

PHYSICS ADMISSIONS TEST
Wednesday, 31 October 2018

Time allowed: 2 hours

*For candidates applying to Physics, Physics and Philosophy,
Engineering, or Materials Science*

Total 23 questions [100 Marks]

Answers should be written on the question sheet in the spaces provided,
and you are encouraged to show your working.
You should attempt as many questions as you can.

No tables, or formula sheets may be used.

Answers should be given exactly and in simplest terms
unless indicated otherwise.

Indicate multiple-choice answers by circling the best answer.
Partial credit may be given for correct workings in multiple choice questions.

The numbers in the margin indicate the marks expected to be assigned
to each question. You are advised to divide your time according to
the marks available.

You may take the gravitational field strength
on the surface of earth to be $g \approx 10 \text{ m s}^{-2}$

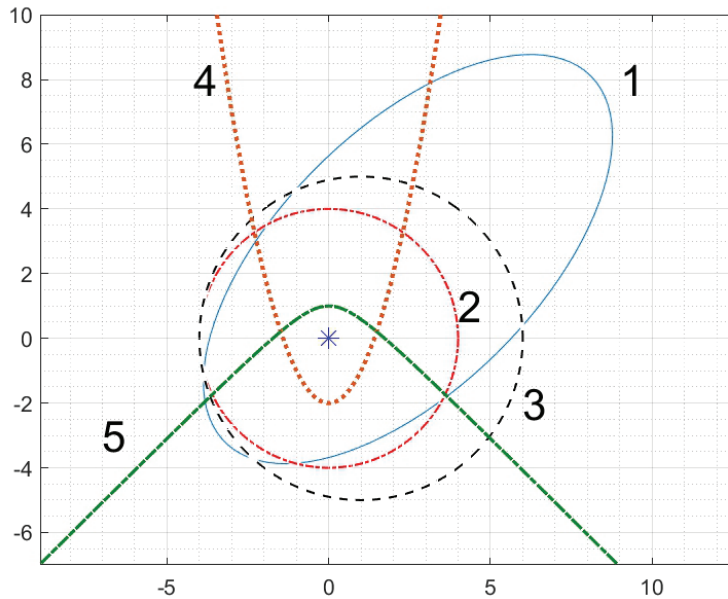
Do NOT turn over until told that you may do so.

1. What is the next number in the sequence? ^{1 4 5 9 14} 4, 5, 9, 14, 23

[2]

A	B	C	D	E
28	34	37	39	42

2. Which of the curves shown in the figure is not a trajectory in the gravitational field of a central star at the point $(x=0, y=0)$ marked by the star symbol?



[2]

A	B	C	D	E
All are tra- jectories	none of these is a trajec- tory	5 is not a tra- jectory	4 and 5 are not trajecto- ries	3 is not a tra- jectory

2 is a circle with the star at the centre ✓
 3 is a circle with the star not at the centre X

3. Which combination of units is the odd one out?

[2]

A	B	C	D	E
C V m^{-1}	A T m	kg m s^{-2}	J m^{-1}	C m s^{-1}

A: $F = QE$

B: $F = BIL$

C: $F = ma$

D: $F = \frac{W}{d}$

4. 90 people enter a maze. At each junction a third will go left and two thirds will go right. After three such junctions, what is the most likely combination of turns people will have taken?

[2]

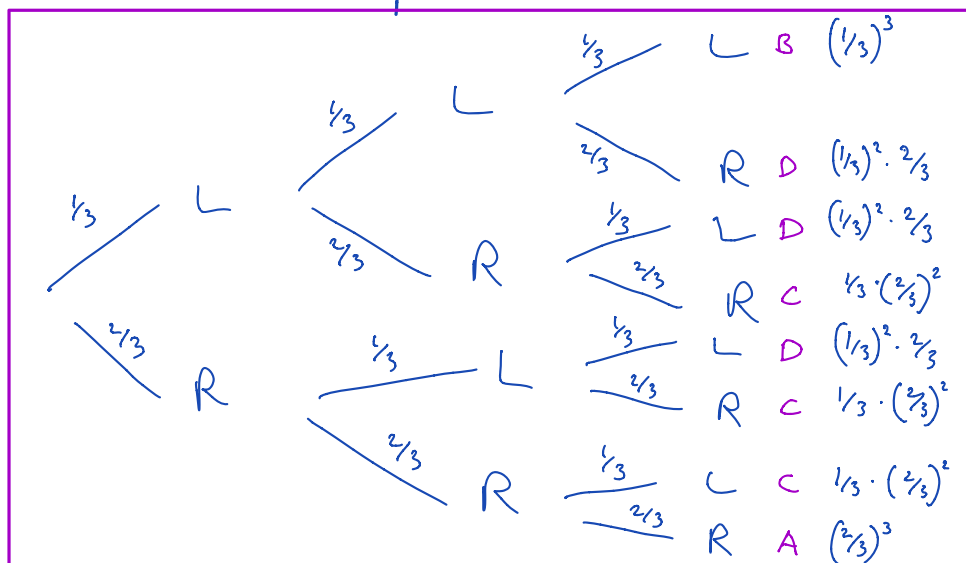
A	B	C	D	E
Gone right three times	Gone left three times	Gone right twice and once left	Gone twice left and once right	It is impossible to tell

