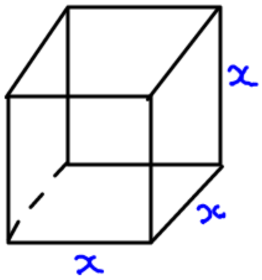
Oxford Physics Aptitude Test (PAT) 2007 Solutions

Physics

1)



$$R = \frac{\rho l}{A} = \frac{\rho x}{x^2} = \frac{\rho}{x}$$

$$R \propto \frac{1}{x}$$

Ans: C

2) Ans: D

3)

$V = IR$

$9 = 100I$

$I = 0.09\,A$

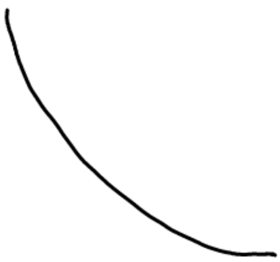
$= 0.09\,C\,s^{-1}$

$$n = \frac{0.09}{1.6\times10^{-19}}$$

$$= 5.625\times10^{17}$$

Ans: A

4)

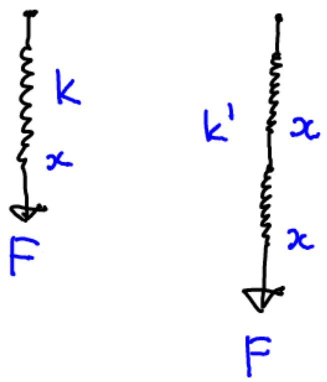


Going down $\Rightarrow$ v increases

less steep $\Rightarrow$ a decreases

Ans: C

5)



$F = kx$; $F = k'(2x)$

$\Rightarrow k' = k/2$

Ans: A

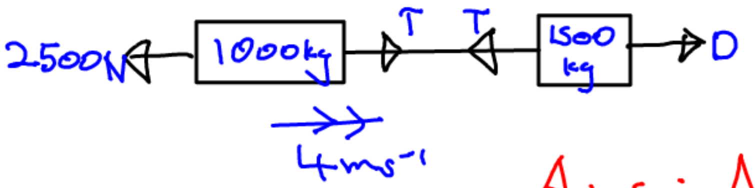
6) $P = \frac{V^2}{R} \Rightarrow P = \frac{V^2}{R_1} + \frac{V^2}{R_2} = V^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

Ans: A

7) Ans: B

8) $p = \frac{F}{A} = \frac{0.125 \times 10}{\pi \times (1\times10^{-3})^2} = 4.0 \times 10^5\ Pa$ Ans: C

9)



$F = ma$

$T - 2500 = 1000(4)$

$T = 6500N$

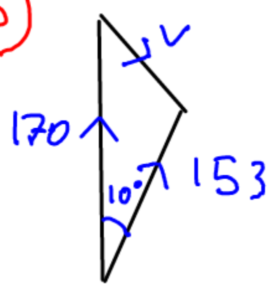
Ans: A

10)

a) $t = \frac{d}{v} = \frac{300}{170} = 1.76h = 106mins$

$\therefore$ Time of arrival: 10:46am

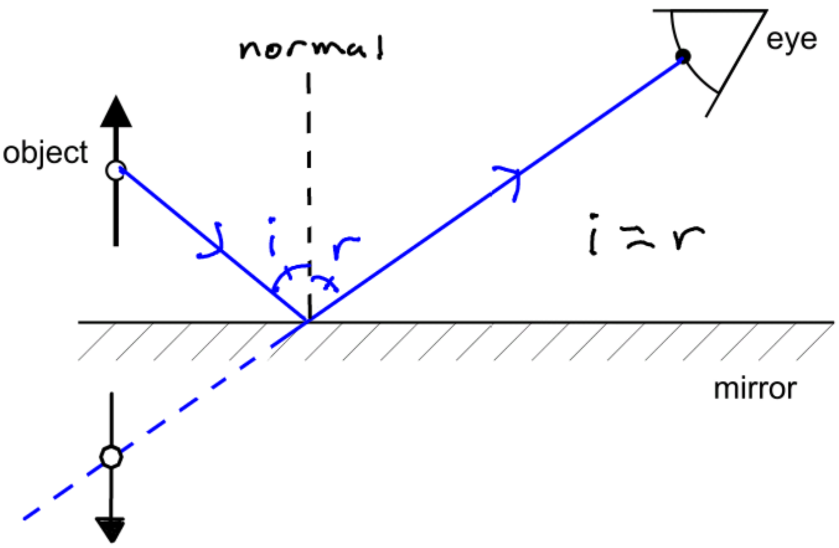
b)



