Oxford Physics Aptitude Test (PAT) 2008 Solutions

Maths

1) $1+2+3+\ldots+99+100 = \frac{n}{2}(n+1) = \frac{100}{2}(101) = 5050$

2) $(0.25)^{-1/2} = \left(\frac{1}{4}\right)^{-1/2} = (4)^{1/2} = 2$

$(0.09)^{3/2} = (0.3)^3 = 0.027$

3) $(1+x)^{m+1}(1-2x)^m \approx \left[1+(m+1)x + \frac{(m+1)(m)x^2}{2}\right]\left[1-2mx+\frac{4m(m-1)x^2}{2}\right]$

$= 1 + (m+1-2m)x + \left(\cancel{2m^2} - 2m - \cancel{2m^2} - 2m + \frac{m^2}{2} + \frac{m}{2}\right)x^2$

$= 1 + (1-m)x + \left(\frac{m^2-7m}{2}\right)x^2$

4) $\frac{x^2+2}{1-x^2} < 3$

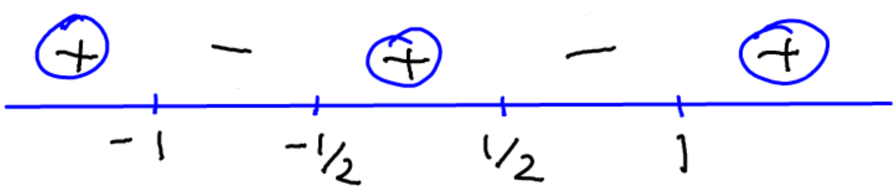
$(x^2+2)(1-x^2) < 3(1-x^2)^2$

$2 - x^2 - x^4 < 3 - 6x^2 + 3x^4$

$0 < 4x^4 - 5x^2 + 1$

$0 < (4x^2-1)(x^2-1)$

$0 < (2x-1)(2x+1)(x-1)(x+1)$



$\{x < -1\} \cup \{-1/2 < x < 1/2\} \cup \{x > 1\}$

5) i) $\log_2 9 = \dfrac{\log_9 9}{\log_9 2} = \dfrac{1}{x}$

ii) $\log_8 3 = \dfrac{\log_9 3}{\log_9 8} = \dfrac{1/2}{\log_9 (2)^3} = \dfrac{1}{2} \cdot \dfrac{1}{3\log_9 2} = \dfrac{1}{6x}$

6) $x^2 - 1 = x - x^2$

$2x^2 - x - 1 = 0$

$(2x+1)(x-1) = 0$

$x = -\dfrac{1}{2}$ or 1

7) $y = x + \dfrac{x^2}{2} + \dfrac{x^3}{3} + \dots$

$y = ax$

$\dfrac{dy}{dx} = 1 + x + x^2 + \dots$

$m = a$

when $x = 0$, $\dfrac{dy}{dx} = 1 \Rightarrow a = 1$

When $x = \dfrac{1}{4}$, $\dfrac{dy}{dx} = 1 + \dfrac{1}{4} + \left(\dfrac{1}{4}\right)^2 + \dots$ G.P. with $r = \dfrac{1}{4}$,

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{1 - 1/4} = \dfrac{4}{3} \Rightarrow a = \dfrac{4}{3}$

8) Centre = mid-point $= \left(\dfrac{5-3}{2}, \dfrac{8+2}{2}\right) = (1, 5)$

Radius, $r = \sqrt{(5-1)^2 + (2-5)^2}$

$r^2 = 25$

$\Rightarrow (x-1)^2 + (y-5)^2 = 25$

9) This is like having a 9-faced die with 2, 3, 4, three 5s and three 6s.

$$P(2) = \frac{1}{9}, \; P(3) = \frac{1}{9}, \; P(4) = \frac{1}{9}, \; P(5) = \frac{3}{9}, \; P(6) = \frac{3}{9}$$

