Natural Sciences Admissions Assessment 2016

Specimen Paper Section 1: explained answers

MATHEMATICS

1

The answer is option **A**.

We can write:

area of shape $=$ area of square $-$ area of semi-circle.

If we let the length of one side of the square be $2x$, so that the radius of the semi-circle is x, then this becomes:

$$100 = 4x^2 - \frac{1}{2}\pi x^2 = x^2\left(4 - \frac{\pi}{2}\right)$$

Rearranging:

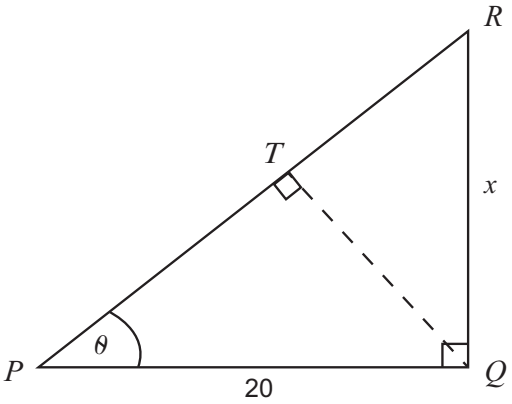
$$x = \sqrt{\frac{200}{8-\pi}} = 10\sqrt{\frac{2}{8-\pi}}$$

So the length of one side of the square is $2x = 20\sqrt{\frac{2}{8-\pi}}$

2

The answer is option **E**.

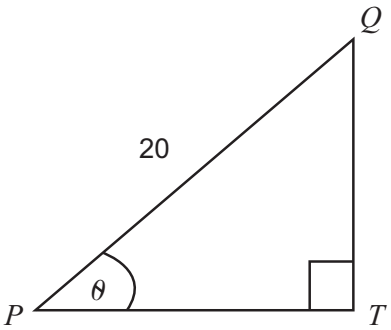
Let $RQ = x$

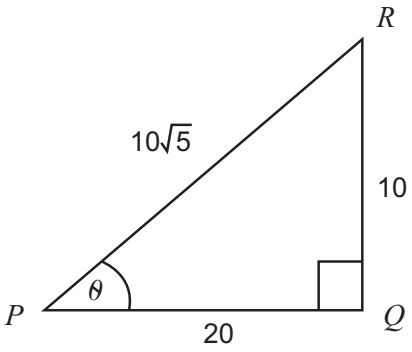


We know $RQ : PQ$ is $1 : 2$ so we can deduce $x = 10$

By Pythagoras' Theorem, we have $PR = 10\sqrt{5}$

We notice triangles PQT and PRQ are similar:



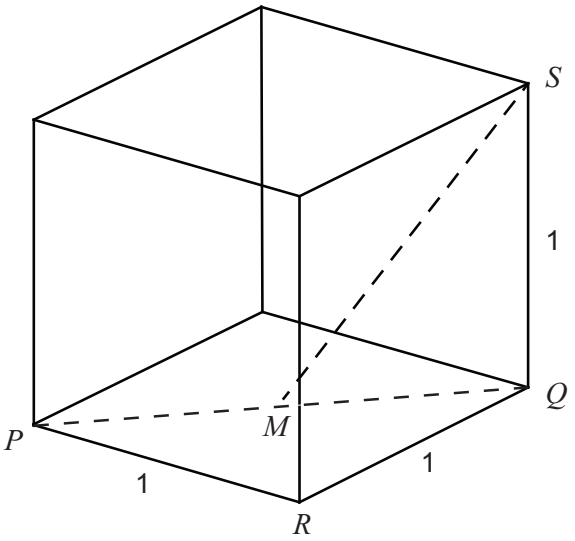


So we can write: $\frac{QT}{10} = \frac{20}{10\sqrt{5}}$

Giving: $QT = \frac{20}{\sqrt{5}} = 4\sqrt{5}$

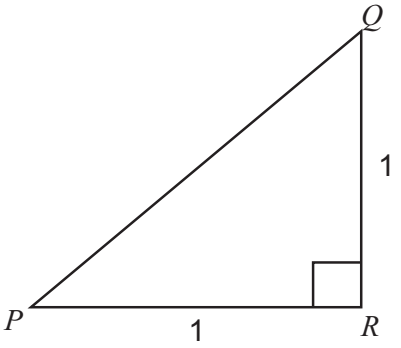
3

The answer is option **A**.



[diagram not to scale]

In triangle PQR, the angle at R is a right angle so we can use Pythagoras' Theorem to find PQ:

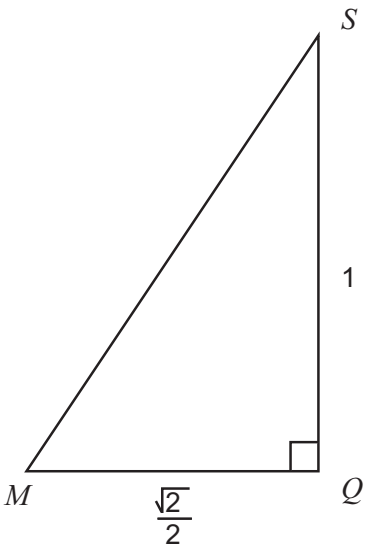


$PQ^2 = 1^2 + 1^2 = 2$

Giving $PQ = \sqrt{2}$

QM is half of PQ so $QM = \frac{\sqrt{2}}{2}$

Now looking at triangle MQS:



The angle at Q is a right angle so we can use Pythagoras' Theorem:

$$MS^2 = \left(\frac{\sqrt{2}}{2}\right)^2 + 1^2 = \frac{2}{4} + 1 = \frac{3}{2}$$

Giving $MS = \sqrt{\frac{3}{2}}$

4

The answer is option **D**.

When the minute hand is pointing to the 9 [indicating 45 minutes past the hour], the hour hand will have moved three-quarters of the angle between the 9 and the 10 on the clock dial. The angle between the 9 and the 10 is 30 degrees so the hour hand will have moved three-quarters of 30 degrees, or 22.5 degrees. The angle between the hands is, therefore, 22.5°.

5

The answer is option **B**.

To answer this we shall use:

area of triangle $= \frac{1}{2} \times$ base $\times$ height

This gives:

$$\text{area} = \frac{1}{2}\left(4+\sqrt{2}\right)\left(2-\sqrt{2}\right) = \frac{1}{2}\left(8-4\sqrt{2}+2\sqrt{2}-\sqrt{2}\sqrt{2}\right) = \frac{1}{2}\left(6-2\sqrt{2}\right) = 3-\sqrt{2}$$

6

The answer is option **D**.

As the cylinder has the same internal diameter as the sphere, it has the same radius, r. As the length of the cylinder is the same as the diameter of the sphere, it has length $2r$. So we can write:

Volume of the sphere $= \frac{4}{3}\pi r^3$

Volume of cylinder $=$ base area $\times$ height $= \pi r^2 \times 2r = 2\pi r^3$

This then allows us to calculate the fraction of the space in the cylinder taken up by the sphere:

$$\frac{\frac{4}{3}\pi r^3}{2\pi r^3} = \frac{2}{3}$$

7

The answer is option **A**.

For $0 < x < 1$, we have:

$0 < x^2 < 1$

$0 < \sqrt{x} < 1$

and also:

$1 < \frac{1}{x}$

$\frac{1}{\sqrt{x}} < \frac{1}{x}$

$\frac{1}{1+x} < \frac{1}{x}$

so $\frac{1}{x}$ has the largest value in $0 < x < 1$

8

The answer is option **C**.