$v^2 = 170^2 + 153^2 - 2(170)153\cos 10$

$v = 32.9 kmh^{-1}$

11)



$i = r$

12) a $\Rightarrow 5m = 7v$ ①

b $\Rightarrow 3l + m = 8v$ ②

c $\Rightarrow 5l + 5m + 2v = 1$ ③

① in ②: $3l + \frac{7v}{5} = 8v \Rightarrow 15l = 33v$ ④

(8) and (9) in (3): $5\left(\frac{33}{15}v\right) + 7v + 2v = 1$

$$20v = 1$$

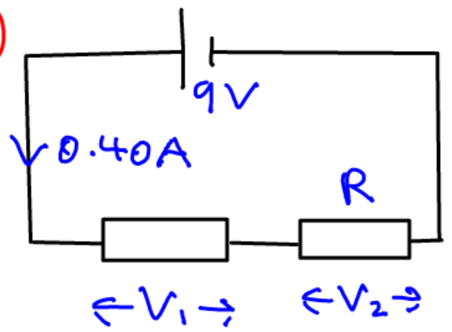
$$v = 0.05\,m$$

d $\Rightarrow$ speed of lavendar, $V_l = 0.005\,ms^{-1}$

e $\Rightarrow$ $V_m = 0.01\,ms^{-1}$

$$\therefore\ t = \frac{d}{v} = \frac{2\pi \times 1180 \times 10^3}{0.01} = 7.4 \times 10^8 s$$

13)



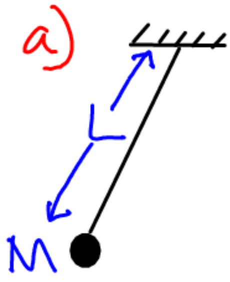
$$I = 0.05 V_1^3$$

$$\frac{0.4}{0.05} = V_1^3$$

$$V_1 = 2V \Rightarrow V_2 = 7V$$

$$R = \frac{V_2}{I} = \frac{7}{0.4} = 17.5\,\Omega$$

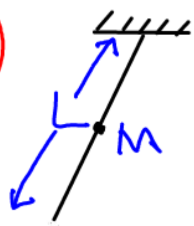
14) a)



$L_{cm} = L$; $I = ML^2$

$$\therefore P = 2\pi\sqrt{\frac{ML^2}{gML}} = 2\pi\sqrt{\frac{L}{g}}$$

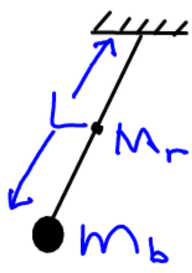
b)



$L_{cm} = \frac{L}{2}$; $I = \frac{1}{3}ML^2$

$$\therefore P = 2\pi\sqrt{\frac{\frac{1}{3}mL^2}{gML/2}} = 2\pi\sqrt{\frac{2L}{3g}}$$

c)



$$L_{cm} = \frac{LM_b + \frac{L}{2}M_r}{M_b + M_r}$$ (taking moments)

$$I = M_bL^2 + \frac{1}{3}M_rL^2$$

$$\therefore P = 2\pi\sqrt{\frac{\frac{L^{\cancel{2}}}{3}(3M_b+M_r)}{g\cancel{(M_b+M_r)}\frac{\cancel{L}(2M_b+M_r)}{2\cancel{(M_b+M_r)}}}} = 2\pi\sqrt{\frac{2L(3M_b+M_r)}{3g(2M_b+M_r)}}$$

For an ideal pendulum, $M_r \to 0$

$$\therefore P \to 2\pi\sqrt{\frac{2L(3M_b)}{3g(2M_b)}} = 2\pi\sqrt{\frac{L}{g}}$$

