

**PHYSICS ADMISSIONS TEST
SAMPLE PAPER**

(2017 style, issued June 2017)

Time allowed: 2 hours

*For candidates applying to Physics, Physics and Philosophy,
Engineering, or Materials*

Total 100 Marks

Answers should be written on the question sheet in the spaces provided,
and you should attempt as many questions as you can.

Indicate multiple-choice answers by circling the best answer.

The numbers in the margin indicate the marks expected to be assigned
to each question. You are advised to divide your time according to
the marks available.

No calculators, tables, or formula sheets may be used.

Answers should be given exactly and in simplest terms
unless indicated otherwise.

Do NOT turn over until told that you may do so.

1. What is the coefficient of x^5 in the expansion of $(1+x)^4(1-x)^6$? [2]

A	B	C	D	E
-12	-8	-6	6	8

$$(1+4x+6x^2+4x^3+x^4) \times (1-6x+15x^2-20x^3+15x^4-6x^5+x^6)$$

$$1(-6) + 4(15) + 6(-20) + 4(15) + 1(-6) \\ = -6 + 60 - 120 + 60 - 6 = -12$$

$$\begin{array}{ccccccc} & & & & 1 & & \\ & & & & & 1 & \\ & & & 1 & & 2 & & 1 \\ & & 1 & & 3 & & 3 & & 1 \\ & 1 & & 4 & & 6 & & 4 & & 1 \\ 1 & & 5 & & 10 & & 10 & & 5 & & 1 \\ 1 & 6 & & 15 & & 20 & & 15 & & 6 & & 1 \end{array}$$

2. What is x in the equation $\log_2 x + \log_4 16 = 2$? [2]

A	B	C	D	E
0	1	2	4	8

$$\log_2 x + 2 = 2$$

$$\log_2 x = 0$$

$$x = 2^0$$

$$= 1$$

3. Differentiate the expression $x \sin(x^2)$ with respect to x . [2]

A	B	C	D	E
$\sin(2x)$	$\cos(2x)$	$\sin(x^2)$	$\sin(x^2) + 2x^2 \cos(x^2)$	$\sin(x^2) + x \cos(2x)$

$$\sin(x^2) + x \cos(x^2) \cdot 2x$$

- [2]

A	B	C	D	E
-10	-6	-2/3	-1/6	-1/10

$$m = \frac{8 - (-2)}{4 - 5} = \frac{10}{-1}$$

- [2]

A	B	C	D	E
4950	5000	5050	10000	10100

$$\begin{aligned} S &= \frac{n}{2} (a + l) \\ &= \frac{100}{2} (1 + 100) \\ &= 5050 \end{aligned}$$

- [2]

A	B	C	D	E
120	144	192	240	256

4 digits: $4 \times 3 \times 2 = 24$

$5 - - -$

$6 - - -$

$7 - - -$

$3 \times 24 = 72$

5 digits: $5 \times 4 \times 3 \times 2 \times 1 = 120$

$\frac{72}{+}$

192

7. Excluding Pluto, for the planets in our solar system, in order of increasing mean distance from the Sun, which of the following statements is/are correct? [2]

1. the duration of the day on each planet increases \times
2. the duration of the year on each planet increases \checkmark Kepler's 3rd
3. the size/volume of the planets increases \times Jupiter
4. the number of moons of each planet increases \times
5. the planets change from rocky to gas giants \checkmark

A	B	C	D	E
1 and 2 and 5	2 only	3 and 4	2 and 5	1 and 3

8. In which part of the electromagnetic spectrum do waves have a frequency of approximately 100 GHz? [2]

A	B	C	D	E
X rays	visible light	microwave	radio waves	infrared

$$\lambda = \frac{10^8}{10^{11}} = 1 \text{ mm}$$

9. An object with small mass becomes detached from the International Space Station (ISS) while it orbits the Earth. Its relative velocity with respect to the ISS can be neglected. What happens to the object? [2]

A	B	C
It will follow ISS in its orbit.	It will go straight along a direction tangential to the ISS orbit at the point when it became detached.	It will fall straight down towards the Earth.
D		E
It will stay still with respect to the Earth.		It will orbit around the ISS.

10. Two satellites are in orbit around the Earth. The first is in a geostationary orbit, the second satellite at radius half that of the first. What is the (approximate) period, in hours, of the second satellite?