RRR
 $\left(\frac{2}{3}\right)^3 = \frac{8}{27}$

LLL
 $\left(\frac{1}{3}\right)^3 = \frac{1}{27}$

RRL
 RLR
 LRR
 $3\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)$
 $= \frac{4}{9}$

RLL
 LRL
 LLR
 $3\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^2$
 $= \frac{2}{9}$



5. A person drinks many cups of tea. The first cup the person drinks is filled completely. They don't want to drink too much tea in total so the second cup is filled with only a fraction (α) of the tea in the first cup, the third cup contains the same fraction α of the second cup and so on. What is the maximum value of α so that the person drinks no more than 3 times the amount of tea in the first cup however many drinks they take? [2]

A	B	C	D	E
$\alpha = \frac{1}{3}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{2}{3}$	$\alpha = \frac{1}{4}$	$\alpha = \frac{3}{4}$

$$1 + \alpha + \alpha^2 + \dots = 3$$

G.P. with $a=1$, $r=\alpha$

$$S_{\infty} = \frac{a}{1-r}$$

$$3 = \frac{1}{1-\alpha}$$

$$\alpha = \frac{2}{3}$$

6. A stationary wave is set up on a string of length L . If the centre and the ends of the string are held fixed with zero displacement, what wavelengths (λ_m) can this stationary wave have? In the table below m denotes any positive non zero integer. [2]

A	B	C	D	E
$\lambda_m = \frac{2L}{m}$	$\lambda_m = \frac{L}{m}$	$\lambda_m = \frac{L}{2m}$	$\lambda_m = mL$	$\lambda_m = 2mL$



$$\lambda = L$$



$$\lambda = \frac{L}{2}$$



$$\lambda = \frac{L}{3}$$

7. A car of mass m is traveling along a straight and narrow road at speed u . A cat walks into the road a distance d in front of the car and stops in the middle of the road. What constant force must be applied to the car so that it does not hit the cat? The road is too narrow for the car to drive past the cat without hitting it. [2]

A	B	C	D	E
$F = \frac{-mu^2}{2d}$	$F = \frac{-u^2}{2d}$	$F = \frac{-mu^2}{2}$	$F = \frac{-mu^2}{d}$	the car cannot avoid hitting the cat

Work done by the force needs to be equal to the kinetic energy of the car.

$$WD = Fd = \frac{1}{2} mu^2$$

$$F = \frac{mu^2}{2d}$$

8. What is the value of x for which y has a minimum in the function $y = (x-3)(x+1)$? [2]

A	B	C	D	E
$x = 1$	$x = -1$	$x = 3$	$x = 0$	$x = -3$

$$y = x^2 - 2x - 3$$

$$\frac{dy}{dx} = 2x - 2 = 0$$

$$x = 1$$

9. What is the equation of the line which intersects $y = 2x - 2$ at right angles and at position $x = 1$?

[2]

A	B	C	D	E
$y = -\frac{1}{2}x$	$y = -\frac{1}{2}x + \frac{1}{2}$	$y = \frac{1}{2}x - \frac{1}{2}$	$y = x$	$y = 2x$

Gradient of $y = 2x - 2$ is 2

Gradient of perpendicular line is $-\frac{1}{2}$

When $x = 1$, $y = 2(1) - 2 = 0$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -\frac{1}{2}(x - 1)$$

$$y = -\frac{1}{2}x + \frac{1}{2}$$

10. For which range of x is the inequality $x^3 - x^2 - x + 1 \geq 0$ satisfied?

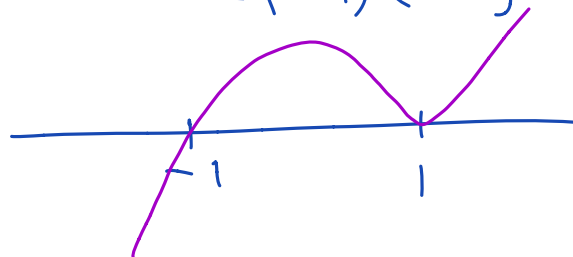
[2]