For a rod pendulum, $M_b \to 0$

$$\therefore P \to 2\pi\sqrt{\frac{2LM_r}{3gM_r}} = 2\pi\sqrt{\frac{2L}{3g}}$$

d) $P = 2\pi\sqrt{\frac{L}{g}} \Rightarrow P' = 2\pi\sqrt{\frac{L+L\alpha\delta T}{g}}$

$$\Delta P = P' - P = 2\pi\sqrt{\frac{L+L\alpha\delta T}{g}} - 2\pi\sqrt{\frac{L}{g}}$$

$$= 2\pi\sqrt{\frac{L}{g}}\left(\sqrt{1+\alpha\delta T} - 1\right) = P\left(\sqrt{1+\alpha\delta T} - 1\right)$$

For an accuracy of 1s in 24h,

$$\frac{\Delta P}{P} = \frac{1}{24\times 60\times 60} = \frac{P\left(\sqrt{1+\alpha\delta T}-1\right)}{P} = \sqrt{1+\alpha\delta T} - 1$$

For $\alpha = 19\times10^{-6}$,

$$\left[\left(\frac{1}{86400}+1\right)^2 - 1\right] \div 19\times10^{-6} = \delta T = 1.22\,K$$

e) For $\alpha = 1.2\times10^{-6}$, $\delta T = 19.3\,K$

Maths

1) $6667^2 - 3333^2 = (6667 + 3333)(6667 - 3333)$

$= 10000 \times 3334 = 33340000$

2) $y = x^4$

$y' = 4x^3$

At $(-2, 16)$, $y' = -32$

$\Rightarrow y - 16 = -32(x+2)$

$y = -32x - 48$

3) $\dfrac{2\log 125}{3\log 25} = \dfrac{2\log 5^3}{3\log 5^2} = \dfrac{6\log 5}{6\log 5} = 1$

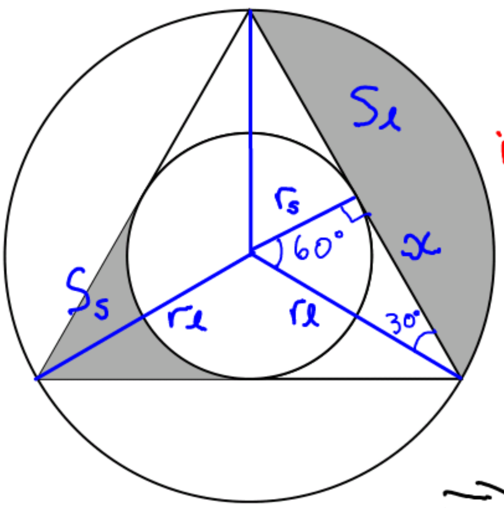
4) i) $(1,5), (2,4), (3,3), (4,2), (5,1) \Rightarrow \dfrac{5}{36}$

ii) $(5,6), (6,5) \Rightarrow \dfrac{2}{36} = \dfrac{1}{18}$

5) $(2+x)^5 = 2^5\left(1+\frac{x}{2}\right)^5 \simeq 2^5\left[1 + \frac{5x}{2} + \frac{5(\cancel{4})}{2}\frac{x^2}{\cancel{4}} + \frac{5(\cancel{4})(\cancel{3})x^3}{{}_2\cancel{6}\;{}_2\cancel{8}}\right]$

$= 32 + 80x + 80x^2 + 40x^3$

6)



$\sin 30 = \dfrac{r_s}{r_\ell} = \dfrac{1}{2} \Rightarrow r_\ell = 2r_s$ *

i) $\dfrac{A_\ell}{A_s} = \dfrac{\pi r_\ell^2}{\pi r_s^2} = \left(\dfrac{r_\ell}{r_s}\right)^2 = 2^2 = 4$

ii) Height of triangle $= r_\ell + r_s$

$x = r_\ell \cos 30 = \dfrac{\sqrt{3}\, r_\ell}{2}$

$\Rightarrow$ Area of triangle $= \frac{1}{2} \times \sqrt{3}\, r_\ell (r_\ell + r_s) = K$