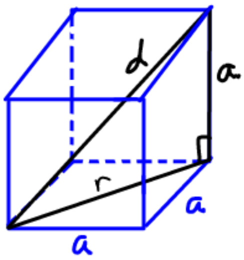
i) $\frac{1}{9}$

ii) $P(S \geq 10) = P(4,6) + P(5,5) + P(5,6) + P(6,4) + P(6,5) + P(6,6)$

$$= \left(\frac{1}{9} \times \frac{3}{9}\right) + \left(\frac{3}{9} \times \frac{3}{9}\right) + \left(\frac{3}{9} \times \frac{3}{9}\right) + \left(\frac{3}{9} \times \frac{1}{9}\right) + \left(\frac{3}{9} \times \frac{3}{9}\right) + \left(\frac{3}{9} \times \frac{3}{9}\right)$$

$$= \frac{1}{81}(3 + 9 + 9 + 3 + 9 + 9) = \frac{42}{81} = \frac{14}{27}$$

10)



$$r^2 = a^2 + a^2 = 2a^2$$

$$d^2 = a^2 + r^2 = 3a^2$$

$$d = \sqrt{3}\, a$$

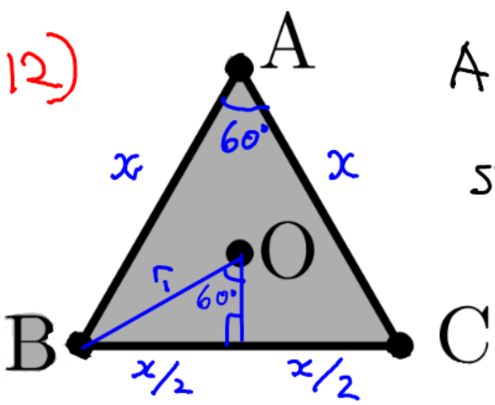
11) i) $\int_{-1}^{1} (x + x^3 + x^5 + x^7)\, dx = \left[\frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \frac{x^8}{8}\right]_{-1}^{1}$

$$= 0$$

ii) $\int_0^1 \frac{x^9 + x^{99}}{11}\, dx = \frac{1}{11}\left[\frac{x^{10}}{10} + \frac{x^{100}}{100}\right]_0^1 = \frac{0 \cdot 11}{11}$

$$= 0.01$$

12)

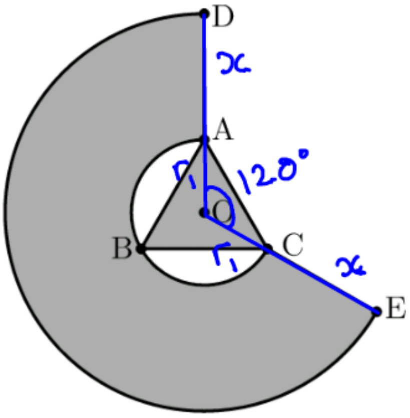


$$A_{ABC} = \frac{1}{2} x^2 \sin 60 = \frac{\sqrt{3}}{4} x^2$$

$$\sin 60 = \frac{x}{2} \cdot \frac{1}{r}$$

$$r_1 = \frac{x}{\sqrt{3}}$$

Area of small circle, $A_1 = \pi r_1^2 = \frac{\pi x^2}{3}$



Area of big circle,

$$A_2 = \pi(x+r_1)^2 = \pi x^2 + \frac{x^2\pi}{3} + \frac{2x^2\pi}{\sqrt{3}}$$

$$= 2\pi x^2\left(\frac{2+\sqrt{3}}{3}\right)$$

$$A_{ADEC} = \frac{240}{360}(A_2 - A_1)$$

$$= \frac{2}{3}\left[2\pi x^2\left(\frac{2+\sqrt{3}}{3}\right) - \frac{\pi x^2}{3}\right]$$

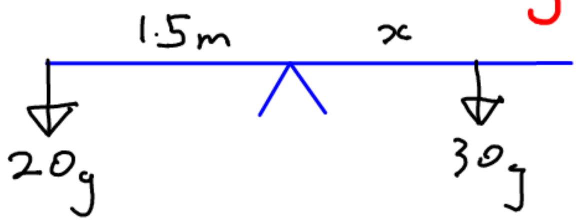
$$= \frac{2\pi x^2}{9}\left(4+2\sqrt{3}-1\right) = \frac{2\pi x^2}{9}\left(3+2\sqrt{3}\right)$$

$$\frac{A_{AEDC}}{A_{ABC}} = \frac{2\pi x^2}{9}\left(3+2\sqrt{3}\right) \div \frac{\sqrt{3}}{4}x^2$$

$$= \frac{8\pi\left(3+2\sqrt{3}\right)}{9\sqrt{3}} = \frac{8\pi\left(\sqrt{3}+2\right)}{9}$$

Physics

13)



$$20g(1.5) = 30g\,x$$

$$x = 1\text{ m}$$

∴ She must sit $1.5 + 1 = 2.5$ m away

Ans: D

14) Energy released comes from the decrease in mass. Ans: B

15) $M = 2\times10^{30}\times 250\times10^{9}\times 400\times10^{9}\times 21$

$$= 20\times10^{4}\times 21\times10^{48} = 4.2\times10^{54}\text{kg}$$

Ans: D

16) Moon between the Earth and the Sun. Ans: A

17) $\rho = \frac{m}{V} \Rightarrow \rho$ stays constant.

