**THE COLLEGES OF OXFORD UNIVERSITY**

**PHYSICS**

**Wednesday 4 November 2009**

**Time allowed: 2 hours**

*For candidates applying for Physics, and Physics and Philosophy*

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**There are two parts (A and B) to this test, carrying equal weight.**

Answers should be written on the question sheet in the spaces provided and you should attempt as many questions as you can from each part.

Marks for each question are indicated in the right hand margin. There are a total of 100 marks available and total marks for each section are indicated at the start of a section. You are advised to divide your time according to the marks available, and to spend equal effort on parts A and B.

**No calculators, tables or formula sheets may be used.**

Answers in Part A should be given exactly unless indicated otherwise. Numeric answers in Part B should be calculated to 2 significant figures.

Use $g = 10\,\mathrm{m\,s^{-2}}$.

**Do NOT turn over until told that you may do so.**

# Part A: Mathematics for Physics [50 Marks]

1. If $x = \sin t$ and $y = \tan t$, express $y$ in terms of $x$. [3]

$$y = \tan t = \frac{\sin t}{\cos t} = \frac{\sin t}{\sqrt{1-\sin^2 t}}$$

$$= \frac{x}{\sqrt{1-x^2}}$$

2. Sketch the function $y = x + \dfrac{4}{x^2}$ over the range $-4 < x < +4$. Find the stationary point of this function. [5]

$$y = x + \frac{4}{x^2}$$

$$\frac{dy}{dx} = 1 - \frac{8}{x^3} = 0$$

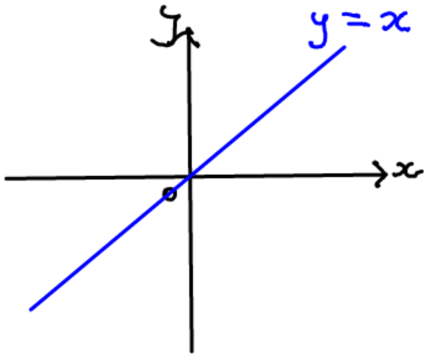
$$x^3 = 8$$

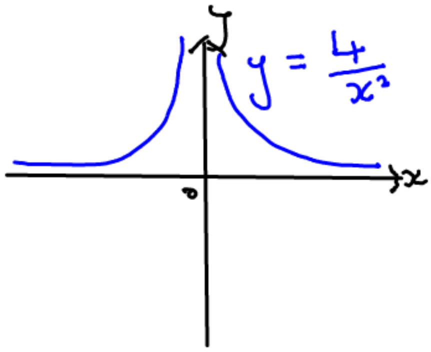
$$x = 2$$

When $x = 2$,

$y = 3$

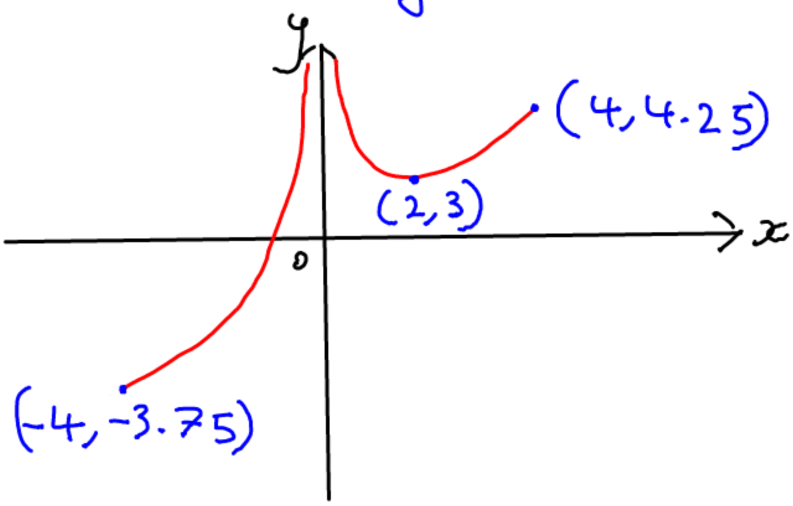
$\therefore$ $(2, 3)$ is a stationary pt.