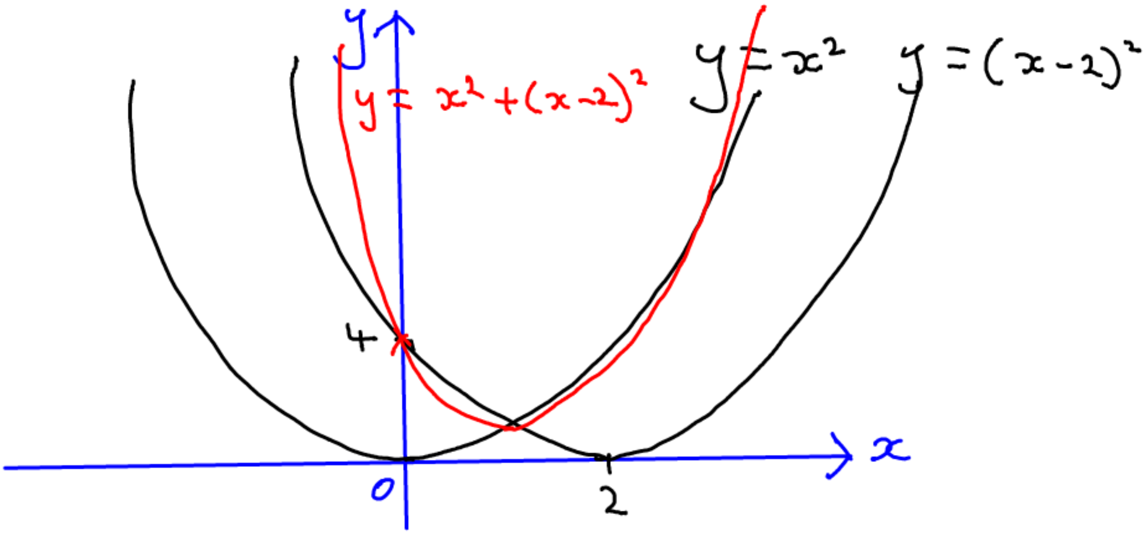
$S_s = \dfrac{K - A_s}{3}$; $S_\ell = \dfrac{A_\ell - K}{3}$

$\Rightarrow \dfrac{S_\ell}{S_s} = \dfrac{A_\ell - K}{K - A_s} = \dfrac{\pi r_\ell^2 - \frac{\sqrt{3}}{2} r_\ell^2 - \frac{\sqrt{3}}{2} r_\ell r_s}{\frac{\sqrt{3}}{2} r_\ell^2 + \frac{\sqrt{3}}{2} r_\ell r_s - \pi r_s^2}$

using * $= \dfrac{2\pi(4\cancel{r_s^2}) - \sqrt{3}(4\cancel{r_s^2}) - \sqrt{3}(2\cancel{r_s^2})}{\sqrt{3}(4\cancel{r_s^2}) + \sqrt{3}(2\cancel{r_s^2}) - 2\pi\cancel{r_s^2}}$

$= \dfrac{8\pi - 6\sqrt{3}}{6\sqrt{3} - 2\pi} = \dfrac{4\pi - 3\sqrt{3}}{3\sqrt{3} - \pi}$

7)



8) $\tan\theta = 2\sin\theta, \quad 0 \leq \theta \leq 2\pi$

$\sin\theta = 2\sin\theta\cos\theta$

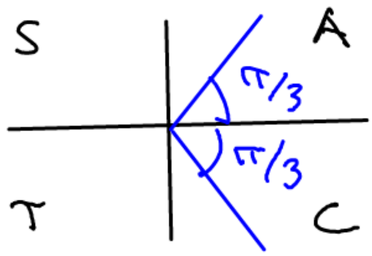
$0 = \sin\theta(2\cos\theta - 1)$

$\sin\theta = 0$ or $2\cos\theta - 1 = 0$

$\theta = 0, \pi, 2\pi$

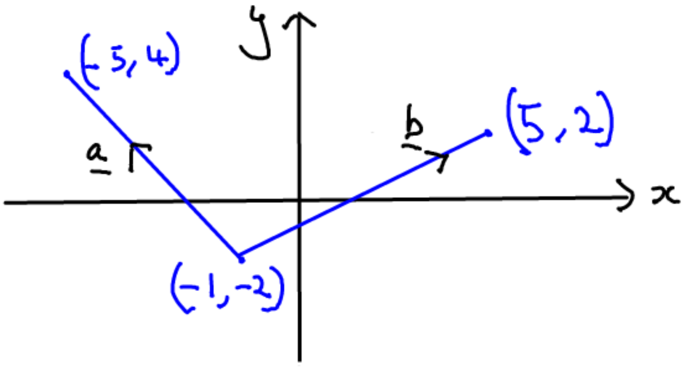
$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$