$p \propto T \Rightarrow$ pressure increases

Ans: C

18) $GPE = mgh = 60 \times 10 \times 4 = 2400\,J$ Ans: B

19) $E = IVt$

$0.1 \times (1 \times 10^3) \times 0.25^2 \times t\,(\text{hours}) = 0.7 \times 3.6 \times 1\,(\text{hour})$

Ans: C $\quad t = \dfrac{3.6 \times 0.7 \times 16}{100} = 0.40$ hours

20) $P = \dfrac{V^2}{R}$ max P when R is min $\Rightarrow$ noon

Ans: C

21) $m_w c \Delta T = \frac{1}{2} m_b v^2$

$2^3 \times 1000 \times 4.2 \times 10^3 \Delta T = 0.5 \times 0.01 \times 400^2$

Ans: A $\quad \Delta T = \dfrac{800}{8 \times 4.2 \times 10^6} = \dfrac{1}{4.2 \times 10^{-4}} = 2.4 \times 10^{-5}$

22) Both sides $\times 10^{-3} \Rightarrow$ mJ Ans: A

23) Given: $q = CV$, $C = \dfrac{\rho A}{d}$

$$W = \frac{q^2}{2C} = \frac{C^2V^2}{2C} = \frac{CV^2}{2} = \frac{\rho A V^2}{2d}$$

Given $V_{max} = Bd$,

$$E_{max} = \frac{\rho A V_{max}^2}{2d} = \frac{\rho A B^2 d^2}{2d} = \frac{\rho A B^2 d}{2} = \frac{\rho B^2 V}{2}$$

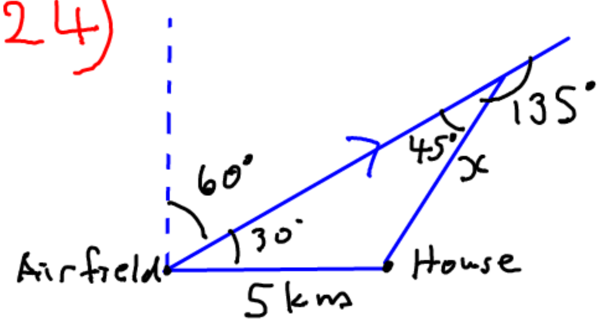
Using $m = DV$, $\quad E_{max} = \dfrac{m \rho B^2}{2D}$

For $p = 2\times10^{-11}\ Fm^{-1}$, $B = 2\times10^{7}\ Vm^{-1}$, $D = 1000\ kg\,m^{-3}$, $m = 1kg$;

$$E_{max} = \frac{1\times \cancel{2}\times10^{-11}\times(2\times10^{7})^2}{\cancel{2}\times 1000} = 4\ J$$

$1kW = 1000\ J/s \Rightarrow 4J$ capacity is too small to have a useful effect

24)



$$\frac{x}{\sin 30} = \frac{5}{\sin 45}$$

$$x = \frac{5\sin 30}{\sin 45} = \frac{5}{2}\cdot\sqrt{2}$$

$$= 2.5\times1.4 = 3.5\ km$$

25) a $\Rightarrow c + r = 2s \Rightarrow r = 2s - c$ (1)

b $\Rightarrow c^2 + s^2 = r^2$ (2)

c $\Rightarrow 2c + s = 1 \Rightarrow s = 1 - 2c$ (3)