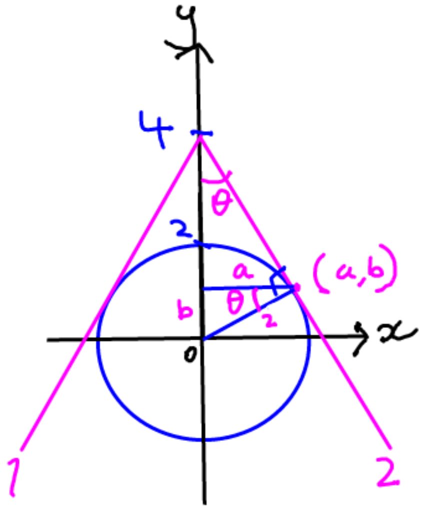

when $x = 4$, $y = 4.25$

$x = -4$, $y = -3.75$



3. Find the equations of the two lines which pass through the point (0,4) and form tangents to a circle of radius 2, centred on the origin.

[5]




$\sin\theta = \frac{2}{4} = \frac{b}{2}$

$b = 1$

$2^2 = b^2 + a^2$

$a = \sqrt{4-1} = \sqrt{3}$

$\Rightarrow m_1 = \frac{3}{\sqrt{3}} = \sqrt{3}, \quad m_2 = -\sqrt{3}$

$\therefore y = \sqrt{3}x + 4$ and $y = -\sqrt{3}x + 4$

4. (i) Find $x$ where $\log_2 \sqrt[3]{x} = \frac{1}{2}$,

[2]

$\frac{1}{3}\log_2 x = \frac{1}{2}$

$\log_2 x = \frac{3}{2}$

$2^{3/2} = x = 2\sqrt{2}$

(ii) Calculate $\sqrt{\log_8 16}$.

[3]

$\sqrt{\log_8 16} = \sqrt{\log_8 (8)^{4/3}} = \sqrt{(4/3)\log_8 8}$

$= \sqrt{\frac{4}{3}}$

$= \frac{2}{\sqrt{3}}$

5. Find all the solutions to $x^4 - 13x^2 + 36 = 0$. [4]

$(x^2-9)(x^2-4)=0$

$x^2=9 \qquad x^2=4$

$x=\pm 3, \quad x=\pm 2$

6. Find the range of values for $x$ (where $x > 0$) for which

$$x + 2 < 1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \cdots$$

[5]

$S_\infty = \frac{1}{1-\frac{1}{x}} = \frac{x}{x-1}$

$x+2 < \frac{x}{x-1}$

$(x-1)^2(x+2) < x(x-1)$

$x^3-3x+2 < x^2-x$

$x^3-x^2-2x+2<0$

$(x-1)(x^2+0x-2)<0$

$(x-1)(x+\sqrt{2})(x-\sqrt{2})<0$

$- \quad -\sqrt{2} \quad + \quad 1 \quad - \quad \sqrt{2} \quad +$

But given $x>0$

$\therefore\ 1 < x < \sqrt{2}$

7. If two identical dice are thrown, what is the probability that the total of the numbers is 10 or higher? [Hint: list the combinations that can give a total of 10 or higher.] [2]

$P(T\geq 10) = P(4,6)+P(5,5)+P(5,6)+P(6,4)$
$+P(6,5)+P(6,6)$
$= \left(\frac{1}{6}\times\frac{1}{6}\right)\times 6 = \frac{1}{6}$

Two dice have been thown, giving a total of at least 10. What is the probability that the throw of a third die will bring the total of the three numbers shown to 15 or higher? [3]

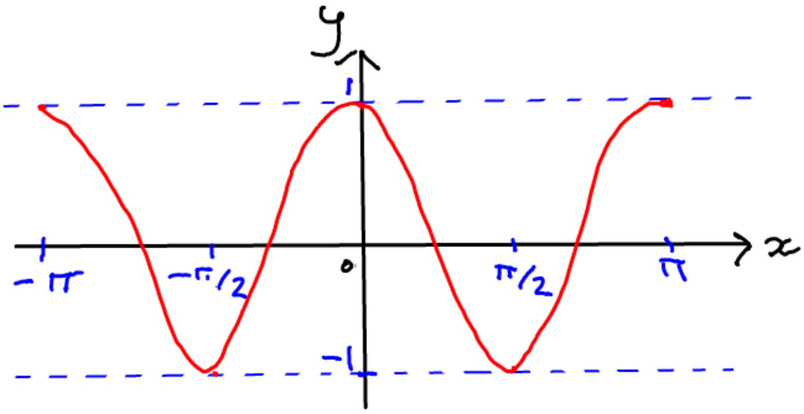
| | | Two | third | |
|---|---|---|---|---|
| $\frac{3}{6}$ | 4,6 / 5,5 / 6,4 | 10 | 5,6 | $\Rightarrow \frac{3}{6}\times\frac{2}{6} = \frac{6}{36}$ |
| $\frac{2}{6}$ | 5,6 / 6,5 | 11 | 4,5,6 | $\Rightarrow \frac{2}{6}\times\frac{3}{6} = \frac{6}{36}$ |
| $\frac{1}{6}$ | 6,6 | 12 | 3,4,5,6 | $\Rightarrow \frac{1}{6}\times\frac{4}{6} = \frac{4}{36}$ |