$\theta = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$

9)



Let $\underline{a} = \begin{pmatrix}-5\\4\end{pmatrix} - \begin{pmatrix}-1\\-2\end{pmatrix} = \begin{pmatrix}-4\\6\end{pmatrix}$

$\underline{b} = \begin{pmatrix}5\\2\end{pmatrix} - \begin{pmatrix}-1\\-2\end{pmatrix} = \begin{pmatrix}6\\4\end{pmatrix}$

$|\underline{a}| = \sqrt{(-4)^2 + 6^2} = \sqrt{52}$

$|\underline{b}| = \sqrt{6^2 + 4^2} = \sqrt{52} = |\underline{a}|$

$\underline{a} \cdot \underline{b} = -24 + 24 = 0$ $\therefore$ Two sides are perpendicular and have the same length

Fourth corner: $\begin{pmatrix}-5\\4\end{pmatrix} + \begin{pmatrix}6\\4\end{pmatrix} = \begin{pmatrix}1\\8\end{pmatrix}$ Area $= (\sqrt{52})^2 = 52$

10) $\int_1^9 \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx = \int_1^9 \left(x^{1/2} + x^{-1/2}\right) dx = \left[\frac{2}{3}x^{3/2} + 2x^{1/2}\right]_1^9$

$$= (18 + 6) - \left(\frac{2}{3} + 2\right) = \frac{64}{3}$$

11) $ar = a + d \Rightarrow d = a(r-1)$

$ar^2 = 2(a + 2d) \Rightarrow d = \frac{a(r^2 - 2)}{4}$

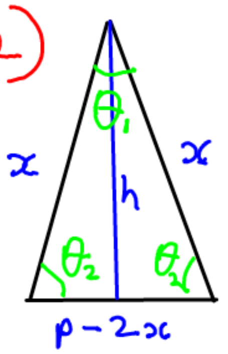
$$a(r-1) = \frac{a(r^2-2)}{4}$$

$$4r - 4 = r^2 - 2$$

$$r^2 - 4r + 2 = 0$$

$$r = \frac{4 \pm \sqrt{16-8}}{2} = 2 \pm \sqrt{2}$$

12)



$$h^2 + \left(\frac{p-2x}{2}\right)^2 = x^2$$

$$h = \left[x^2 - \frac{1}{4}(p-2x)^2\right]^{1/2}$$

$$A = \frac{1}{2}(p-2x)h = \frac{1}{2}(p-2x)\left[x^2 - \frac{1}{4}(p-2x)^2\right]^{1/2}$$

$$= \frac{1}{2}(p-2x)\left[x^2 - \frac{p^2}{4} - x^2 + px\right]^{1/2}$$

$$= \frac{1}{2}(p-2x)\left[\frac{1}{4}(4px - p^2)\right]^{1/2}$$

$$= \frac{1}{4}(p-2x)(4px - p^2)^{1/2}$$

$$\frac{dA}{dx} = \frac{1}{\cancel{4}}(p-2x)\cancel{\frac{1}{2}}(4px-p^2)^{-1/2} \cdot \cancel{4}p - \cancel{\frac{1}{2}}(4px - p^2)^{1/2} = 0$$

$$\cancel{p}(p-2x) = 4\cancel{p}x - p^{\cancel{2}} \Rightarrow x = \frac{p}{3}$$

$$\cos\theta_2 = \frac{\frac{1}{2}\left(p - \frac{2p}{3}\right)}{p/3}$$

$$\theta_2 = 60^\circ \qquad \theta_1 = 180 - (2 \times 60) = 60^\circ$$