[2]

$$T^2 \propto r^3$$

A	B	C	D	E
4.3	8.5	17	48	72

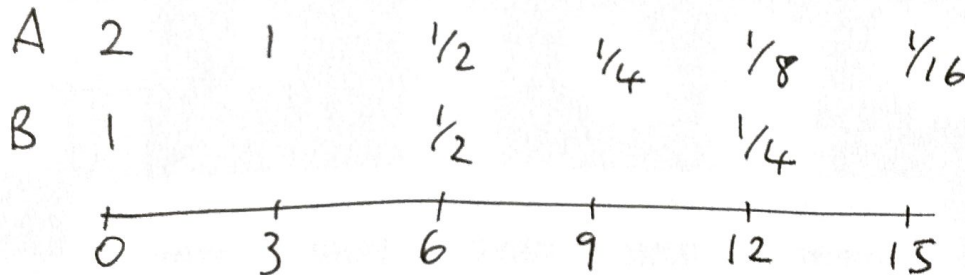
$$\frac{24^2}{R^3} = \frac{T^2}{(R/2)^3}$$

$$T = \frac{24 \times 24}{8} = \sqrt{72} = 6\sqrt{2} = 6 \times 1.41 = 8.46$$

11. A radioactive sample contains two different isotopes, A and B. A has a half-life of 3 days, and B has a half-life of 6 days. Initially in the sample there are twice as many atoms of A as of B. At what time will the ratio of the number of atoms of A to B be **reversed**?

[2]

A	B	C	D	E
3 days	6 days	9 days	12 days	never



12. A cube painted black is cut into 125 identical cubes. How many of them are not painted at all?

[2]

A	B	C	D	E
9	21	25	27	30

13. A jar contains buttons of four different colours. There are twice as many yellow as green, twice as many red as yellow, and twice as many blue as red. What is the probability of taking from the jar

- (a) a blue button;
- (b) a red button;
- (c) a yellow button; or
- (d) a green button?

You may assume that you are only taking one button at a time and replacing it in the jar before selecting the next one.

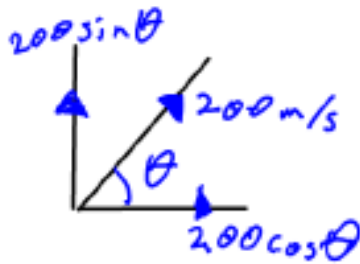
[4]

$$\begin{array}{ccccc} \frac{Y}{2} & \frac{G}{1} & \frac{R}{4} & \frac{B}{8} & \frac{\text{Tot}}{15} \end{array}$$

$$\begin{array}{cccc} \text{a) } \frac{8}{15} & \text{b) } \frac{4}{15} & \text{c) } \frac{2}{15} & \text{d) } \frac{1}{15} \end{array}$$

14. A gun is designed that can launch a projectile of mass 10 kg at a speed of 200 m/s. The gun is placed close to a straight, horizontal railway line and aligned such that the projectile will land further down the line. A small rail car of mass 200 kg and travelling at a speed of 100 m/s passes the gun just as it is fired. Assuming the gun and the car are at the same level, at what angle upwards must the projectile be fired in order that it lands in the rail car?

[3]



$$200 \cos \theta = 100$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = 60^\circ$$

15. What is the sum of the following terms:

$$1 + e^{-x} + e^{-2x} + \dots$$

Over what range of x is the solution valid?

[4]

G.P. with $a=1$, $r=e^{-x}$

$$S_{\infty} = \frac{a}{1-r} = \frac{1}{1-e^{-x}}$$

Valid for $|r| < 1$

$$|e^{-x}| < 1$$

$$e^{-2x} < 1$$

$$-2x < \ln 1$$

$$\underline{x > 0}$$

16. An electron gun in a cathode ray tube accelerates an electron with mass m and charge $-e$ across a potential difference of 50 V and directs it horizontally towards a fluorescent screen 0.4 m away. How far does the electron fall during its journey to the screen? Take $m \approx 10^{-30}$ kg and $e \approx 1.6 \times 10^{-19}$ C. [5]

$$\frac{1}{2}mv^2 = eV$$

$$v^2 = \frac{2 \times 1.6 \times 10^{-19} \times 50}{10^{-30}} = 16 \times 10^{12}$$

$$v = 4 \times 10^6 \text{ ms}^{-1}$$

$$\text{Time to screen, } t = \frac{0.4}{4 \times 10^6} = 1 \times 10^{-7} \text{ s}$$

$$\downarrow \text{ +ve } s = ?, u = 0, v = x, a = 10 \text{ ms}^{-2}, t = 1 \times 10^{-7} \text{ s}$$

$$s = ut + \frac{1}{2}at^2$$

$$= \frac{1}{2} \times 10 \times (1 \times 10^{-7})^2$$

$$= 5 \times 10^{-14} \text{ m}$$

17. Evaluate the following integrals:

(a)

$$\int_0^{\pi/2} \frac{\cos x}{1 + \sin x} dx$$

(b)

$$\int_0^2 \frac{x}{x^2 + 6x + 8} dx$$

[6]