A	B	C	D	E
$x \geq 1$	$x \leq 1$	$x \geq 1$ and $x \leq -1$	$-1 \leq x \leq 1$	$x \geq -1$

$$(1)^3 - (1)^2 - (1) + 1 = 0 \Rightarrow (x-1) \text{ is a factor}$$

$$x^3 - x^2 - x + 1 = (x-1)(x^2 + 0x - 1)$$

$$= (x-1)(x-1)(x+1)$$



11. A rectangular building with sides 50 m and 100 m long has a flat roof on top of it. The roof has a mass per unit area of 100 kg m^{-2} . The walls are 10 cm thick and can take a maximum stress of S_{max} which depends on the material of the wall. The values of S_{max} for five building materials are:

moist wood: $S_{max} = 25.5 \text{ N mm}^{-2}$

dry wood: $S_{max} = 48.3 \text{ N mm}^{-2}$

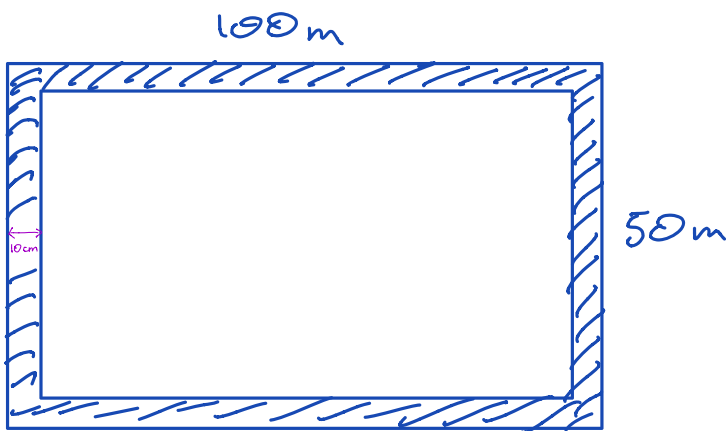
concrete: $S_{max} = 300 \text{ N mm}^{-2}$

solid cardboard: $S_{max} = 15 \text{ N mm}^{-2}$

brick: $S_{max} = 7 \text{ N mm}^{-2}$

Which materials could be used so that the walls will support the building, assuming that the mass of the walls is negligible? [2]

A	B	C	D	E
only concrete	only brick	any of the materials above	concrete or dry wood	concrete, dry or moist wood



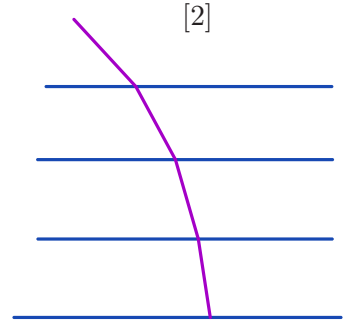
$$\begin{aligned} \text{Area of roof} &= 50 \times 100 \\ &= 5000 \text{ m}^2 \\ \text{Mass of roof} &= 5000 \times 100 \\ &= 5 \times 10^5 \text{ kg} \\ \text{Weight of roof} &= 5 \times 10^5 \times 10 \\ &= 5 \times 10^6 \text{ N} \end{aligned}$$

$$\begin{aligned} \text{Cross sectional area of walls} &= 0.1 \times 2 (100 + 50) \\ &= 30 \text{ m}^2 \\ &= 30 \times 10^6 \text{ mm}^2 \end{aligned}$$

$$\text{Stress} = \frac{\text{force}}{\text{area}} = \frac{5 \times 10^6 \text{ N}}{30 \times 10^6 \text{ mm}^2} = 0.17 \text{ N mm}^{-2}$$

12. A light ray passes through an infinite stack of thin transparent plates. The refractive index of these plates increases slightly by a constant factor from one plate to the next. The ray enters the first plate making an angle θ_0 with the surface normal. The ray's angle to the normal changes at each new interface. At the i^{th} interface between plates the ray makes an angle θ_i to the normal. Find the limiting value θ_∞ for the angle θ_i when the ray has traversed an infinite number of plates.

A	B	C	D	E
$\theta_\infty = \theta_0$	$\theta_\infty = -\theta_0$	$\theta_\infty = 0^\circ$	$\theta_\infty = 90^\circ$	$\theta_\infty = 180^\circ$

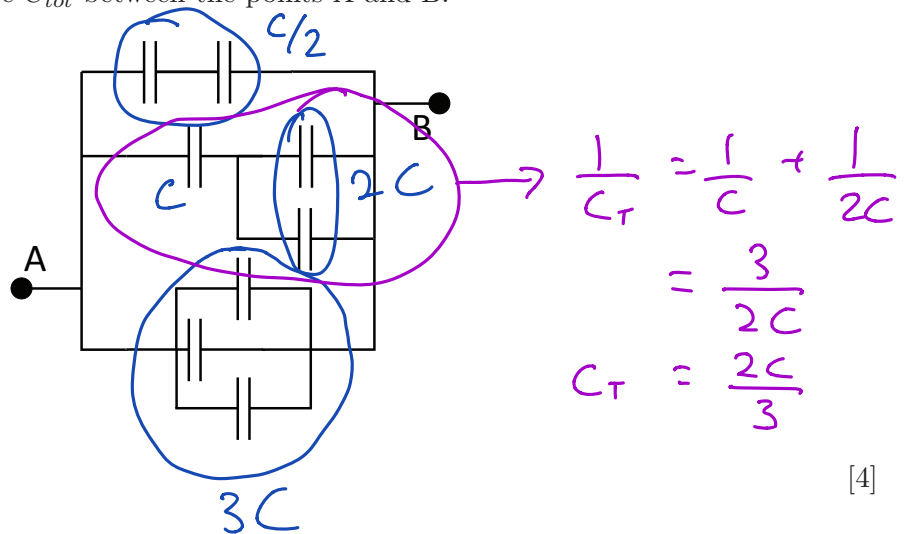


[2]