Using the fact that triangle ABC is similar to triangle ADE [they have the same angles], we can write:

$$\frac{AB}{AD} = \frac{BC}{DE}$$

This gives:

$$\frac{4}{x} = \frac{x}{x+3}$$

From which we get:

$$4(x+3) = x^2$$

Rearranging:

$$x^2 - 4x - 12 = 0$$

Factorising:

$$(x-6)(x+2) = 0$$

Giving $x = 6$ [as we cannot have a negative value for x as it represents a length].

And so $DE = x + 3 = 6 + 3 = 9$

9

The answer is option **C**.

$P \propto \frac{1}{Q^2}$ so $P = \frac{k}{Q^2}$ for some k

We can use multiplying factors to work out how each symbol changes; for instance, as Q increases by 40% its new value is $1.4Q$

For this question, we have:

$Q_{new} = 1.4Q$

Giving:

$P_{new} = \frac{k}{(1.4Q)^2} = \frac{k}{1.96Q^2} = \frac{1}{1.96}P$

We then notice:

$\frac{1}{1.96} \approx \frac{1}{2}$ and $\frac{1}{1.96} > \frac{1}{2}$

so P_{new} is a little over 50% of P and so we can deduce that P has decreased by a little under 50%.

10

The answer is option **D**.

We are given: $x \propto z^2$ and $y \propto \frac{1}{z^3}$ which allows us to write: $x = kz^2$ and $y = \frac{c}{z^3}$

We want to eliminate z so we need both expressions to have the same power of z in them:

$x^3 = k^3 z^6$ and $y^2 = \frac{c^2}{z^6}$

Eliminating z and ignoring the constants gives:

$x^3 \propto \frac{1}{y^2}$

11

The answer is option **C**.

To make the question easier to answer, we can let $QX = 2$

We then find, using the given ratios, that $PX = 12$ and $XR = 3$ and so $PR = 15$

As M is the midpoint of PR then $PM = 7.5$ and so $MX = 4.5$

We can then calculate the ratio asked for:

$$\frac{QX}{MX} = \frac{2}{4.5} = \frac{4}{9}$$

12

The answer is option **D**.

$x^2 \geq 8 - 2x$

Rearranging:

$x^2 + 2x - 8 \geq 0$

Factorising:

$(x + 4)(x - 2) \geq 0$

Giving:

$x \leq -4$ or $x \geq 2$

13

The answer is option **A**.

We are told:

surface area of cylinder $=$ volume of cylinder

We can write this as follows:

$2\pi r^2 + 2\pi rh = \pi r^2 h$

Rearranging:

$2r = rh - 2h$

$2r = h(r-2)$

Giving:

$\frac{2r}{r-2} = h$

14

The answer is option **D**.

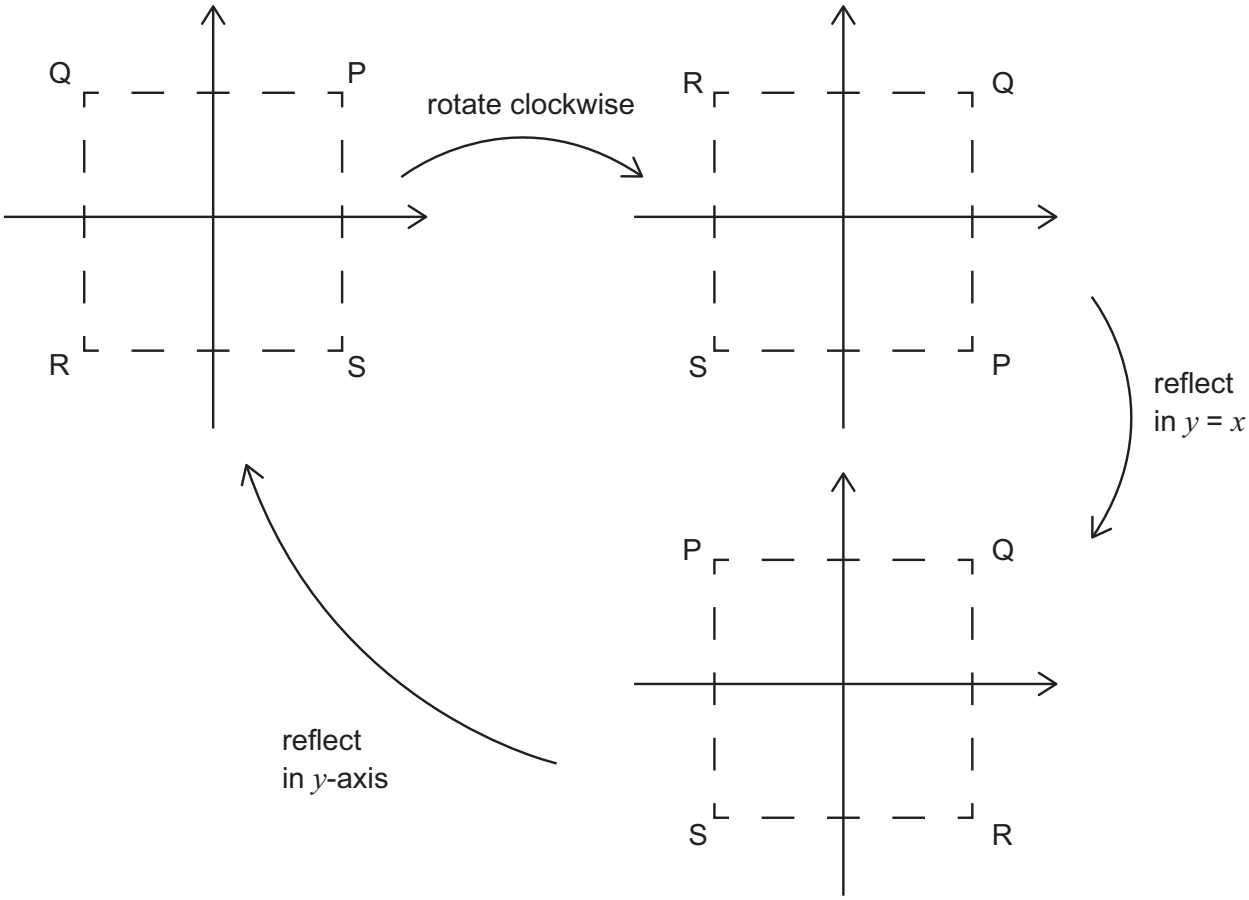
We require: $6 < 2\sqrt{n} < 8$

Which gives, on squaring: $36 < 4n < 64$ or $9 < n < 16$

This is satisfied when $n = 10,11,12,13,14,15$, i.e. there are 6 integers.

15

The answer is option **B**.



16

The answer is option **C**.

We can let the length of a side of the large square be 3 to help us find the area scale factor.

First we use Pythagoras' theorem to find the length of one side of the first inscribed square:

Length of side of inscribed square $= \sqrt{1^2 + 2^2} = \sqrt{5}$

This allows us to work out the area scale factor using:

Area of main square $= 9$

Area of inscribed square $= 5$

Area scale factor $= \frac{5}{9}$

Therefore the area of the fourth square [the third inscribed square] as a fraction of the area of the first square must be:

$$\left(\frac{5}{9}\right)^3 = \frac{125}{729}$$

17

The answer is option **C**.

We can use multiplying factors to work out how each symbol changes; for instance, as x increases by 50% its new value is $1.5x$

After the percentage changes the expression becomes:

$$\frac{\left(1.5x + 1.5y^2\right)0.8z}{2P}Q$$

This simplifies:

$$\frac{1.5^2\,(x+y)^2\,0.8z}{2P}Q = \frac{1.5^2 \times 0.8}{2} \times \frac{(x+y)^2 z}{P}Q = 0.9\,\frac{(x+y)^2 z}{P}Q$$

giving a 10% decrease in the value of M.