$\therefore P(T\geq 15)$
$= \frac{6}{36}+\frac{6}{36}+\frac{4}{36}$
$= \frac{4}{9}$

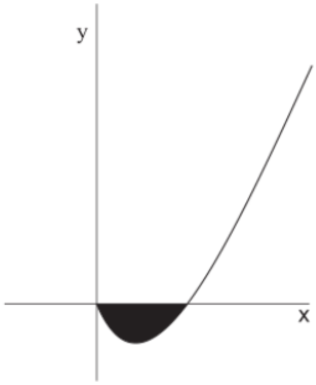
8. Sketch the function $y = 1 - 2\sin^2 x$ over the range $-\pi < x < +\pi$. [3]

- $y = \sin x$ squared
- Reflection in $y$-axis
- Stretch along $y$-axis by a factor 2
- Translation along $y$-axis by 1 units

Alternative:
$1 - 2\sin^2 x = \cos 2x$




9. The plot below shows the function $y = \dfrac{(x^2 - 4x)}{\sqrt{x}}$. Calculate the area of the shaded regions between the $x$-axis and the section of the line below the $x$-axis. [5]




When $y = 0$,

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0$ or $4$

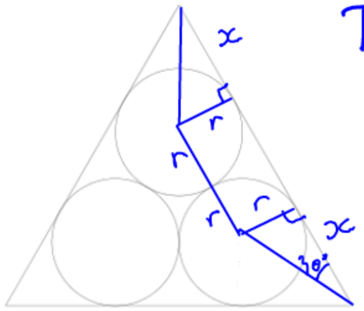
$$-A = \int_0^4 \frac{x^2 - 4x}{\sqrt{x}}\,dx = \int_0^4 \left(x^{3/2} - 4x^{1/2}\right)dx$$

$$= \left[\frac{2}{5}x^{5/2} - \frac{8}{3}x^{3/2}\right]_0^4 = \left(\frac{2}{5}\times 32\right) - \left(\frac{8}{3}\times 8\right)$$

$$= \frac{64}{5} - \frac{64}{3} = -\frac{128}{15}$$

$$A = \frac{128}{15}$$

10. The figure below shows three circles arranged within a triangle. Find an expression for the ratio of the area of the circles to the area of the triangle. [5]




Triangle is equilateral.

$\tan 30 = \frac{r}{x}$

$x = \sqrt{3}\, r$

One side of triangle $= x + 2r + x = (2 + 2\sqrt{3})r$

Area of triangle $= \frac{1}{2} \times (2+2\sqrt{3})r \times (2+2\sqrt{3})r \sin 60$

$= (4 + 12 + 8\sqrt{3})r^2 \frac{\sqrt{3}}{4} = (4\sqrt{3}+6)r^2$

$\frac{A_{circles}}{A_{triangle}} = \frac{3\pi r^2}{(4\sqrt{3}+6)r^2} = \frac{3\pi}{4\sqrt{3}+6}$

11. Determine the value of $b$ for which the sequence $-a, -\frac{a}{b}, \frac{a}{b}, a$ is an arithmetic progression (where $a \neq 0$). [2]

$a - \frac{a}{b} = \frac{a}{b} - \left(-\frac{a}{b}\right)$

$a = \frac{3a}{b}$

$b = 3$

12. Evaluate $2.1^5$ to one decimal place. [3]

$(2.1)^5 = (2 + 0.1)^5 = 2^5 + 5(2)^4 0.1 +$

$= 2^5 + 5(2)^4 0.1 + 10(2)^3(0.1)^2 + 10(2)^2(0.1)^3 + \ldots$

$= 32 + 8 + 0.8 + 0.04$

$= 40.8$

## Part B: Physics [50 Marks]

### Multiple choice (10 marks)

Please circle **one** answer to each question only.

$E = mc^2$

$m = \frac{E}{c^2}$

$= \frac{3.8 \times 10^{26}}{(3 \times 10^8)^2}$

13. The sun produces $3.8 \times 10^{26}$ W through fusion. How much mass is it losing every second? (The speed of light is $c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$.)