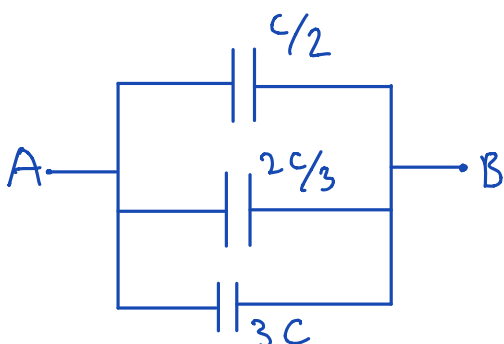
13. Consider the network of capacitors shown below. All capacitors in the network have the same capacitance C . Assume that capacitances combine as follows:

- Two capacitors in series combine their capacitance in the same way that two resistors in parallel combine their resistance.
- Two capacitors in parallel combine their capacitance in the same way that two resistors in series combine their resistance.

Find the total capacitance C_{tot} between the points A and B.



[4]



$$C_{tot} = \frac{C}{2} + \frac{2C}{3} + 3C$$

$$= \frac{25C}{6}$$

14. Solve the following equation for x : $\log_x 25 = \log_5(x)$

[4]

$$\frac{\log_5 25}{\log_5 x} = \log_5 x$$

$$2 = (\log_5 x)^2$$

$$\pm \sqrt{2} = \log_5 x$$

$$x = 5^{\pm\sqrt{2}}$$

15. A car is initially stationary. At time t_0 it starts to move at speed v_1 for a time Δt_1 . Immediately after this it moves at speed $v_2 > v_1$ for a time $\Delta t_2 < \Delta t_1$. After this time it is stationary again.

(a) Using this information sketch a graph of speed v versus time t . Label all axes sufficiently so that the graph is quantitatively accurate. [1]

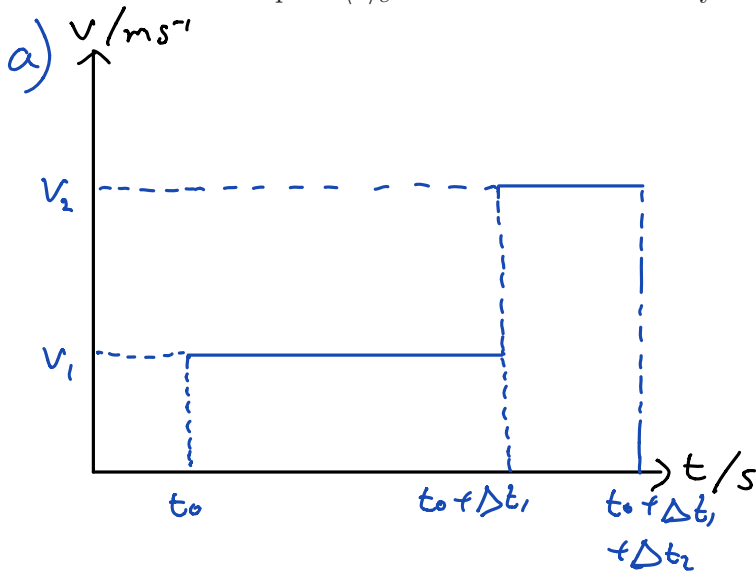
(b) From your graph determine the average speed $\langle v \rangle_t$ of the car while it is moving. Express $\langle v \rangle_t$ as a formula using the symbols $v_1, \Delta t_1, v_2, \Delta t_2$. $\langle v \rangle_t$ is known as the time weighted average. Compute a numerical value for $\langle v \rangle_t$ given the following: $v_1 = 1 \text{ ms}^{-1}, \Delta t_1 = 2 \text{ s}, v_2 = 2 \text{ ms}^{-1}, \Delta t_2 = 1 \text{ s}, t_0 = 0 \text{ s}$ [2]

(c) Now draw a graph of speed v versus distance s using the numerical values for speeds and times given in section (b) and label the axes as before. [1]

(d) From the second graph find the average speed $\langle v \rangle_s$ known as the distance weighted average. Again express $\langle v \rangle_s$ as a formula using the symbols $v_1, \Delta s_1, v_2, \Delta s_2$, where Δs_1 and Δs_2 are the distances that the car travels at v_1 and v_2 . Also provide a numerical value for $\langle v \rangle_s$ using the values given in section (b) [2]