18

The answer is option **B**.

Prob of scoring $12 = (\text{Prob of scoring 6 on fair dice}) \times (\text{Prob of scoring 6 on biased dice})$

As the question tells us that the Prob of scoring $12 = \frac{1}{18}$

We can write: $\frac{1}{6} \times (\text{Prob of scoring 6 on biased dice}) = \frac{1}{18}$

So: $(\text{Prob of scoring 6 on biased dice}) = \frac{1}{3}$

As the probability of scoring a 1 or 2 or 3 or 4 or 5 is the same on the biased dice, we can deduce that:

$(\text{Prob of scoring 1 on the biased dice}) = \frac{1}{5} \times \frac{2}{3} = \frac{2}{15}$

And so this gives:

Prob of scoring 2 $=$ $(\text{Prob of scoring 1 on fair dice}) \times (\text{Prob of scoring 1 on biased dice})$

or

Prob of scoring $2 = \frac{1}{6} \times \frac{2}{15} = \frac{1}{45}$

PHYSICS

19

The answer is option **C**.

The forces on the parachutist are 600 N downwards and 900 N upwards.

These cause a resultant force on the parachutist of $900\,\text{N} - 600\,\text{N} = 300\,\text{N}$ upwards.

acceleration = resultant force / mass $= 300/60 = 5.0\,\text{m s}^{-2}$ upwards.

20

The answer is option **C**.

From the graph, period $T = 2.0\,\text{s}$

$f = \frac{1}{T} = \frac{1}{2.0} = 0.50\,\text{Hz}$

$v = f\lambda = 0.50 \times 1.5 = 0.75\,\text{cm s}^{-1}$

21

The answer is option **B**.

momentum $p = mv$ and kinetic energy $\text{KE} = \frac{1}{2}mv^2$

rearranging:

$v = \frac{p}{m}$ and so $KE = \frac{1}{2}\frac{p^2}{m}$

therefore:

$m = \frac{p^2}{2KE} = \frac{900}{300} = 3.0\,\text{kg}$

[Finally, it is good practice to verify this answer by substituting it back into the kinetic energy equation:

kinetic energy $= \frac{1}{2}mv^2 = \frac{1}{2} \times 3 \times 10^2 = 150\,\text{J}$, which is correct.]

22

The answer is option **E**.

Looking at each option in turn:

A $V = IR$ so volts are amp $\times$ ohm not amp per ohm.

B The volt is defined as a joule per coulomb.

Therefore (coulomb per joule) $=$ 1 / volt. This is not equal to the volt.

C The joule per second is the watt.

This is not equal to the volt.

D Since the volt is defined as a joule per coulomb it is worth expressing the newton as a joule per metre.

Therefore newton per coulomb $=$ (joule per metre) per coulomb $=$ (joule per coulomb) per metre $=$ volt per metre. This is not equal to the volt.

E $P = IV$, therefore $V = \frac{P}{I}$ so volts are watt per amp.

23

The answer is option **D**.

Consider each source in turn.

Source X: 24 hours is 5 half-lives ($24 / 4.8 = 5$). So, in 24 hours the activity of source X will halve 5 times, from 320 to 160 to 80 to 40 to 20 to 10. After 24 hours the activity of X will have fallen to 10 Bq.

Source Y: 24 hours is 3 half-lives ($24 / 8 = 3$). So, in 24 hours the activity of Y will halve 3 times, from 480 to 240 to 120 to 60. After 24 hours the activity of Y will have fallen to 60 Bq.

The combined count rate after 24 hours is therefore $10 + 60 = 70$ Bq.

24

The answer is option **B**.

Since air resistance is negligible, all of the initial GPE is transferred to KE so $mgh = \frac{1}{2}mv^2$

Rearranging the equation, we obtain: $h = \frac{v^2}{2g} = 400 / 20 = 20$ m

(Notice that m cancels so the result is independent of the mass of the falling body.)

25

The answer is option **E**.

As the cyclist loses 100 m of height, he loses gravitational potential energy

$= mgh$
$= 100 \times 10 \times 100$
$= 100\,000\,\text{J}$

The cyclist is descending the slope at a constant speed, which means that all of this potential energy becomes heat as work is done on the cyclist by the resistive forces.

Therefore, work done by resistive forces $= 100\,000\,\text{J}$

distance travelled along the slope whilst descending 100 m height $= 100 \times 10 = 1000\,\text{m}$

Therefore, resistive force $=$ work done / distance $= 100\,000 / 1000 = 100\,\text{N}$

Alternatively, since the cyclist is moving at a constant velocity he must be in equilibrium. Therefore the drag force must be equal and opposite to the component of weight acting down the slope ($mg\sin\theta$). In this case, $\sin\theta = 10/100 = 0.1$ so the drag force is $0.1mg = 0.1 \times 100 \times 10 = 100\,\text{N}$.

26

The answer is option **D**.

Work done by the motor in lifting the load through 5.0 m $= mgh = 3.0 \times 10 \times 5.0 = 150\,\text{J}$

Therefore, output power of motor $=$ work done / time taken $= 150 / 1.5 = 100\,\text{W}$

Motor is 100% efficient so the electrical input power to the motor is also 100 W

Therefore, average current in motor $=$ power / voltage $= 100 / 25 = 4.0\,\text{A}$

Alternatively, since the process is 100% efficient, the power output of the motor ($P = IV$) will equal the power used to lift the mass ($P = mgh/t$).

Equating and rearranging we have: $IV = mgh/t$ so $I = mgh/Vt = (3 \times 10 \times 5)/(25 \times 1.5) = 4.0\,\text{A}$

27

The answer is option **A**.

Let the mass of the ball in kg be *m*. (Its value is not needed to do the question.)

kinetic energy at the point of release $= \frac{1}{2}mv^2 = \frac{1}{2} \times m \times 12^2 = \frac{1}{2} \times 144 \times m = 72m$ joules

At the highest point, all of this kinetic energy has turned into gravitational potential energy

Therefore, at the highest point, gravitational potential energy $= 72m$ joules

This is also equal to *mgh*, where *h* is the height reached.

Therefore, $10mh = 72m$. Notice that *m* cancels out here (as expected), leaving $10h = 72$

Therefore, height reached $h = 72/10 = 7.2$ metres

Alternatively, since all of the initial KE $\left(\frac{1}{2}mv^2\right)$ is transferred to GPE (*mgh*) at the top of the motion:

$mgh = \frac{1}{2}mv^2$, so $h = v^2/2g = 12^2/20 = 7.2\,\text{m}$

28

The answer is option **C**.

kinetic energy of lorry $= \frac{1}{2}mv^2$

As the lorry is moving along a horizontal road, the resistive forces need to absorb this energy and no more. Work done by resistive forces in stopping the lorry is therefore $\frac{1}{2}mv^2$

distance travelled by lorry while stopping $=$ work done / stopping force $= \left(\frac{1}{2}mv^2\right)/F = mv^2/2F$

29

The answer is option **B**.

An alpha particle consists of two protons and two neutrons so alpha emission results in the reduction of atomic number (bottom) by 2 and the reduction of nucleon number (top) by 4.

A beta-minus particle is an electron emitted when a neutron in the nucleus changes into a proton. This increases atomic number (bottom) by 1 but has no effect on nucleon number (since both protons and neutrons are nucleons).

The decay from X to Y involves the atomic number going down by 2 (from R to $R - 2$). This means that this stage must be alpha decay, and therefore that the mass number must drop by 4. The mass number of Y must therefore be $N - 4$, and this is the value of P.