**A** $4.2 \times 10^9\,\mathrm{kg\,s^{-1}}$ **B** $4.2 \times 10^{12}\,\mathrm{kg\,s^{-1}}$
**C** $3.4 \times 10^8\,\mathrm{kg\,s^{-1}}$ **D** $1.3 \times 10^7\,\mathrm{kg\,s^{-1}}$ [1]

14. A battery is replaced by two identical batteries connected in parallel. The combination can deliver

**A** the same maximum voltage and the same maximum current
**B** the same maximum voltage and a lower maximum current
**C** the same maximum voltage and a higher maximum current
**D** a higher maximum voltage and a lower maximum current [1]

15. The moon Titan has an angular diameter of 4.4 mrad as seen from the surface of Saturn. The Sun has an angular diameter of 9.3 mrad as seen from the surface of the Earth. Which of the following eclipses **cannot** be seen from the surface of Saturn?

**A** A lunar eclipse of Titan by Saturn.
**B** A partial solar eclipse due to Titan.
**C** A total solar eclipse due to Titan.
**D** An annular solar eclipse due to Titan [1]

16. A yacht on a lake drops its anchor overboard. What happens to the water level in the lake?

**A** It rises very slightly. **B** It stays exactly the same.
**C** It falls very slightly. **D** It's impossible to say. [1]

17. Estimate the change in temperature of the water in Fell Beck before and after it falls into the Gaping Gill pothole (depth 105 m). The specific heat capacity of water is $4.2\,\mathrm{kJ\,kg^{-1}\,K^{-1}}$.

**A** 4 °C **B** 0.25 °C
**C** 0.025 °C **D** $9.2 \times 10^{-4}$ °C [1]

$mgh = mc\Delta T$

$\Delta T = \frac{10 \times 105}{4.2 \times 10^3}$

18. A time-of-flight mass spectrometer can be used to determine the mass of charged molecules through the equation $t = d\sqrt{m/2qU}$, where $t$ is the time-of-flight, $d = 1.5\,\mathrm{m}$ is the length of the tube, $m$ is the mass of the molecule, $q$ is its charge, and $U = 16\,\mathrm{kV}$ is the accelerating voltage. Assuming that $q$ is a single elementary charge ($1.6 \times 10^{-19}\,\mathrm{C}$) what is the mass that corresponds with a time-of-flight of $30\,\mu\mathrm{s}$?

**A** $1.4 \times 10^{-12}\,\mathrm{kg}$ **B** $1.0 \times 10^{-23}\,\mathrm{kg}$
**C** $1.0 \times 10^{-24}\,\mathrm{kg}$ **D** $2.0 \times 10^{-24}\,\mathrm{kg}$ [1]

19. When an object moves at high velocity in a fluid the drag force on it is given by $F = K\,v^2\,A$, where $v$ is the object's velocity and $A$ its area. What sort of quantity is $K$?

$K = \frac{F}{v^2 A} = \frac{\mathrm{kg\,m\,s^{-2}}}{\mathrm{m^2 s^{-2}\,m^2}}$

$= \mathrm{kg\,m^{-3}}$

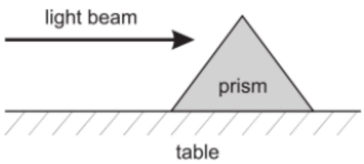
**A** A mass **B** An acceleration
**C** A length **D** A density [1]

20. A battery is connected across two identical resistors in series. If one of the resistors is instantaneously replaced by an uncharged capacitor, what happens to the current in the circuit?

Resistance decreases so initially I increases. Then capacitor charges up

**A** It rises
**B** It falls
**C** It initially rises, but then falls
**D** It initially falls, but then rises [1]

21. A triangular glass prism sits on a table point upwards, and a beam of coloured light is directed horizontally near the top of the prism. What happens to the light beam at the prism?




Light moving into a more dense medium
Refraction towards normal

**A** It is bent up **B** It is bent down
**C** It continues horizontally **D** It depends on the colour [1]

22. The moon orbits the earth at a distance of 400,000 km with a period of $2.4 \times 10^6$ s. What is its acceleration towards the earth?