(e) Which of the two averages $\langle v \rangle_t$ or $\langle v \rangle_s$ is equivalent to the conventional definition of average speed which we assume to be $\langle v \rangle_c = \frac{\text{total distance travelled}}{\text{total time taken}}$? [1]

(f) State the mathematical operation that should be applied to the curve of $v(t)$ to compute $\langle v \rangle_c$ if we assume v to be any function of time or distance. [1]



b)

$$\langle v \rangle_t = \frac{v_1 \Delta t_1 + v_2 \Delta t_2}{\Delta t_1 + \Delta t_2}$$

$$= \frac{1(2) + 2(1)}{2+1}$$

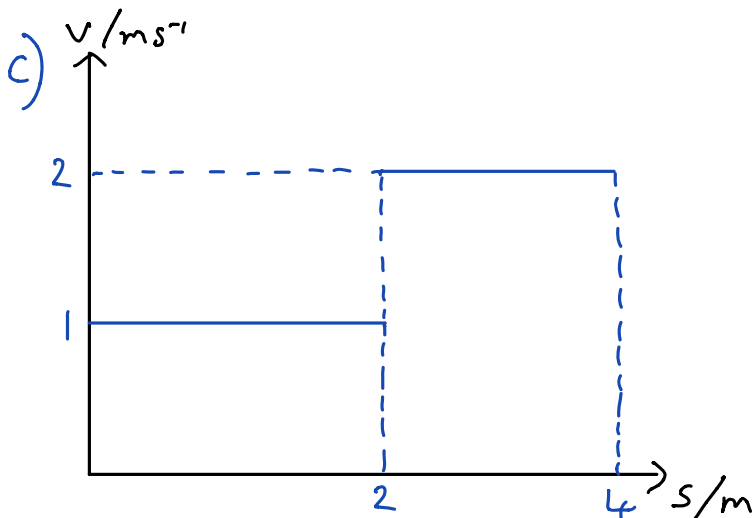
$$= 1.33 \text{ ms}^{-1}$$

d)

$$\langle v \rangle_s = \frac{v_1 \Delta s_1 + v_2 \Delta s_2}{\Delta s_1 + \Delta s_2}$$

$$= \frac{1(2) + 2(2)}{2+2}$$

$$= 1.5 \text{ ms}^{-1}$$



e)

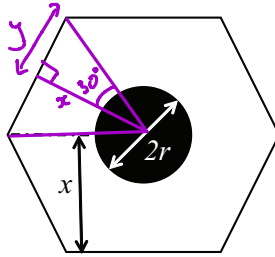
$$\langle v \rangle_c = \langle v \rangle_t$$

f) Integrate w.r.t. time and divide by the total time

16. Consider the cross section of a pencil, where a concentric circle of radius r sits at the centre of regular hexagon. If the black area is a quarter of the total area of the hexagon, what is the length of the dimension x in terms of r ?

$$\tan 30^\circ = \frac{y}{x} = \frac{1}{\sqrt{3}}$$

$$y = \frac{x}{\sqrt{3}}$$



$$\text{Area of hexagon} = 6 \times xy = \frac{6x^2}{\sqrt{3}}$$

[6]

$$\text{Black area} = \pi r^2$$

$$\text{Given } \frac{6x^2}{\sqrt{3}} = 4\pi r^2$$

$$x^2 = \frac{2\sqrt{3}\pi r^2}{3}$$

$$x = \left(\frac{2\sqrt{3}\pi}{3}\right)^{1/2} r$$

17. To which order should you expand $(1+x)^9$ in the integral $\int_0^{0.1} (1+x)^9 dx$ so that the approximate integral agrees with the exact result to better than 10%? [7]

$$\int_0^{0.1} (1+x)^9 dx = \left[\frac{(1+x)^{10}}{10} \right]_0^{0.1} = \frac{1.1^{10} - 1}{10} = 0.1593742$$

$$0.1593742 \times 1.1 = 0.175311 \quad (\text{upper limit})$$

$$0.1593742 \times 0.9 = 0.143437 \quad (\text{lower limit})$$

$$\int_0^{0.1} (1+x)^9 dx \approx \int_0^{0.1} \left[1 + 9x + \frac{9(8)x^2}{2} + \frac{9(8)(7)x^3}{6} + \dots \right] dx$$