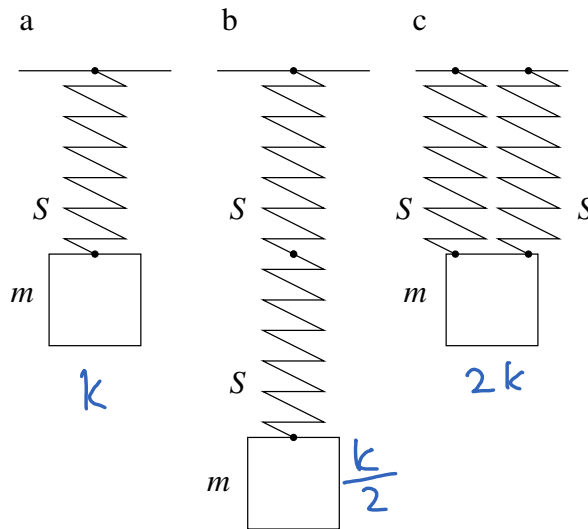
$$\begin{aligned} \text{a) } \int_0^{\pi/2} \frac{\cos x}{1 + \sin x} dx &= \left[\ln |1 + \sin x| \right]_0^{\pi/2} \\ &= \ln 2 - \ln 1 \\ &= \underline{\ln 2} \end{aligned}$$

$$\begin{aligned} \text{b) } \int_0^2 \frac{x}{x^2 + 6x + 8} dx &= \int_0^2 \left(\frac{-4/-2}{x+4} + \frac{-2/2}{x+2} \right) dx \\ &= 2 \int_0^2 \frac{1}{x+4} dx - \int_0^2 \frac{1}{x+2} dx \\ &= \left[2 \ln |x+4| - \ln |x+2| \right]_0^2 \\ &= \left[\ln \left| \frac{(x+4)^2}{x+2} \right| \right]_0^2 = \ln 9 - \ln 8 \\ &= \underline{\ln \left(\frac{9}{8} \right)} \end{aligned}$$

18. A mass m is attached to a spring S (as sketched in figure **a** below) and oscillates with a period T . What would be the period of the oscillation if two springs S are connected in series (figure **b**) or in parallel (figure **c**)?

What would be the period of the oscillations in figure **a** on a planet with surface gravity $2g$?

[4]



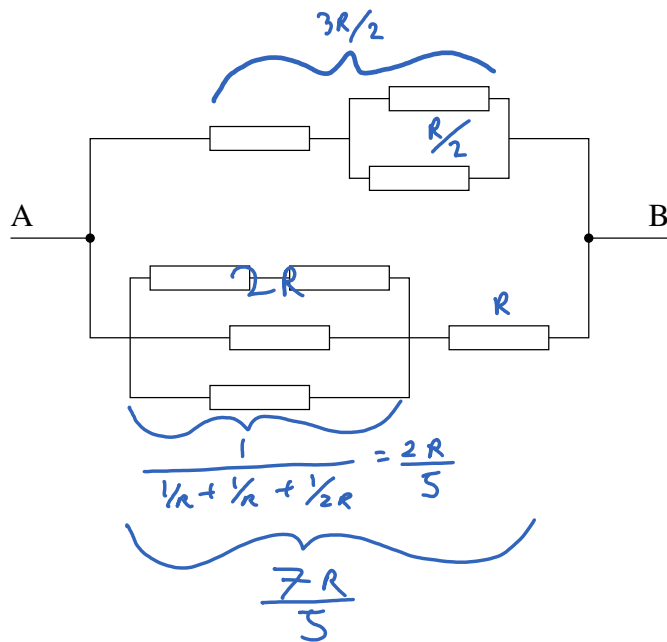
$$T = 2\pi \sqrt{\frac{m}{k}}$$

series: $T' = 2\pi \sqrt{\frac{m}{k/2}} = \underline{\underline{\sqrt{2} T}}$

parallel: $T' = 2\pi \sqrt{\frac{m}{2k}} = \underline{\underline{\frac{T}{\sqrt{2}}}}$

other planet: same because period
doesn't depend on g

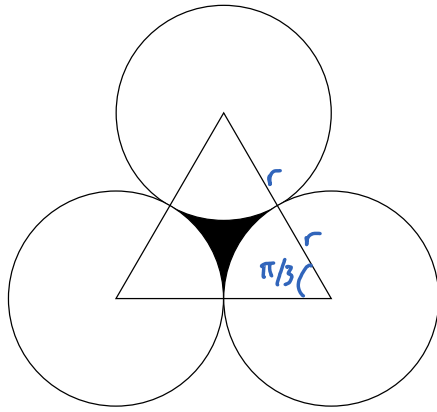
19. Given the circuit below, where all the resistors have the same resistance R , what is the resistance between A and B ? [5]



$$\begin{aligned}
 R_T &= \frac{1}{\frac{2}{3R} + \frac{5}{7R}} \\
 &= \frac{1}{\frac{14 + 15}{21R}} \\
 &= \frac{21R}{29}
 \end{aligned}$$

