candidates' logic from their prose.

Part (ii) was less well done in general, with many candidates giving very little detail. When considering squares of non-multiples of 5, a common error was to deal with the case $5k \pm 1$ but not $5k \pm 2$. A significant number guessed the correct relationship $a^4 + b^4 = 26a^2b^2$, but no credit was given for simply writing this without any reasoning. Very few candidates got the final mark for explaining why divisibility by 3 was not sufficient for this case, although some mentioned the key fact that 26 is 2 more than a multiple of 3, without saying why that is relevant.

Question 8

This was a fascinating question to mark, particularly as many candidates couldn't seem to figure out what to do with integrals that they *weren't* required to integrate. Another confusing feature of the question for many was the rather different role played by the variable x … a lot of candidates had clearly not encountered the notion that x doesn't have to be the (dummy) variable of the integrand; even amongst those who did make good progress with the question, checking what happened to the upper limit was a major stumbling-block. Almost 850 candidates attempted this question, 350 of whom failed to score more than 2/20 on it.

Those candidates who realised that this was a test of substitution integration generally coped very well with parts (i) and (ii), where the change-of-variable was more obvious; indeed, (ii) fell very readily to either one of two obvious tactics. Part (iii) required either a very keen appreciation of what was going on in the background or a careful build-up to the problem that started generally and then compared the result with the desired outcome. Quite a few candidates seemed to resort to guesswork and the accompanying working could often be vague, at best.

Part (iv), requiring a quadratic substitution, proved a step too far for many candidates, who – in spirit if not on paper – gave up at this point. Many appreciated that a quadratic substitution was 'on the cards' but couldn't make the conceptual leap of turning the $u^2\,\mathrm{d}u$ involved into $\sqrt{u^2}.\,u\,\mathrm{d}u$ so that the integrand could now be made to look more like the one in part (ii)'s integral.

Overall, full attempts were usually accompanied by high marks.

Question 9

There were fewer than 500 starts at Q9, with almost two-thirds of these attempts earning no more than 2/20 overall. Very few candidates were able to make a substantial attempt at this question; most were stymied by not having any physical intuition regarding the direction and position of the reaction forces at the moment of toppling. Once this was done then the question became mainly angle chasing and taking moments about appropriate points but most candidates lacked the confidence to make much progress.

Even the opening result in (i) – which required nothing more than taking moments (once) about a suitably chosen point – offered four marks for obtaining a given answer; this, however, proved too much for the majority of candidates. The real obstacle lay in the widespread reluctance among candidates to draw a good diagram, of a suitable size for labelling all necessary points, angles and forces (including their directions), and clearly labelled; a good diagram is *the* most important thing that a candidate can do … everything else follows so much more easily from a good diagram. In principle, all that is required to complete this question is to resolve twice, take moments (twice) and then apply the '*Law of Friction*' in its $F \le \mu R$ form and then sort out the resulting mix of algebra and trigonometry.

Question 10

This mechanics question was poorly done, despite being on the topic most frequently occurring on the early STEPs. Many candidates knew a general result for the trajectory of a projectile, but some could not adapt it when the angle given was with the vertical. Candidates did not seem to be sufficiently familiar with the sums and products of roots of a quadratic, which is new to the syllabus.

This question introduced two inequalities, and this is always a problem for candidates. The first came from the quadratic discriminant and the second came from the given point being not greater than the maximum height. However, candidates often resorted to algebraic meandering rather than clear thinking about how an inequality might arise.

Those candidates who really engaged with the question generally did well, although very high marks were seldom acquired.

Question 11

A relatively small number of candidates attempted this question, which was generally not well done. In part (i), candidates often failed to explain clearly how to get the given answer, with some simply calculating the probability for the first few values of n explicitly and spotting a pattern. Many candidates identified the given value as corresponding to $p = 0.5$, but lost marks for failing to explain why the probability was minimised at this point. Successful approaches to this part included differentiating, completing the square, or the AM-GM inequality.

Most candidates struggled with the setup for part (ii). Most got the correct probability of finishing in the first round, but generally didn't properly account for the different cases in the second round depending on the first round tosses. One occasional misconception was to think that the process is similar to part (i), i.e. that all three coins are tossed again if the first round is inconclusive. This incorrect approach fortuitously produces the same probability as a correct approach, so candidates who successfully analysed this situation could still get most of the marks. Again, relatively few candidates explained clearly why $p = 0.5$ gives the minimal probability.