**A** $2.7 \times 10^{-3}\,\mathrm{m\,s^{-2}}$ **B** $2.7 \times 10^{-6}\,\mathrm{m\,s^{-2}}$
**C** $10\,\mathrm{m\,s^{-2}}$ **D** $6.6 \times 10^{3}\,\mathrm{m\,s^{-2}}$ [1]

$$a = \omega^2 r = \left(\frac{2\pi}{T}\right)^2 r = \frac{4\pi^2 \times 4 \times 10^8}{(2.4\times10^6)^2} \approx \frac{16\times10}{2.4^2} \times 10^{-4}$$

## Written answers (20 marks)

23. A hot object will glow, emitting radiation with a total power approximately given by $P = A\,k\,T^4$ where $A$ is the surface area, $T$ is the temperature in Kelvin, and $k \approx 6 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$ is a constant (the Stefan–Boltzmann constant). Estimate the surface area of the filament of a 75 W incandescent light bulb working at 5000 K. [3]

$$75 = A \times 6 \times 10^{-8} \times (5000)^4$$

$$A = \frac{75 \times 10^{8}}{6 \times 625 \times 10^{12}}$$

$$= 2 \times 10^{-6}\,\mathrm{m}^2$$

24. Freight transport on Titan is mostly by ship, with three types of ship called pangs, quizzers, and roodles in common use. All three ships have the same shape and design but differ in size. The cargo capacity depends on the hold volume, while the number of crew required is proportional to the surface area of the deck. A quizzer and a roodle taken together have the same length as two pangs, and the crew of a quizzer is just sufficient to provide crew for two pangs and a roodle.

A fully loaded quizzer wishes to transfer all its cargo to smaller pangs and roodles, while minimising the number of crew required for the resultant fleet. How many pangs and roodles are needed?

[Hint: Note that for objects of any shape the surface area is proportional to the square of the object's size, and the volume is proportional to the cube of its size.] [6]

Let $p, q, r$ equal the lengths of pangs, quizzers and roodles respectively

$$q + r = 2p \Rightarrow q = 2p - r \quad (1)$$

$$q^2 = 2p^2 + r^2 \quad (2)$$

(1) in (2): $(2p - r)^2 = 2p^2 + r^2$

$$4p^2 - 4pr + r^2 = 2p^2 + r^2$$

$$2p^2 = 4pr$$

$$\underline{p = 2r}$$

In (1): $\underline{q} = 2(2r) - r = \underline{3r}$

Let $q^3 = ap^3 + br^3$, where $a$ = no. of pangs used, $b$ = no of roodles used

$$27r^3 = a8r^3 + br^3$$

$$\boxed{27 = 8a + b}$$

We need to minimise:

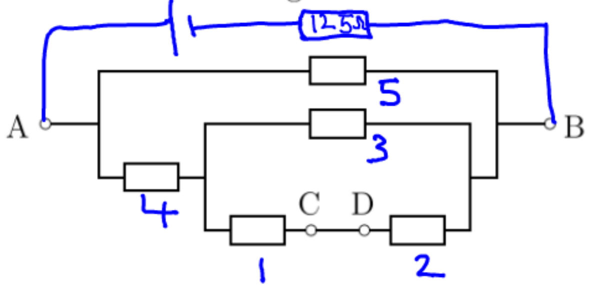
$ap^2 + br^2 = (4a + b)r^2$

Possible solutions:

| a | b | 4a+b |
|---|---|---|
| 1 | 19 | 23 |
| 2 | 11 | 19 |
| 3 | 3 | 15 |

$\therefore$ Pangs: 3 Roodles: 3

25. Five identical 1 k$\Omega$ resistors are arranged in a network as shown




What resistance would be measured between terminals A and B? If a 6 V battery with an internal resistance of 125 $\Omega$ is connected across terminals A and B, what current flows in the wire connecting terminals C and D? [6]

$$R_{1-3} = \frac{(1+1)(1)}{(1+1)+1} = \frac{2}{3}\,k\Omega \qquad R_{1-4} = \frac{2}{3}+1 = \frac{5}{3}\,k\Omega$$

$$R_{1-5} = R_{AB} = \frac{(5/3)\times 1}{(5/3)+1} = \frac{5/3}{8/3} = \frac{5}{8}\,k\Omega = 625\,\Omega$$

When battery is connected, $R_T = 625 + 125 = 750\,\Omega$

$$V_{AB} = V_T \times \frac{R_{AB}}{R_T} = 6 \times \frac{625}{750} = 5\,V = V_{1-4}$$

$$V_{1-3} = V_{1-4} \times \frac{R_{1-3}}{R_{1-4}} = 5 \times \frac{2/3}{5/3} = 2\,V = V_{1-2}$$

$$I_{CD} = \frac{V_{1-2}}{R_{1-2}} = \frac{2}{(1+1)\times 10^3} = 1\,mA$$

26. An electron gun in a cathode ray tube accelerates an electron with mass $m$ and charge $-e$ across a potential difference of 50 V and directs it horizontally towards a fluorescent screen 0.4 m away. How far does the electron fall during its journey to the screen? Take $m \approx 10^{-30}$ kg and $e \approx 1.6 \times 10^{-19}$ C.