20. Consider the shape shown below, consisting of three circles, each with radius r . An equilateral triangle with sides of length $2r$ joins the centres of the circles. What is the area of the triangle which is not enclosed within the circles, and which is shown shaded black in the figure?

[5]



$$A = A_{\text{triangle}} - 3 A_{\text{sector}}$$

$$A_{\text{sector}} = \frac{\pi}{3} \times \frac{1}{2} \times r^2 = \frac{\pi r^2}{6}$$

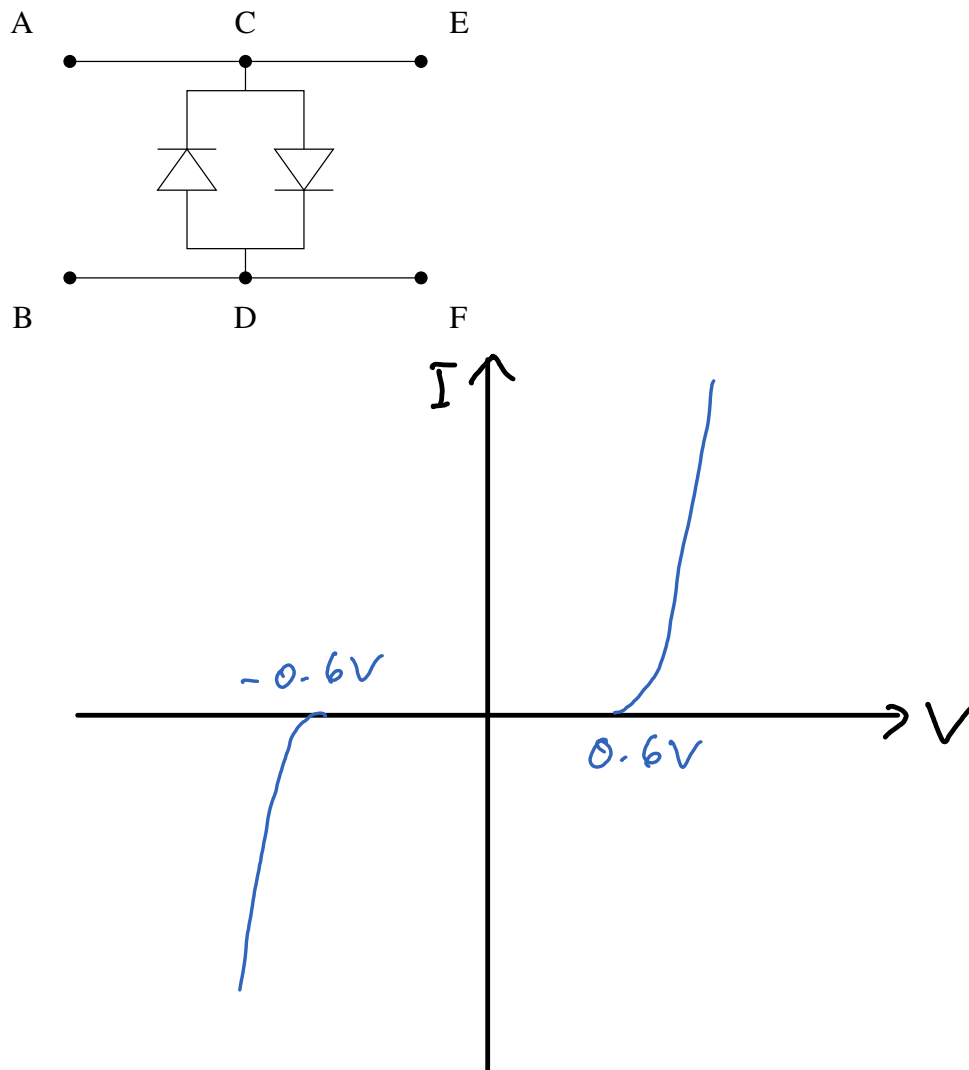
$$\begin{aligned} A_{\text{triangle}} &= \frac{1}{2} (2r)^2 \times \sin \frac{\pi}{3} \\ &= \frac{1}{2} \cdot 4r^2 \cdot \frac{\sqrt{3}}{2} \\ &= \sqrt{3} r^2 \end{aligned}$$

$$\begin{aligned} \Rightarrow A &= \sqrt{3} r^2 - 3 \times \frac{\pi r^2}{6} \\ &= \underline{\underline{\left(\sqrt{3} - \frac{\pi}{2}\right) r^2}} \end{aligned}$$

21. Two diodes are connected as sketched below. Sketch the current flowing between points C and D as a function of voltage applied between points C and D.