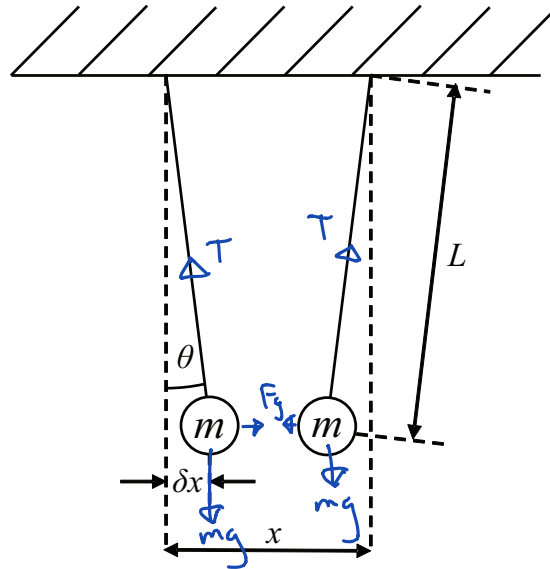
$$= \left[x + \frac{9x^2}{2} + 12x^3 + 21x^4 + \dots \right]_0^{0.1}$$

$$= \underbrace{0.1 + 0.045 + 0.012 + 0.0021 + \dots}$$

$$0.145 > 0.143437$$

\therefore Expanding up to the x -term is enough

18. The diagram shows two spheres of mass m suspended from wires of negligible mass. The length between the pivot points and the centres of each sphere is L . The two suspension points lie on a horizontal line and are separated by a distance x . The system is in equilibrium. Take the total acceleration due to gravity to be g .
Hint: You may assume that $\delta x \ll L$ as well as $\delta x \ll x$.



- (a) Show that the small deflection δx by which the gravitational force F_g between the spheres will deflect them from the vertical is given by:

$$\delta x \approx \frac{x}{8} - \sqrt{\left(\frac{x}{8}\right)^2 - \frac{LmG}{4xg}}$$

where G is Newton's gravitational constant. Clearly state all assumptions you make and show all your workings.

[7]

- (b) The spheres are now both electrically charged with the same positive charge Q . How big must Q be so that the wires hang vertically again?
Hint: The force between two spheres, electrically charged with charges Q_1 and Q_2 is given by:

$$F_Q = k \frac{Q_1 Q_2}{r^2}$$

where k is a constant and r is the separation of their centres.

[2]

This page intentionally left blank for working.

$$a) \uparrow T \cos \theta = mg$$

$$\rightarrow T \sin \theta = F_g = \frac{Gmm}{(x-2\delta x)^2}$$

$$L \sin \theta = \delta x$$

$$\frac{\delta x}{L} = \sin \theta \approx \tan \theta$$

$$\frac{T \sin \theta}{T \cos \theta} = \frac{Gmm}{(x-2\delta x)^2} \cdot \frac{1}{mg}$$

$$\tan \theta = \frac{Gm}{x^2 - 4x\delta x + 4(\delta x)^2} \cdot \frac{1}{mg}$$

$$\frac{\delta x}{L} \approx \frac{Gm}{gx^2 - 4xg\delta x}$$

$$LmG = gx^2\delta x - 4xg(\delta x)^2$$

$$4gx(\delta x)^2 - gx^2(\delta x) + LmG = 0$$

$$\delta x = \frac{gx^2 \pm \sqrt{g^2x^4 - 16gxLmG}}{8gx}$$

$$= \frac{x}{8} \pm \sqrt{\frac{g^2x^4}{64g^2x^2} - \frac{16gxLmG}{64g^2x^2}}$$

$$= \frac{x}{8} \pm \sqrt{\left(\frac{x}{8}\right)^2 - \frac{LmG}{4gx}}$$

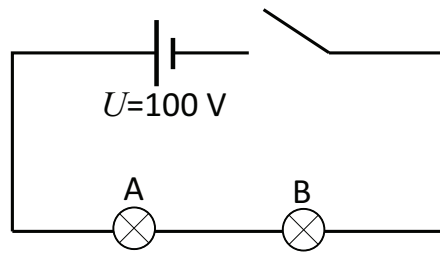
$$\text{As } m \rightarrow 0, \delta x \rightarrow 0 \quad \therefore \delta x = \frac{x}{8} - \sqrt{\left(\frac{x}{8}\right)^2 - \frac{LmG}{4gx}}$$

$$b) \frac{Gmm}{r^2} = \frac{kQQ}{r^2}$$

$$Q = \sqrt{\frac{Gr}{k}} m$$

[Turn over]

19.



Consider the above circuit. The filament lamp A consumes an electrical power $P_A = 100\text{ W}$ when it alone is connected to a voltage of $U = 100\text{ V}$. Filament lamp B consumes a power $P_B = 20\text{ W}$ when it alone is connected to a voltage of $U = 100\text{ V}$.

- (a) When the switch is closed, which lamp will be brighter and why?

Find the ratio of the levels of brightness of the two lamps assuming their resistances are constant. Assume further that the brightness of a lamp is proportional to the power it consumes.

[5]

- (b) How would your answer change if the lamps were wired in parallel rather than series?