[5]

$\frac{1}{2} m v^2 = eV$

$v^2 = \frac{2 \times 1.6 \times 10^{-19} \times 50}{10^{-30}} = 16 \times 10^{12}$

$v = 4 \times 10^{6}\ ms^{-1}$

Time to screen, $t = \frac{0.4}{4 \times 10^{6}} = 1 \times 10^{-7} s$

$\downarrow$ +ve $s = ?, u = 0, v = x, a = 10 ms^{-2}, t = 1 \times 10^{-7} s$

$s = ut + \frac{1}{2} a t^2$

$= \frac{1}{2} \times 10 \times (1 \times 10^{-7})^2$

$= 5 \times 10^{-14} m$

## Long question (20 marks)

27. In this question you will use a simple model to estimate how the energy used by a car depends on its design and how it is driven. Begin by neglecting air and ground resistance, and assume that the car travels at constant velocity between regular equally spaced stops.

(a) A stationary car of mass $m$ is rapidly accelerated to a velocity $v$, driven for a distance $s$, and is rapidly brought to a halt by its brakes. Calculate the energy dispersed by the brakes. [1]

$$E = \frac{1}{2}mv^2$$

(b) Assuming the car restarts immediately, calculate the time between subsequent stops and hence the average power dissipated. [2]

$$t = \frac{s}{v} \qquad P = \frac{E}{t} = \frac{1}{2}mv^2 \times \frac{v}{s}$$

$$= \frac{mv^3}{2s}$$

(c) Hence or otherwise calculate the energy used in travelling a total distance $d$. [2]

$$E = Pt = P \times \frac{d}{v} = \frac{mv^2 d}{2s}$$

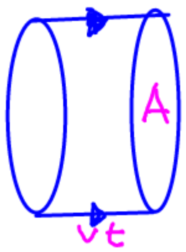
(d) Taking $m = 1000\,\mathrm{kg}$, $v = 10\,\mathrm{m\,s^{-1}}$ and $s = 100\,\mathrm{m}$ calculate the energy used in travelling 1 km. What would be the effect of doubling the speed to $20\,\mathrm{m\,s^{-1}}$? [3]

$$E_{10} = \frac{1000 \times 10^2 \times 1000}{2 \times 100} = 5 \times 10^5\,\mathrm{J}$$

$$E_{20} = 4E_{10} = 2 \times 10^6\,\mathrm{J}$$

Now consider the effect of air resistance. This can be estimated by assuming that the car has to accelerate all the air it travels through to the same average velocity as itself. (You may ignore the rapid random motion of individual air molecules in this calculation.)

(e) Treating the car as a disc with cross sectional area $A$ travelling at velocity $v$ calculate the volume of air swept out in a time $t$. If the air has density $D$ calculate the kinetic energy transferred to the air in this time, and hence the power needed to overcome the air resistance. [5]




$$V = Avt$$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}VDv^2 = \frac{1}{2}ADtv^3$$

$$P = \frac{E}{t} = \frac{1}{2}ADv^3$$

(f) Taking $A = 1\,\mathrm{m}^2$, $v = 10\,\mathrm{m\,s^{-1}}$ and $D = 1\,\mathrm{kg\,m^{-3}}$ calculate the energy used in travelling 1 km. [2]

$$E = \left(\frac{1}{2} \times 1 \times 1 \times \frac{1000}{10} \times 10^3\right) + 5 \times 10^5 = 5.5 \times 10^5\,\mathrm{J}$$

(g) Using the data above calculate the distance between stops at which the energy dissipated in the brakes is the same as that lost to air resistance. [2]

$$\frac{1}{2}mv^2 = \frac{1}{2}ADtv^3 = \frac{1}{2}ADv^2s$$

$$s = \frac{m}{AD} = \frac{1000}{1 \times 1} = 1000\,\mathrm{m}$$

(h) Comment on the significance of these calculations for the design of cars optimised for driving in cities and cars optimised for driving on highways. [3]

Highways: s is large, more energy is lost to air resistance. Aerodynamics are important: Smaller A desirable.

Cities: s is small, more energy is lost to braking. Lighter designs are important! Smaller m.