(3) in (1): $r = 2(1-2c) - c = 2 - 5c$

In (2): $c^2 + (1-2c)^2 = (2-5c)^2$

$$c^2 + 1 - 4c + 4c^2 = 4 - 20c + 25c^2$$

$$0 = 20c^2 - 16c + 3$$

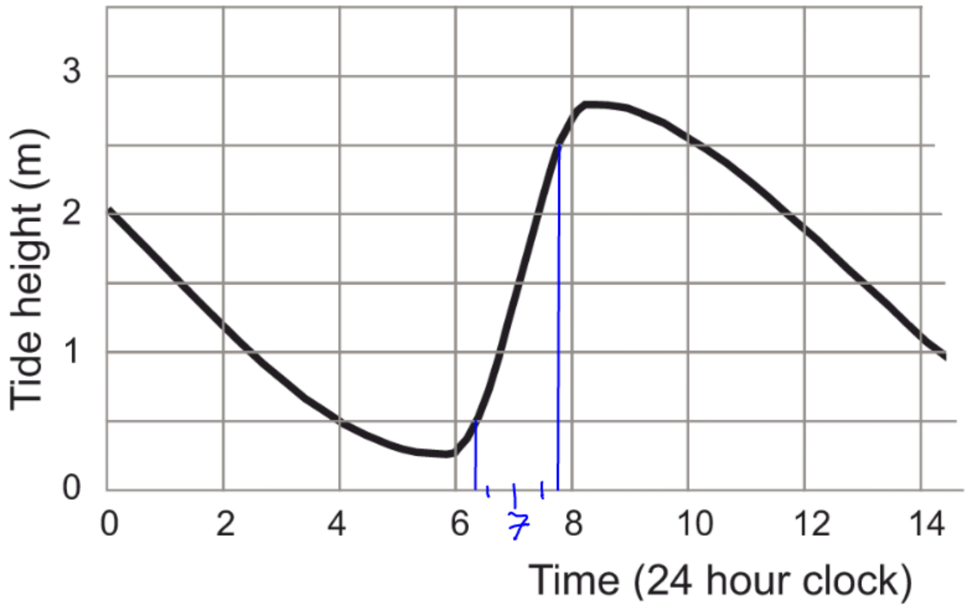
$$= (10c - 3)(2c - 1)$$

$$c = \frac{3}{10} \text{ or } \cancel{\frac{1}{2}}$$

From (3), c cannot be $\frac{1}{2}$, as this would give $s = 0$

$\therefore\ c = 0.3m,\ s = 0.4m,\ r = 0.5m$

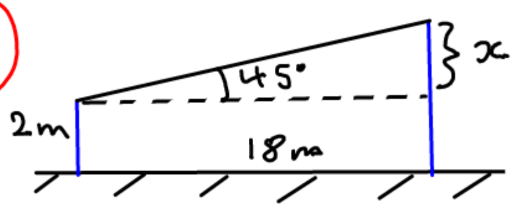
26) Most rapid change is when the gradient is the steepest $\Rightarrow$ 7am



$$m = \frac{\Delta h}{\Delta t} = \frac{250 - 50}{105 - 15}$$

$$= 2.2 \text{ cm/min}$$

27)



$$\tan 45 = \frac{x}{18}$$

$$x = 18 \text{ m}$$

a) $s = 20\text{m}$, $u = 0$, $v = x$, $a = 10\text{ms}^{-2}$, $t = ?$

↓+ve $s = ut + \frac{1}{2}at^2$

$$20 = 5t^2$$

$$t = 2\text{s}$$

b) $v^2 = u^2 + 2as$

$$v = \sqrt{2 \times 10 \times 20} = 20\text{ms}^{-1}$$

c) $\frac{1}{2}mv^2 = Fd$

$$F = \frac{0.5 \times 0.02 \times 20^2}{1 \times 10^{-3}} = 4\text{kN}$$

d) $WD = Fd = 4 \times 10^3 \times 1 \times 10^{-3} = 4\text{J}$

$GPE = mgh = 0.02 \times 10 \times 20 = 4\text{J} = WD$

e) $d = 5\,cm \Rightarrow F = \dfrac{0.5 \times 0.02 \times 20^2}{0.05} = 80\,N$

$Ft = mv - mu$

$t = \dfrac{0.02(0-20)}{-80} = \dfrac{1}{200} = 5\,ms$

f) $E_{min} = mgh = (100 + 0.02) \times 10 \times 20 = 2.0 \times 10^4\,J$

g) $E_{min} \times \text{efficiency} = mc\Delta T$

$\text{efficiency} = \dfrac{0.02 \times 4 \times 10^3 \times 80}{2 \times 10^4} = 32 \times 10^{-2}$

$= 32\%$