[2]

$$P = \frac{V^2}{R} \quad R_A = \frac{100^2}{100} = 100\ \Omega \quad R_B = \frac{100^2}{20} = 500$$

$$a) \quad P = I^2 R \quad I_A = I_B \quad \frac{P_A}{P_B} = \frac{100 I^2}{500 I^2} = \frac{1}{5}$$

Bulb B is brighter

$$b) \quad P = \frac{V^2}{R} \quad V_A = V_B \quad \frac{P_A}{P_B} = \frac{V^2/100}{V^2/500} = 5$$

Bulb A is brighter

20. In a crystalline material, planes of symmetry are labeled using triplets of integer numbers of the form (hkl) , e.g. (100) is equivalent to $h = 1, k = 0, l = 0$. For this question it is not important how this labeling scheme works exactly or what h, k and l mean. By performing certain experiments, it is possible to determine these (hkl) triplets for multiple planes of symmetry. From the knowledge of several such triplets the microscopic structure of the material can be inferred using the following rules:

- If for all planes all positive integer values of h, k and l are possible the material has a simple cubic lattice.
- If for all planes $h + k + l = 2n$ where n is any positive integer then the material has a body centred cubic lattice.
- If for all planes h and k and l are either all even or all odd the material has a face centred cubic lattice.

Note: zero is an even number

During the analysis of the data from such an experiment the following four planes of symmetry were found: $(111), (200), (220), (311)$.

- (a) What type of lattice did the material in these experiments have? [2]
- (b) In a material with some form of cubic lattice the distance d between the planes of symmetry is given by:

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

where a is a constant length that is usually of the order of a fraction of a nanometre. Using the formula for d and the data in the table below determine the best estimate for a . [3]

d (nm)	Plane
0.224	(111)
0.195	(200)
0.137	(220)
0.117	(311)

- (c) A cube of the material has a length L in each dimension where $L = Na$ and N is a very large number. Pressure is applied to two opposite sides of the cube. The volume of the cube remains constant but one of the cube's dimensions reduces to a length of $L' = \frac{2}{3}Na$. What is the new length of the deformed object in the other two dimensions? [2]

a) Face centred cubic

b) $a = d\sqrt{h^2 + k^2 + l^2}$

$$a_1 = 0.224\sqrt{3} = 0.3880$$

$$a_2 = 0.195\sqrt{4} = 0.39$$

$$a_3 = 0.137\sqrt{8} = 0.3875$$

$$a_4 = 0.117\sqrt{11} = 0.3880$$

$$a = \frac{a_1 + a_2 + a_3 + a_4}{4} = 0.389$$

This page intentionally left blank for working.

$$c) \text{ Initially } V = L^3 = N^3 a^3 = \text{const}$$

$$\text{After deformation, } V = x^2 L'$$
$$= x^2 \frac{2}{3} Na$$

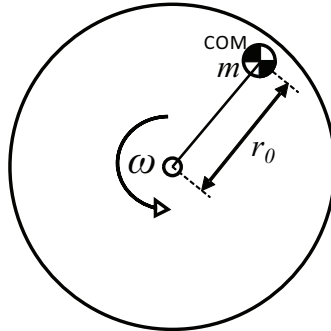
$$x^2 = \frac{3V}{2Na} = \frac{3N^3 a^3}{2Na}$$

$$x = \sqrt{\frac{3}{2}} Na$$

$$= \sqrt{\frac{3}{2}} L$$

[Turn over]

21. A child of mass m stands on a platform which rotates freely at angular speed ω . The child's centre of mass (COM) is initially at a radius of r_0 from the axis of the platform as shown in the figure below. The child now slowly pulls itself closer to the centre of the platform.



- (a) State the minimal force $F(r)$ needed to pull the child closer to the centre as a function of the radius r of the child's COM. [1]
- (b) Find a mathematical expression for the work done by the child when it has reached the centre of the platform.
Hint: You may take the work done along a path to be $W = \int F(s)ds$ where F is the force in the direction of motion and s is the distance. If your expression contains an integral you do not have to perform the integration yet. [1]
- (c) Into which form of energy is the child's work converted and what will qualitatively happen to the angular speed ω as the child approaches the centre? [2]
- (d) The angular momentum J of the system of child and platform is a conserved quantity during the above process. It is defined to be $J = I\omega$ where I is the moment of inertia of the system which is given by $I = mr^2 + I_p$ where I_p is the constant moment of inertia of the platform. Find an expression for the angular speed $\omega(r)$ as a function of the radius r of the child's COM. [2]
- (e) Find the total work done by the child when it has reached $r = 0$.
Hint: $\int \frac{x}{(a+x^2)^2} dx = \frac{-1}{2(a+x^2)}$ [4]

$$a) F(r) = mr[\omega(r)]^2$$

$$b) W = \int_0^{r_0} mr[\omega(r)]^2 dr$$

c) Rotational kinetic energy. ω increases

$$d) J = I\omega \\ = (mr^2 + I_p)\omega$$

$$\omega = \frac{J}{mr^2 + I_p}$$

This page intentionally left blank for working.