The decay from Y to Z involves the mass number remaining the same (P in both cases). This means that this stage must be beta decay, and therefore that the atomic number of Z must be one higher than that of Y. As the atomic number of Y is $R - 2$, the atomic number of Z must be $(R - 2) + 1 = R - 1$, and this is the value of Q.

30

The answer is option **B**.

This question is simply a straightforward application of speed = distance / time, and the frequency of the pulse is irrelevant.

distance travelled by pulse from transmitter to foetus and back again to receiver $= 2 \times 10$
$= 20\,\text{cm}$
$= 0.20\,\text{m}$

Therefore, time taken = distance / speed $= 0.20 / 500 = 0.0004\,\text{s} = 0.40\,\text{ms}$

31

The answer is option **B**.

Both graphs P and Q show an object accelerating with constant acceleration. The value of that acceleration is the gradient of the velocity-time graph.

Graph P: Gradient = 10 / 24, which is not $2.4\,\text{m s}^{-2}$. (You don't need to evaluate 10 / 24 to see that.)

Graph Q: Gradient = $(58 - 10) / 20 = 48 / 20 = 2.4\,\text{m s}^{-2}$. So graph Q is correct.

Graphs R and S are both distance-time graphs which are straight lines, signifying constant speed. Constant speed means zero acceleration, and so neither of these graphs shows an acceleration of $2.4\,\text{m s}^{-2}$.

The correct answer is therefore that graph Q only shows an acceleration of $2.4\,\text{m s}^{-2}$.

32

The answer is option **D**.

This question is all about the significance of the gradient of a straight–line graph. The gradient of the graph shown is $20/100 = 0.20$ in the units of the *y*-axis divided by the units of the *x*-axis.

A As $F = ma$, and therefore $a = F/m$, a graph of acceleration against force will give a straight line with a gradient of $1/m$. If $m = 5\,\text{kg}$ then $1/m = 1/5 = 0.2\,\text{kg}^{-1}$. Graphical representation correct.

B As $V = IR$, and therefore $I = V/R$, a graph of current against voltage will give a straight line with a gradient of $1/R$. If $R = 5\,\Omega$ then $1/R = 1/5 = 0.2\,\Omega^{-1}$. Graphical representation correct.

C As $\text{KE} = \frac{1}{2}mv^2$, a graph of KE against speed squared will give a straight line with a gradient of $\frac{1}{2}m$. If $m = 0.4\,\text{kg}$ then $\frac{1}{2}m = 0.2\,\text{kg}$. Graphical representation correct.

D As $v = f\lambda$, and therefore $\lambda = v/f$, a graph would need to be plotted of wavelength against 1/frequency to get a straight line. A graph of wavelength against frequency will not give a straight line. Graphical representation therefore **not** correct.

E As $W = Fd$, a graph of work done against distance moved will give a straight line with a gradient of F. If $F = 0.2\,\text{N}$ then the graphical representation is correct.

33

The answer is option **D**.

Consider each statement in turn:

A Gamma emission is a form of naturally occurring radioactive decay. It is not the same process as nuclear fission. Statement not correct.

B The half-life of a substance is the time for half the amount of substance remaining at any moment to decay. A substance will never fully decay, and so ‘half the time taken to decay’ has no meaning. Statement not correct.

C The number of neutrons is the mass number minus the atomic number. Statement **C** has this the wrong way round. Statement not correct.

D When a nucleus emits a beta particle, a neutron turns into a proton (and an electron is emitted). The nucleus therefore contains the same number of particles as before, it’s just that one of them has changed into a different particle. Statement correct.

E A neutron becoming a proton and emitting an electron is beta decay, not alpha decay. Statement not correct.

34

The answer is option **D**.

Total resistance of circuit = sum of separate resistances = $R_1 + R_2$

Current in circuit is therefore $I = \frac{V}{R_{\text{total}}}$

$$= \frac{V}{(R_1 + R_2)}$$

This current is the same in both resistors.

Voltage across R_1 is therefore $V = IR_1$

$$= \frac{V}{(R_1 + R_2)} \times R_1$$

$$= \frac{VR_1}{(R_1 + R_2)}$$

Therefore power dissipated in R_1 is $P = VI$

$$= \frac{VR_1}{(R_1 + R_2)} \times \frac{V}{(R_1 + R_2)}$$

$$= \frac{V^2 R_1}{(R_1 + R_2)^2}$$

Alternatively, a solution can be reached more quickly by considering dimensions: the power dissipated by a resistor is given by (voltage across resistor)^2 / (resistance). The only option with a voltage-squared divided by a resistance is **D**.

35

The answer is option **D**.

The distance travelled is the area under the graph, but you also need to match the units. The speeds on the graph are in $\mathrm{m\,s^{-1}}$ but the times are in minutes. For the area under the graph to represent distance in metres, the times therefore need to be converted to seconds. This can be done all in one go by multiplying by 60 at the end.

Now consider the areas under each section of the graph.

0–1 minutes: $\text{Area} = (16+20)/2 \times 1 = 18\ (\mathrm{m\,s^{-1}})\,\mathrm{min}$

1–4 minutes: $\text{Area} = 20 \times 3 = 60\ (\mathrm{m\,s^{-1}})\,\mathrm{min}$

4–5 minutes: $\text{Area} = (20+10)/2 \times 1 = 15\ (\mathrm{m\,s^{-1}})\,\mathrm{min}$

5–6 minutes: $\text{Area} \approx (10+2)/2 \times 1 = 6\ (\mathrm{m\,s^{-1}})\,\mathrm{min}$

6–7 minutes: $\text{Area} \approx (2+0)/2 \times 1 = 1\ (\mathrm{m\,s^{-1}})\,\mathrm{min}$

(Note there are various ways of approximating the areas under the last two curved sections of the graph. For instance, another way could be to consider 5–6 minutes as being a trapezium from 10 down to $5\,\mathrm{m\,s^{-1}}$, effectively importing the last section into the preceding one, and then ignoring the 6–7 minute section. Each method of approximation will produce a slightly different estimate; one such method is shown. All estimates will, however, be within a reasonable range of the key).

Adding these together, total area $\approx 18+60+15+6+1 = 100\ (\mathrm{m\,s^{-1}})\,\mathrm{min}$

At this point it is now convenient to deal with the necessary unit conversion by multiplying by 60, to give a total area $\approx 100 \times 60 = 6000\,\mathrm{m} = 6.00\,\mathrm{km}$.

(Note the technique of waiting until the end to multiply by 60 both saves time and avoids dealing with large numbers along the way. It is perfectly legitimate to multiply each time by 60 before working out each individual area, but this will make the numbers more awkward to work with and will result in the question taking a lot longer to do.)

36

The answer is option **B**.

Consider each statement in turn:

P We have no information about the wavelength of the sound waves produced, and nor do we know how far the waves travel in a given time. It is therefore not possible to deduce the speed from the information given. Statement P cannot be deduced.

Q The maximum distance of oscillation from compression to rarefaction is 5.0 mm, but the amplitude of the wave is from the equilibrium position to the maxima. The amplitude is therefore half of this (2.5 mm) and statement Q is incorrect.

R We cannot deduce the wavelength from the information given, and hence cannot deduce statement R.

S The period of the pulses produced is 0.2 ms = 0.0002 s. This means the frequency of the wave is 1 / period = 1 / 0.0002 = 5000 Hz. Statement S therefore can be correctly deduced from the information given.

The correct answer is that statement S only can be correctly deduced.

CHEMISTRY

37

The answer is option **E**.

The pH scale is a measure of the acidity or alkalinity of an aqueous solution.

All indicators in the mixture experience a pH of 5.0.

Methyl orange (colour change at 4.0) will be yellow at pH 5.0.

Bromothymol blue (colour changes at 6.5) will be yellow at pH of 5.0.