A sensitive amplifier is connected to terminals E and F to measure small electric signals from an instrument connected to terminals A and B. From time to time there are discharges in the instrument which might destroy the amplifier if the amplifier is connected to the instrument directly, without the diodes. Explain briefly how the diodes protect the amplifier.

[4]



For small signals (low voltage), current doesn't flow through C-D. Amplifier operates.

When there's a discharge (high voltage), current flows through C-D, not through the amplifier.

22. You want to make a snowman out of modelling clay. The snowman consists of 2 spheres, where one sphere has a radius r , and the other has a radius $2r$. The modelling clay comes in the form of a cylinder with radius $r/2$. What length of modelling clay is required to make the snowman?

[5]

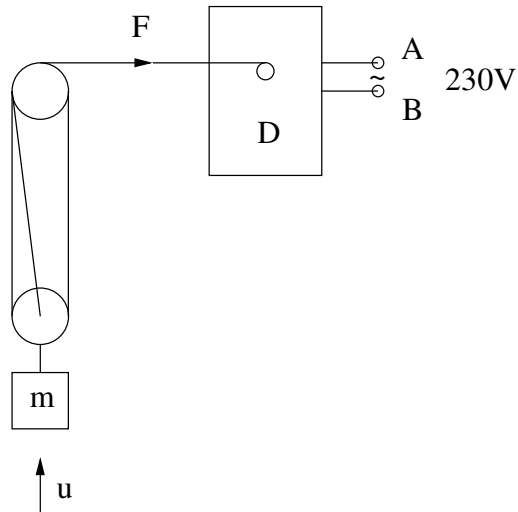
$$V_{\text{snow}} = \frac{4}{3} \pi r^3 + \frac{4}{3} \pi (2r)^3$$
$$= 12 \pi r^3$$

$$V_{\text{clay}} = \pi \left(\frac{r}{2}\right)^2 \cdot l = 12 \pi r^3$$
$$l = \frac{12 \pi r^3}{\pi \frac{r^2}{4}}$$
$$= \underline{48r}$$

23. An electric motor is lifting a mass via a system of pulleys as sketched below. The motor is powered by a voltage source of 230 V. The diameter D of the motor winding reel is 5 cm, and a mass m of 100 kg is being lifted with a speed u of 0.5 m/s. The masses of the pulleys and the string can be neglected.

- (a) What is the electric current driving the motor?
 (b) What is the angular velocity of the motor's winding reel?
 (c) What is the force F with which the motor is pulling?

[5]



$$a) P = IV$$

$$P = \frac{mgh}{t} = \frac{100 \times 10 \times 0.5}{1}$$

$$= 500 \text{ W}$$

$$\Rightarrow I = \frac{500}{230} = \underline{\underline{\frac{50}{23} \text{ A}}}$$

$$b) v = \omega r$$

$$v = 3u = 1.5 \text{ m s}^{-1}$$

$$r = D/2 = 0.025 \text{ m}$$

$$\omega = \frac{1.5}{0.025} = \frac{15}{0.25} = \underline{\underline{60 \text{ rad s}^{-1}}}$$