$$\begin{aligned}
 e) \quad W &= \int_0^{r_0} m r [\omega(r)]^2 dr \\
 &= \int_0^{r_0} \frac{m r J^2}{(m r^2 + I_P)^2} dr \\
 &= \frac{m J^2}{m^2} \int_0^{r_0} \frac{r}{\left(\frac{I_P}{m} + r^2\right)^2} dr \\
 &= \frac{J^2}{m} \left[\frac{-1}{2\left(\frac{I_P}{m} + r^2\right)} \right]_0^{r_0} \\
 &= \frac{-J^2}{2m} \left(\frac{1}{\frac{I_P}{m} + r_0^2} - \frac{m}{I_P} \right) \\
 &= \frac{J^2}{2} \left(\frac{1}{I_P} - \frac{1}{I_P + m r_0^2} \right)
 \end{aligned}$$

[Turn over]

22. Determine the area inside the circle defined by:

$$x^2 + y^2 - 8x + 4y + 4 = 0$$

but outside the triangle bounded by the three lines below.

$$y = x - 7$$

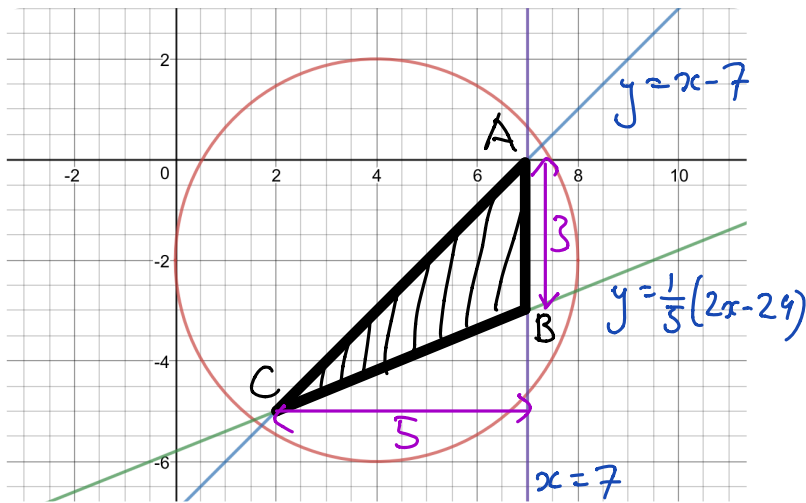
$$y = \frac{1}{5}(2x - 29)$$

$$x = 7$$

[9]

$$x^2 - 8x + 16 + y^2 + 4y + 4 = -4 + 16 + 4$$

$$(x - 4)^2 + (y + 2)^2 = 4^2$$



$$A(7, 0)$$

When $x = 7$, $y = \frac{1}{5}(14 - 29) = -3$

$$\Rightarrow B(7, -3)$$

$$x - 7 = \frac{1}{5}(2x - 29)$$

$$5x - 35 = 2x - 29$$

$$x = 2$$

When $x = 2$, $y = 2 - 7 = -5$

$$\Rightarrow C(2, -5)$$

$$\text{Area of triangle} = \frac{1}{2} \times 5 \times 3 = \frac{15}{2}$$

$$\text{Area of circle} = \pi(4)^2$$

$$\Rightarrow \text{Required area is } 16\pi - \frac{15}{2}$$

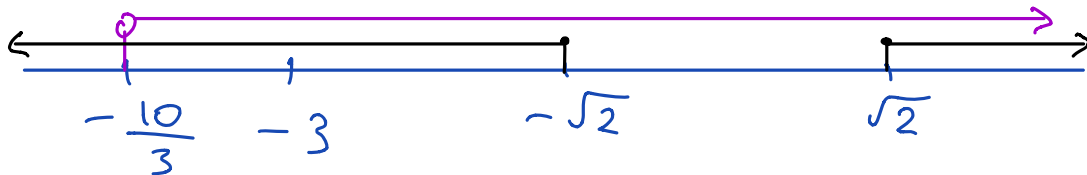
23. Given that

$$f(x) = \frac{\sqrt{x^2 - 2}}{\ln(3x + 10)}$$

Determine the range of x for which $f(x)$ is real and finite.

[5]

- $\ln(3x + 10)$ is real and finite when $3x + 10 > 0$
 $x > \frac{-10}{3}$
- $\sqrt{x^2 - 2}$ is real and finite when $x^2 - 2 \geq 0$
 $x \geq \sqrt{2}$ or $x \leq -\sqrt{2}$
- $f(x)$ is not finite when $\ln(3x + 10) = 0$
 $x = -3$



$$-\frac{10}{3} < x < -3 \quad \text{or} \quad -3 < x \leq -\sqrt{2} \quad \text{or} \quad x \geq \sqrt{2}$$