Phenolphthalein (colour changes at 9.0) will be colourless at pH of 5.0.

In order: yellow + yellow + colourless = yellow.

38

The answer is option **A**.

Aqueous sodium chloride contains two cations (Na^+ and H^+) and two anions (Cl^- and OH^-).

The positive electrode (anode) attracts anions and the negative electrode (cathode) attracts cations.

When two ions are attracted to an electrode, only one can be discharged as atoms. This is called preferential discharge.

In this case:

chlorine is discharged in preference to oxygen at the positive electrode (anode);

hydrogen is discharged in preference to sodium at the negative electrode (cathode).

39

The answer is option **A**.

Assessment of each option:

A: The high melting and boiling points, together with non-conductivity in the solid and molten states, indicates a giant covalent structure.

B: The high melting and boiling points indicate a giant structure. No conductivity when solid but having conductivity when molten is typical of a giant ionic structure.

C: Despite the relatively low melting and boiling points, the fact that it conducts electricity when solid and molten means that it is a giant metallic structure.

D: With low melting and boiling points, no electrical conductivity and being a liquid at room temperature and pressure, the substance is typical of a simple molecular structure.

E: Despite the very low melting point, the good conductivity when solid and molten indicates a metallic structure. In fact, **E** is mercury.

40

The answer is option **A**.

The reaction is stated to go to completion.

At the start of the reaction there is a concentration of A of 1.2 mol dm^{-3}. The equation gives the ratio of A : Z as 2 : 1. This means that for every unit of A that is reacted, only 0.5 as much Z is produced.

In a complete reaction, all of A will be used up and the final concentration of Z will be 0.6 mol dm^{-3}, so graph 1 must be correct.

As the chemical A is reacted, Z is produced. Graph 2 shows that an equilibrium state has been reached with the concentration of A being twice that of Z; this is incorrect.

Graph 3 shows that an equilibrium state has been reached with equal concentrations of A and Z; this is incorrect.

41

The answer is option **B**.

One method for solving the problem would be as follows:

The oxide ion has a charge of -2, O^{2-}.

A compound formed from Fe^{3+} and O^{2-} ions must have the formula Fe_2O_3.

A compound formed from Fe^{2+} and O^{2-} ions must have the formula FeO.

A 1 : 1 mixture of these compounds would give a compound of formula Fe_3O_4 ($Fe_2O_3 + FeO$).

So the molar ratio $Fe^{3+} : Fe^{2+}$ must be 2 : 1, making Fe^{2+} $\frac{1}{3}$ of the iron ion total.

42

The answer is option **E**.

Mass number = number of protons + number of neutrons = 40

Atomic number = number of protons = 20

Number of protons = 20, so number of electrons = 20 to balance charge.

Number of neutrons = 40 − 20 = 20

Using the above information and applying it to each statement:

1: Incorrect. The relative mass of the nucleus is 40.

2: Incorrect. A total of 20 electrons would give an electronic configuration of 2,8,8,2. The incomplete outer shell containing two electrons indicates that is in Group 2. Group 2 elements are not noble gases, they are alkaline earth metals.

3: Incorrect. Group 2 elements form 2+ ions by loss of the outer two electrons.

4: Correct.

5: Incorrect. Group 2 elements are metals.

43

The answer is option **E**.

Bond breaking is an endothermic process, whilst bond making is an exothermic process.

The reaction is exothermic overall, which means that more energy is released when bonds are made than is needed for bonds to be broken.

The bonds to be broken are on the left of the equation $(1 \times \text{N} \equiv \text{N}$ and $3 \times \text{H}-\text{H})$ and those made are on the right of the equation $(2 \times 3 \times \text{N}-\text{H})$. This means that the 6 N—H bond energies must be greater than the N_2 and $3 \times H_2$ bond energies in total.

Looking at the number of bonds from the mole ratios in the equation (1 : 3 : 2), the inequality $6z > x + 3y$ is satisfied.

44

The answer is option **C**.

Equations 2 and 3 are correct.

(Construction of half-equations is another valid solution method with the same conclusion.)

Oxidising agents accept electrons. Reducing agents lose (or donate) electrons.

Looking at each equation for the first reactant in the equation in turn:

1: Mg is transformed into Mg^{2+}. Electrons are being lost and so Mg is a reducing agent.

2: The oxidation state of chromium in the first compound is +6 and it is transformed into Cr^{3+}. Electrons are being accepted and so $Cr_2O_7^{2-}$ is an oxidising agent.

3: Cu^{2+} is transformed into Cu^{+}. Electrons are being accepted and so Cu^{2+} is a oxidising agent.

4: The oxidation state of sulfur in the first compound is +4 and it is transformed into a compound where it is +6. Electrons are being lost and so the first compound is a reducing agent.

45

The answer is option **D**.

1 mole of H_2O has a mass of $(2 \times 1) + 16 = 18\,\text{g}$

The volume of 1 mole of a gas at room temperature and pressure is $24\,\text{dm}^3$

6.00 g of H_2O is $\frac{6}{18} = \frac{1}{3}$ mol, and therefore occupies a volume of $\frac{24}{3} = 8\,\text{dm}^3$

$8\,\text{dm}^3 = 8000\,\text{cm}^3$

46

The answer is option **E**.

There are several methods that will give the correct answer, of which one is shown below:

	I	O
number of moles:	$\frac{63.5}{127} = 0.5$	$\frac{20.0}{16} = 1.25$
ratio ($\div 0.5$)	1	2.5
smallest whole number ratio ($\times 2$)	2	5

Therefore the empirical formula is I_2O_5.

47

The answer is option **C**.

Electronic configurations of atoms can be used to identify the position of an element in the Periodic Table. The number of electrons in the outer shell determines the group. The number of shells determines in which period the element is.

A: 2,4 Group 14, where non-metals are at the top of the group and metals are at the bottom

B: 2,6 Group 16, a non-metal

C: 2,7 Group 17, a non-metal

D: 2,8,1 Group 1, a metal

E: 2,8,6 Group 16, a non-metal

F: 2,8,7 Group 17, a non-metal

Only **D** can be eliminated. Non-metals react to gain electrons and acquire the configurations of noble gases.

The fewer the electrons to gain, the more reactive the element. This narrows the choice down to **C** and **F**, as both atoms only need to gain one electron.

C is the smaller atom (two shells) so there is more attraction between the nucleus and the electron to be gained, compared with **F**. **C** will therefore be more reactive. The element is fluorine.

48

The answer is option **C**.

This question is about identifying which possible isotopes can give rise to CH_2BrCl with $M_r = 128$, and the proportion of these isotopes that exist, to identify which isotope combinations give $M_r = 128$ and those that do not.

The combined total mass of bromine and chlorine (removing CH_2) must total $128 - 14 = 114$

The only combination of isotopes with this total mass is ^{79}Br combined with ^{35}Cl. As these are the isotopes with lowest mass for each element, any other possibilities would result in a higher M_r for the compound.

The fraction occurring is directly related to probability. We want the probability that $Br = {}^{79}Br$ AND $Cl = {}^{35}Cl$, in CH_2BrCl and these events are independent of each other.

^{35}Cl is three times as common as ^{37}Cl, and the question states that there are only two isotopes to be considered (3 : 1), so the relative frequency of ^{35}Cl is $\frac{3}{4}$, and for ^{37}Cl it is $\frac{1}{4}$. We can treat these as probabilities.

For bromine, the relative frequency of ^{79}Br is $\frac{1}{2}$, and it is the same for ^{81}Br.

Hence $P(^{79}Br \text{ AND } ^{35}Cl) = P(Br = {}^{79}Br) \times P(Cl = {}^{35}Cl) = \frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

49

The answer is option **A**.