$$c) P = Fv$$

$$F = \frac{P}{v} = \frac{500}{1.5} = \underline{\underline{333 \text{ N}}}$$

24. Sketch the region defined by the following inequalities:

$$x^2 \geq y$$

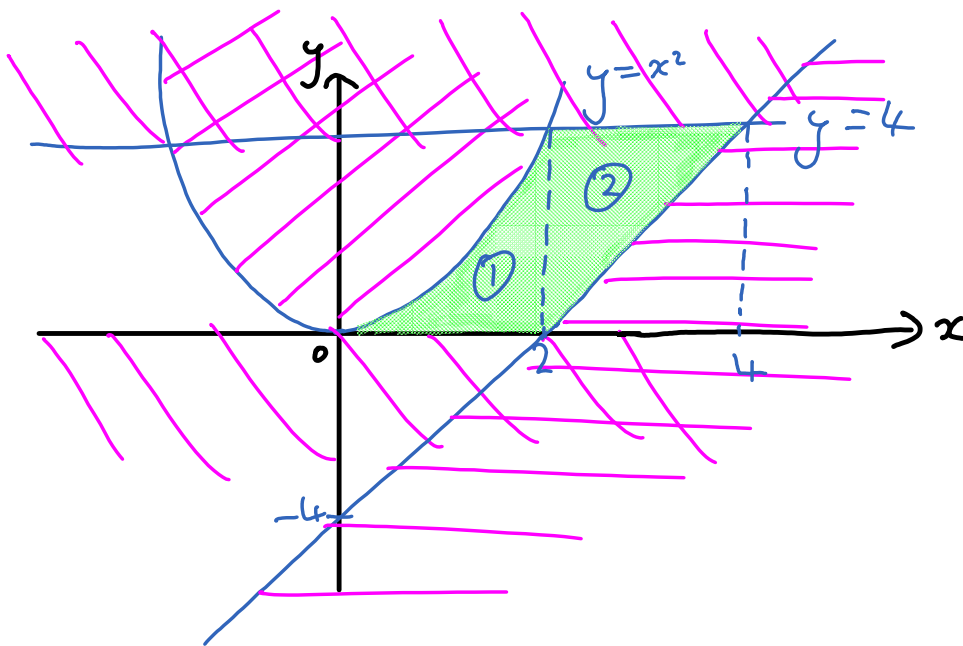
$$4 \geq y \geq 0$$

$$y \geq 2x - 4$$

$$x \geq 0$$

Evaluate the area defined by the above inequalities.

[7]



$$4 = 2x - 4$$

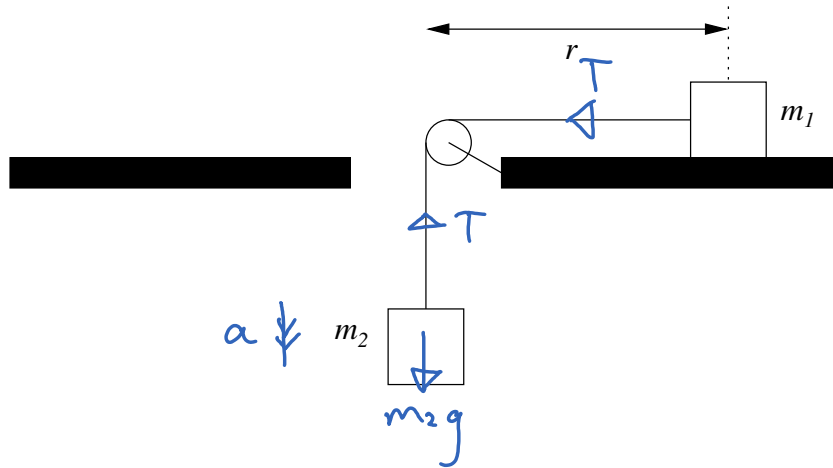
$$x = 4$$

$$\textcircled{1}: \int_0^2 x^2 dx = \left[\frac{x^3}{3} \right]_0^2 = \frac{8}{3}$$

$$\textcircled{2}: \frac{4 \times 2}{2} = 4$$

$$\Rightarrow A = 4 + \frac{8}{3} = \underline{\underline{\frac{20}{3}}}$$

25. Two masses m_1 and m_2 are connected by a massless, non-extensible string supported by a massless pulley attached to a table with a hole in the middle; see the sketch below. Assuming no friction, derive an expression for the acceleration



of the masses and for the tension of the string.

Now consider friction acting on the table, but not on the pulley, in the system described previously, with coefficient of static friction μ_s and coefficient of dynamic friction μ_d . Derive expressions for the acceleration of the masses and for the tension of the string. What condition needs to be satisfied for m_1 to accelerate?

[5]

$$1: [F = ma] \leftarrow$$

$$T = m_1 a$$

$$2: [F = ma] \downarrow$$

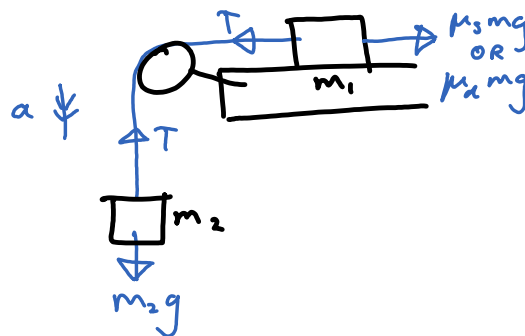
$$m_2 g - T = m_2 a$$

$$(1) \text{ in } (2): m_2 g - m_1 a = m_2 a$$

$$a = \frac{m_2 g}{m_1 + m_2}$$

$$\text{In (1): } T = \frac{m_1 m_2 g}{m_1 + m_2}$$

25 cont.