When using a fractionating column, the flask is at the bottom and is heated. The temperature is highest at the bottom of the column and lowest at the top.

The hexane has the lowest boiling point and so will evaporate first in the flask and will rise to the top of the column first (68 °C). The liquid mixture in the flask must be boiling so the temperature must be at least 68 °C, but less than 98 °C, so that only the hexane is vapourised.

50

The answer is option **D**.

In this question, stage 1 can be ignored.

In stage 2, the stoichiometry of the equation shows that 1 mole of C will produce 2 moles of CO.

12 g of C (A_r 12) is 1 mole

M_r of CO is $12 + 16 = 28$, so 2 moles of CO is $28 \times 2 = 56$ g

In stage 3, the stoichiometry of the equation shows that 3 moles of CO will produce 3 moles of CO_2, i.e. a 1 **:** 1 molar ratio.

56 g (2 moles) of CO will make 2 moles CO_2

M_r of CO_2 is $12 + (16 \times 2) = 44$, so 2 moles of CO_2 is $44 \times 2 = 88$ g

51

The answer is option **E**.

A: False. There is a different number of molecules on the two sides of the equation so altering the pressure will shift the equilibrium position.

B: False. The reaction is exothermic and so increasing the temperature will move the equilibrium position to the left.

C: False. Chemical equilibrium is dynamic so reactants are constantly changing to products and products are constantly changing back to reactants.

D: False. A catalyst increases the rate at which equilibrium is reached but does not shift the position of equilibrium.

E: Correct. Until equilibrium is reached, the forward reaction will be faster than the backward reaction.

52

The answer is option **E**.

The chemical equation for the reaction is: $NaOH + HCl \rightarrow NaCl + H_2O$

Molar ratio is 1 NaOH **:** 1 HCl

Number of moles of HCl $= \frac{\text{vol. in cm}^3}{1000} \times \text{conc.} = \frac{50.0}{1000} \times 0.5 = 0.025$

Therefore, the number of moles of NaOH is also 0.025

A_r of NaOH is $23 + 1 + 16 = 40$, so 1 mole of NaOH has a mass of 40 g

In the question, the number of moles of NaOH present $= 0.025 \times 40 = 1.0\,\text{g}$

Percentage purity $= \frac{1.0}{1.20} \times 100 = 83.3\%$

53

The answer is option **C**.

Various approaches can be used to calculate the correct answer, of which one is shown below.

Number of moles H_2SO_4 used $= \frac{12.5}{1000} \times 2.0 = 0.025$

Molar ratio in the reaction is 2 XOH **:** 1 H_2SO_4

So, the number of moles of XOH present $= 0.025 \times 2 = 0.05$

Number of moles $= \frac{\text{mass in g}}{M_r}$, so $0.05 = \frac{2.8}{M_r}$

Rearranging, $M_r = \frac{2.8}{0.05} = \frac{28}{0.05} = 56$

M_r of XOH $= A_r(\text{X}) + 16 + 1 = 56$

$A_r(\text{X}) = 39$

54

The answer is option **D**.

The number of moles of each element on each side of the equation is:

	H	N	O
left-hand side	r	r	$3r$
right-hand side	$2s$	$2 + t$	$6 + s + 2t$

These numbers must balance for each element:

For H, $r = 2s$

For N, $r = 2 + t$

For O, $3r = 6 + s + 2t$

Substituting $r = 2s$ into $3r = 6 + s + 2t$, we get $3 \times (2 + t) = 6 + s + 2t$

Rearranging: $6 + 3t = 6 + s + 2t$, therefore $t = s$

Moles of H and N are equal (r), so $2s = 2 + t$

Substituting $t = s$ into this, we get $2s = 2 + s$ and therefore $s = 2$

It follows that $t = 2$ and $r = 4$

BIOLOGY

55

The answer is option **D**.

Statement 1 is incorrect as gamete formation occurs due to meiosis. Both statement 2 and statement 3 are also incorrect. However, they are common misconceptions held by students. Growth of an organism such as a human occurs by mitosis as the number of cells increases. The size of a cell does not increase due to mitosis. Mitosis results in the production of new cells which can be used to replace damaged cells in tissues such as muscles and organs such as skin. However, mitosis cannot repair cells, so statement 3 is incorrect.

Therefore the only correct statement is statement 4 – replacement of skin cells.

56

The answer is option **D**.

As oxygen is being supplied and used, aerobic respiration is occurring in these muscle cells. However, as the oxygen demand between 0 and 11 minutes is greater than the supply, as shown by the higher position of the oxygen demand line on the graph, the muscle cells are also carrying out anaerobic respiration.

57

The answer is option **F**.

Competition for limiting resources can occur both within a species (intraspecific – statement 1) and between species (interspecific – statement 2). Likewise, natural selection can lead to evolution due to individuals with advantageous adaptations being more likely to survive and pass on their alleles, thus passing on the advantageous adaptations to their offspring (statement 3). For a species that competes less well, natural selection may lead to extinction (statement 4). Therefore all four statements are correct.

58

The answer is option **D**.

The main factors affecting the rate of photosynthesis are light intensity, temperature and carbon dioxide concentration. Any change in the level of a factor will affect the rate of reaction and the process will be limited by the factor which is in the shortest supply.

In the case of temperature, the rate of reaction will increase until the optimum temperature for the enzymes controlling the reaction is reached, as long as the other factors are present in sufficient amounts. Therefore statement 1 is correct. After this point the enzymes start to become denatured and no longer work, so statement 2 is correct. An increase in temperature produces an increase in kinetic energy and, up to a temperature of 22 °C the rate of photosynthesis also increases. However temperature does not increase the rate across the whole of the range studied as after the optimum temperature has been reached, the rate of photosynthesis decreases. Therefore statement 3 is incorrect.

59

The answer is option **F**.

Statement 1 is incorrect as at position 21 there are three chromosomes rather than two. As such, this is an example of a chromosome mutation and leads to Down's syndrome. Statement 2 is correct as the 23rd pair of chromosomes are the sex chromosomes and as they are XY this means the cell has been donated by a male. White blood cells are diploid body cells and so they contain two copies of each chromosome with two sex chromosomes as shown in the karyogram, so statement 3 is also correct.

60

The answer is option **F**.

Bone marrow stem cells will divide by mitosis, rather than meiosis, to produce genetically identical offspring, so statement 1 is incorrect. Human stem cells, including bone marrow stem cells, are diploid and so contain 46 chromosomes (statement 2 is correct). Stem cells of the bone marrow are able to differentiate into white blood cells so statement 3 is also correct.

61

The answer is option **C**.

The frequency of occurrence is the proportion of quadrats containing at least one individual of a given species.

In this study there are 25 daisies, occurring in a total of 10 quadrats.

In order to calculate the frequency of occurrence you need to divide the total number of quadrats in which the species (daisy) occurs (which is 10) by the total number of quadrats (in this case 25).

This equals 0.40.

62

The answer is option **F**.

The two females with attached ear lobes are homozygous recessive. The female in the first generation must be heterozygous, as she has the dominant phenotype, but must have inherited the recessive allele from her mother (who is homozygous recessive). The male parent, the male in the first generation, and the people in the second generation who have unattached ear lobes could be either heterozygous or homozygous dominant. So the maximum possible number of heterozygous individuals is 7.

63

The answer is option **A**.

A change from 0.12 mol dm^{-3} to 0.084 mol dm^{-3} gives an increase in percentage haemolysis (read graph from right to left) from 2% up to 72%, which is an increase of 70%. The increase is due to the uptake of water through osmosis into the cells, which occurs as a result of the reduction in the concentration of the NaCl solution.

64

The answer is option **H**.

A positive phototropic response is shown by shoots when they grow towards a source of light.

In the diagram, three of the shoots have shown phototropism but the shoot without a tip has not responded, although it was uncovered. Statement 1 is therefore incorrect.

The diagram shows that when the shoot tip is covered by an opaque cap, or if the whole shoot is placed in an opaque box, there is no phototropic response. However, both the shoot with its tip covered by a transparent cap, and the shoot with its base covered by an opaque sheath, do show growth towards the light. Statement 2 is therefore incorrect.

Statement 3 is also incorrect – the shoot enclosed in an opaque box has grown taller.

65

The answer is option **D**.

In order to interpret this data correctly you need to ensure that you understand what is represented by the graph.

The *x*-axis shows the years studied from 1959 to 2008.

The left-hand vertical axis represents the number of wolves, with a range from 0 to just over 50.

The right-hand vertical axis represents the number of moose, with a range from 0 to just under 2500.

Trend 1 is quickly determined to be incorrect, since from 1959 to 1965, overall both the wolf and moose populations are increasing. Additionally, with no knowledge of what has happened before 1959 it cannot be said that there has been an increase in the wolf population as there was an earlier increase in moose population.

Trend 2 is correct. You need to remember that the two vertical scales are different. Reading from the graph, the maximum wolf population is 50, whilst the minimum moose population is about 400.

Trend 3 is incorrect. Whilst the point plotted for wolves is higher than that for moose in 1965, you need to remember that the two vertical scales are different. There are about 30 wolves but there are about 750 moose.

Trend 4 is correct. When wolf populations are low on the graph, such as from 1968 to 1973 and from 1982 to 1995, there is a corresponding increase in the size of the moose population.

This means that trend 4 must be correct, so you only need to identify which of trend 2 or trend 3 is correct.

66

The answer is option **G**.

The ratio of X**:**A in columns 1 and 2 is 0.5**:**1 which makes the flies male, irrespective of the presence or absence of the Y chromosome. The ratio of X**:**A in columns 3, 4 and 5 is 1**:**1 which makes the flies female, irrespective of the presence or absence of the Y chromosome.

67

The answer is option **H**.

Process 1 is photosynthesis and therefore should not feature in any answers. For carbon to be incorporated into animals and decomposers, digestion of carbon-rich compounds must initially occur (processes 2 and 3). Respiration can release carbon into the air as CO_2 which is illustrated by process 4.

68

The answer is option **B**.

The *y*-axis represents the variable being measured as the enzyme-catalysed reaction progresses with time. The only variable which increases with the progress of the reaction will be the enzyme-substrate complexes which will then decrease as the substrate is broken down so fewer complexes can form.

69

The answer is option **B**.

In order to genetically modify bacteria to produce human insulin you need to cut out the DNA coding for human insulin from a normal, healthy human's DNA. A special enzyme is used to cut the DNA. Therefore statement 1 is correct.

This DNA is then inserted into the DNA of a bacterium. Bacteria do not have a nucleus, so statement 2 is incorrect.

The modified bacteria have to be cultured. This means that they are grown in a fermenter so that large numbers of bacteria are produced. Therefore statement 3 is correct.

The bacteria are used to produce human insulin, which is extracted from them and purified. This insulin is then used to treat diabetics. Therefore, statement 4 is incorrect, since bacteria are not injected into humans with diabetes.

70

The answer is option **E**.

Mutations are random changes in the DNA sequence of an organism. If this occurs within the sequence of a gene that codes for a protein then it can change the bases in the triplet sequence of that gene and may alter the amino acids that they code for (statement 1). If the mutation occurs within an enzyme that is part of an essential cell process, like respiration, then the cell may no longer be able to function correctly and may die (statement 3).

Some mutations do produce positive changes in cells that could be beneficial to the organism. If they take place in a cheek cell instead of a gamete however, they will not be passed on to any offspring of the organism so statement 2 is incorrect.

71

The answer is option **D**.

Using the information in the question, as the condition is caused by at least one allele it must be a dominant allele (statement 1). Therefore a person with the condition can be heterozygous or homozygous. For U to show the condition she must have received the allele for this condition from one of her parents and this would be T as he shows the condition (statement 2). In this case U would be heterozygous. Although U's mother does not show the condition a mutation could occur in the egg that is fertilised by T's sperm (statement 3). In this case U would be homozygous for the condition.

72

The answer is option **F**.

Starch is found in plant cells, not animal cells such as a liver cell, so statement 1 is incorrect. Within the liver cell's DNA there would be one pair of sex chromosomes. If the cell was from a male these would be XY and in a female these would be XX. In either case there is at least one X chromosome, so statement 2 is correct. An adult liver cell is a diploid body cell so it will contain the same genes as all other diploid body cells, including the gene for amylase, so statement 3 is correct.

ADVANCED MATHEMATICS AND ADVANCED PHYSICS

73

The answer is option **F**.

Take logs of each side and separate out the left-hand side:

$x\log_{10} a + 2x\log_{10} b + 3x\log_{10} c = \log_{10} 2$

$x(\log_{10} a + 2\log_{10} b + 3\log_{10} c) = \log_{10} 2$

$x\log_{10}(ab^2c^3) = \log_{10} 2$, so $x = \dfrac{\log_{10} 2}{\log_{10}(ab^2c^3)}$

74

The answer is option **C**.

Let the positive direction be the direction of initial motion of particle P.

momentum of particle P before the collision $= +(2\times 3) = +6\,\text{kg m s}^{-1}$

momentum of particle Q before the collision $= -5\times r$

Therefore, total momentum of the particles before the collision $= 6 - 5r$

momentum of particle P after the collision $= -(2\times 1) = -2\,\text{kg m s}^{-1}$

momentum of particle Q after the collision $= +(5\times 0.5r) = +2.5r$

Therefore, total momentum of the particles after the collision $= 2.5r - 2$

Therefore, because momentum is conserved in the collision:

$(6 - 5r) = (2.5r - 2)$

Rearranging:

$(6 + 2) = (2.5r + 5r)$

$8 = 7.5r$

$16 = 15r$

$r = \dfrac{16}{15}$

75

The answer is option **D**.

A: $\tan\left(\frac{3\pi}{4}\right) = -1$

B: $\log_{10} 100 = 2$

C: $\sin^{10} x \leq 1$ for any x; in this case, it equals 1

D: $\log_2 10 > \log_2 8 = 3$

E: $0 < \sqrt{2} - 1 < 1$ (as $1 < \sqrt{2} < 2$), so raising it to the power of 10 still gives a number less than 1

So the largest of the five numbers is **D**: $\log_2 10$

76

The answer is option **A**.

When travelling at terminal velocity, the force of air resistance (drag) on the parachutist will be equal and opposite to his weight. This is true when travelling at a high terminal velocity before opening the parachute and when travelling at a lower terminal velocity after opening the parachute. Terminal velocity means zero acceleration which means zero resultant force.

During the act of opening the parachute there is a rapid deceleration which means that momentarily there must be a resultant upwards force acting on the parachutist. During that instant there must therefore be an air resistance force which is much greater than the weight.

The correct graph is therefore one that shows the same air resistance force, in the same direction, throughout apart from during the moment of opening the parachute. And during that moment there must be a much larger air resistance force, again all in the same direction. The only graph that shows all this is graph **A**.

B and **D** are incorrect because they show the air resistance force changing direction, which it cannot do. It will always be upwards. **C** and **D** are incorrect because they show different air resistance forces at terminal velocity before and after opening the parachute.

77

The answer is option **E**.