$$1: [F = ma] \leftarrow$$

$$T - \mu_d m_1 g = m_1 a$$

$$2: [F = ma] \downarrow$$

$$m_2 g - T = m_2 a$$

$$(1) + (2): m_2 g - \mu_d m_1 g = (m_2 + m_1) a$$

$$a = \frac{g(m_2 - m_1 \mu_d)}{m_2 + m_1}$$

$$\ln (2): T = m_2 g - m_2 g \frac{(m_2 - m_1 \mu_d)}{m_2 + m_1}$$

$$= m_2 g \left(1 + \frac{m_1 \mu_d - m_2}{m_1 + m_2} \right)$$

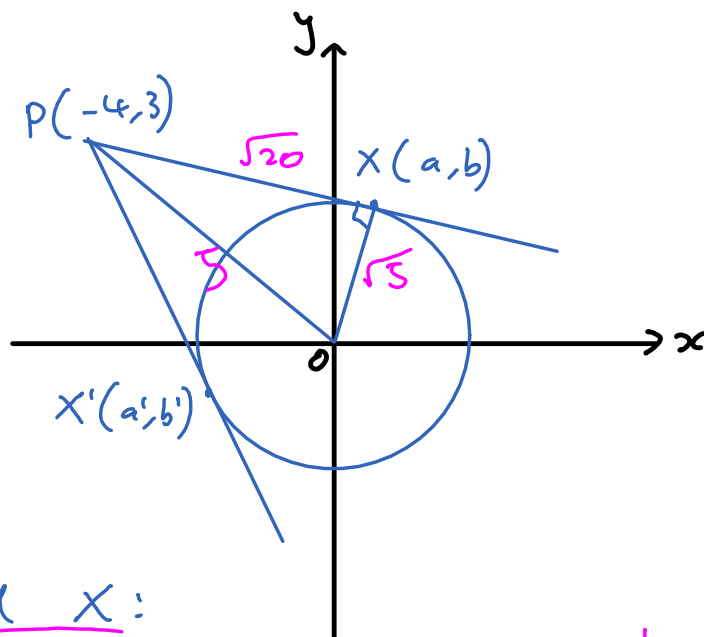
For acceleration, $T > \mu_s m_1 g$ and $T < m_2 g$

$$\Rightarrow \mu_s m_1 g < T < m_2 g$$

$$\therefore \mu_s m_1 < m_2$$

26. Find the equations of the tangents drawn from the point $(-4, 3)$ to the circle $x^2 + y^2 = 5$.

[7]



$$OP^2 = 3^2 + (-4)^2 = 25$$

$$PX^2 = OP^2 - OX^2$$

$$= 25 - 5$$

$$= 20$$

Find X:

$$PX: (a+4)^2 + (b-3)^2 = 20$$

$$a^2 + 8a + 16 + b^2 - 6b + 9 = 20$$

$$a^2 + b^2 + 8a - 6b = -5 \quad (1)$$

$$OX: a^2 + b^2 = 5 \quad (2)$$

$$(1) - (2): 8a - 6b = -10$$

$$b = \frac{4a+5}{3} \quad (3)$$

$$(1): a^2 + \left(\frac{4a+5}{3}\right)^2 + 8a - 6\left(\frac{4a+5}{3}\right) = -5$$

$$9a^2 + 16a^2 + 40a + 25 + 72a - 72a - 90 = -5$$

$$25a^2 + 40a - 20 = 0$$

$$5a^2 + 8a - 4 = 0$$

$$(5a-2)(a+2) = 0$$

$$a = \frac{2}{5} \quad \text{or} \quad a' = -2$$

$$(3): b = \frac{4\left(\frac{2}{5}\right) + 5}{3} = \frac{11}{5}$$

$$b' = \frac{4(-2) + 5}{3} = -1$$

Find eqn's:

$$PX: m = \frac{\frac{11}{5} - 3}{\frac{2}{5} - (-4)} = \frac{11-15}{2+20} = -\frac{2}{11}$$

$$y - 3 = -\frac{2}{11}(x + 4)$$

$$\underline{2x + 11y - 25 = 0}$$

$$PX': m = \frac{-1 - 3}{-2 - 4} = -2$$

$$y - 3 = -2(x + 4)$$

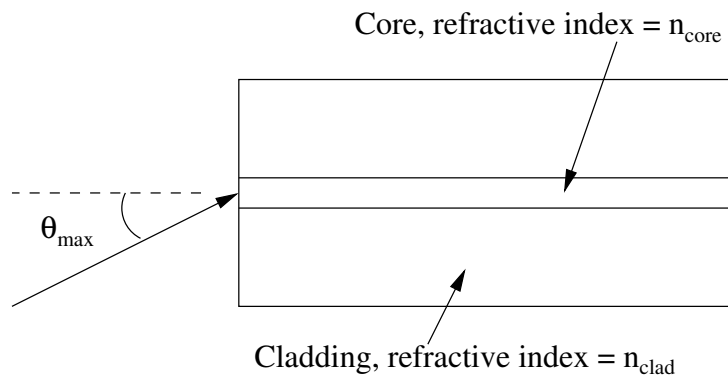
$$\underline{2x + y + 5 = 0}$$

27. This question concerns total internal reflection, optical fibres, and refraction. You may assume that the refractive index of glass is larger than that of water, and that the refractive index of water is larger than that of air.

Explain what is meant by the phrases “total internal reflection” and “critical angle”. (You are encouraged to use a diagram to explain your answer.)

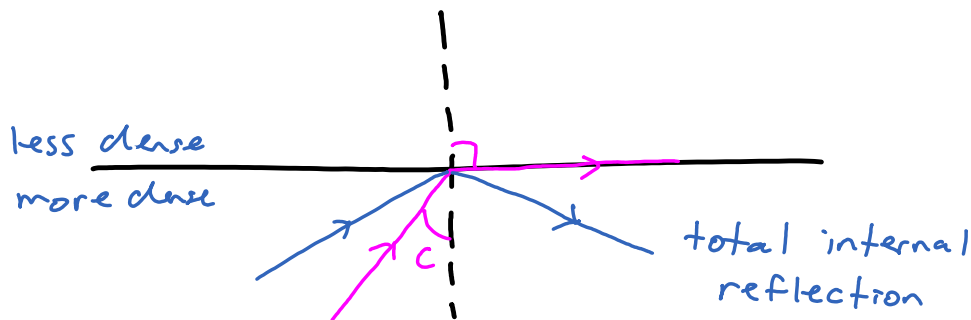
Derive an equation relating the critical angle and the refractive indices of two materials, n_1 and n_2 , where $n_2 < n_1$.

An optical fibre is usually made of two materials, a core and a cladding, as shown in the diagram below (not drawn to scale).



Light may only be transmitted along the fibre if the incident angle of the light is less than a maximum angle θ_{max} . By using your equation from above and Snell's Law, or otherwise, derive an expression for θ_{max} in terms of the core and cladding refractive indices only.

[7]

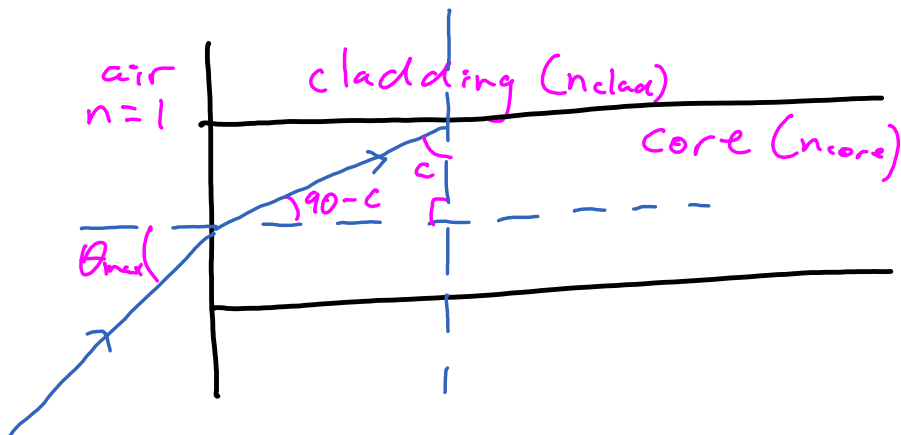


$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$n_1 \sin c = n_2 \sin 90$$

$$\sin c = \frac{n_2}{n_1}$$

27 cont.



part b: $\sin c = \frac{n_2}{n_1} = \frac{n_{core}}{n_{clad}}$

Snell's law: $\sin \theta_{max} = n_{core} \sin (90 - c)$

$$= n_{core} \cos c$$

$$= n_{core} \sqrt{1 - \sin^2 c}$$

$$= n_{core} \sqrt{1 - \left(\frac{n_{core}}{n_{clad}}\right)^2}$$

$$\theta_{max} = \arcsin \left[n_{core} \sqrt{1 - \left(\frac{n_{core}}{n_{clad}}\right)^2} \right]$$