Let $y = 2^x$, then $y^2 - 8y + 15 = 0$. Solve to give $y = 3$ or 5

$2^x = 3$ implies that $x = \frac{\log_{10} 3}{\log_{10} 2}$. Similarly, the other solution is $\frac{\log_{10} 5}{\log_{10} 2}$

Sum of roots $= \frac{\log_{10} 3 + \log_{10} 5}{\log_{10} 2} = \frac{\log_{10} 15}{\log_{10} 2}$

78

The answer is option **C**.

The friction acting on the block cannot exceed the applied force. At first the friction cancels the applied force and the block is stationary. When the friction reaches its maximum magnitude, the net force is zero but as the applied force continues to increase, the block moves. The net force is increasing as the applied force increases and so the block accelerates with an increasing acceleration.

Only the options where the block remains stationary at first can be possible. In two of these cases, once the block moves, it moves with either constant velocity (zero acceleration) or with constant acceleration.

The correct statement is: It remains stationary at first and then accelerates forwards with an increasing acceleration.

79

The answer is option **E**.

Statement 1 subtracts $a + b$ from both sides.

Statement 2 can be written as $(a - b)^2 \geq 0$. This is always true.

Statement 3 can be false if c is negative, for example $a = 2$, $b = 1$, $c = -1$

80

The answer is option **C**.

Use the conservation of momentum to calculate the speed of the second ball.

momentum before $= 0.2 \times 3.0 = 0.6 \text{ kg m s}^{-1}$

Let the speed of the second ball after the collision be $v \text{ ms}^{-1}$

total momentum after collision $= (0.2 \times 1.0) + (0.2v) \text{ kg m s}^{-1}$

total momentum after collision $=$ total momentum before collision

$(0.2 \times 1.0) + (0.2v) = 0.6 \text{ kg m s}^{-1}$

$0.2v = 0.6 - 0.2 = 0.4$

$v = \frac{0.4}{0.2} = 2.0 \text{ m s}^{-1}$

81

The answer is option **A**.

The sequence begins $a_1 = 2,\ a_2 = 2 + (-1) = 1,\ a_3 = 2 = 1 + (-1)^2 = 2,\ a_4 = 1,$ etc.

So the sum of the first 100 terms is $2 + 1 + 2 + 1 + \ldots = 50 \times 2 + 50 \times 1 = 150$

82

The answer is option **A**.

Initial potential energy of the ball $=$ mgh $=$ $16mg$

Kinetic energy just before first bounce is therefore $16mg$

Kinetic energy just after first bounce $= \frac{1}{2} \times 16mg$

Therefore height of rebound after first bounce $= \frac{1}{2} \times 16 = 8\,\text{m}$

The height of rebound halves each time.

So height of rebound after second bounce $= 4\,\text{m}$

So height of rebound after third bounce $= 2\,\text{m}$

So height of rebound after fourth bounce $= 1\,\text{m} = 100\,\text{cm}$

It is after the fourth bounce, therefore, that the height of rebound first falls below 160 cm, so the 4th bounce is the correct answer.

83

The answer is option **C**.

To determine how many roots the equation has, we can investigate how many times the graph of $y = x^4 - 4x^3 + 4x^2 - 10$ intersects the x-axis. To help us do this we can find where the graph has turning points:

Differentiating gives: $4x^3 - 12x^2 + 8x = 0$

Solving this: $4x\left(x^2 - 3x + 2\right) = 4x(x-1)(x-2) = 0$

Giving $x = 0$, 1, 2

The coordinates of the turning points are: $(0, -10)$, $(1, -9)$, $(2, -10)$.

From this, and knowing the general shapes of quartics, we can deduce that the curve will intersect the x-axis at two distinct places and so the original quartic must have two distinct real roots.

84

The answer is option **E**.

The fact that the reading on the scales has decreased means that the normal contact force acting on the man has decreased. As his weight is now greater than the normal contact force there is a resultant force acting on the man which is downwards. This in turn means that the man has a downwards acceleration. This could mean either that he is moving upwards but slowing down or moving downwards and speeding up. The first of these is the one to appear in the options, and the correct answer is that the elevator is moving upwards with decreasing speed.

85

The answer is option **D**.

The answer is $y = ax^b$

We can see this by taking logs of the equation and comparing it with the standard equation of a line, $Y = mX + c$

Taking logs gives:

$\log y = \log a + b \log x$

Comparing with $Y = mX + c,$ we can see that plotting $\log y$ on the Y-axis and $\log x$ on the X-axis would give us a straight line with a gradient of b and a Y-intercept of $\log a$

86

The answer is option **B**.

The deceleration to rest is constant and so the average velocity is $\left[\frac{(12.0 + 0)}{2}\right]$ m s^{-1} = 6.0 m s^{-1}.

Secondly, acceleration $=$ $\frac{\text{velocity change}}{\text{time taken}}$, so:

time taken $= \frac{\text{velocity change}}{\text{acceleration}} = \frac{12.0}{1.5} = 8.0$ s to come to rest.

In 8.0 s at an average velocity of 6.0 m s^{-1} along a straight, horizontal track, the tram travels a distance of (6.0×8.0) m. This gives an answer of 48.0 m.

Alternatively, using $v^2 = u^2 + 2as$

Re-arranging:

$$s = \frac{v^2 - u^2}{2a} = \frac{0^2 - 12^2}{2 \times (-1.5)} = \frac{-144}{-3} = 48.0\,\text{m}$$

87

The answer is option **D**.

For real distinct roots the discriminant condition gives:

$(a-2)^2 > 4a(-2)$ or $(a-2)^2 - 4a(-2) > 0$

$a^2 + 4a + 4 > 0$

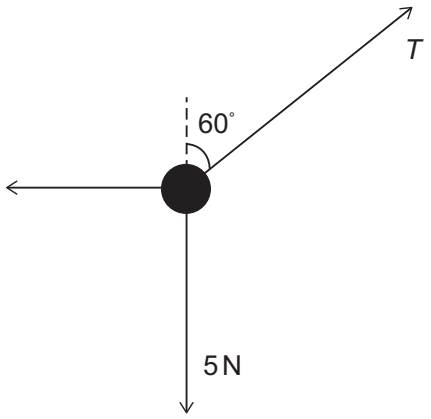
$(a+2)^2 > 0$

This is true for all values of a except -2

88

The answer is option **C**.

You will find it helpful to draw a force diagram to help you to visualise the information given in this question:



T is the tension in the diagonal rope.

By resolving forces in the vertical direction:

$T\cos 60° = 5$

and resolving forces in the horizontal direction:

tension in horizontal rope $= T\sin 60°$

Since $\cos 60°$ is 0.5, we can see that $0.5T = 5$, and so $T = 10$

This means the tension in the horizontal rope $= 10\sin 60° = 10 \times \left(\frac{\sqrt{3}}{2}\right) = 5\sqrt{3}$

89

The answer is option **B**.

$-1 \leq \tan x \leq 1$ is satisfied when $0 \leq x \leq \frac{\pi}{4}$ and $\frac{3\pi}{4} \leq x \leq \pi$

$\sin 2x \geq 0.5$ is satisfied when $\frac{\pi}{6} \leq 2x \leq \frac{5\pi}{6}$ so $\frac{\pi}{12} \leq x \leq \frac{5\pi}{12}$

So both are satisfied together when $\frac{\pi}{12} \leq x \leq \frac{\pi}{4}$

The length of the interval $= \frac{\pi}{6}$

90

The answer is option **A**.

First, consider the whole train:

mass of train $= 20\,000 + 5000 + 5000 = 30\,000$ kg

The only horizontal force acting on the train is the 15 000 N thrust force from the engine, so this is also the resultant force acting on the train.

Acceleration of train is therefore:

resultant force / mass $= \frac{15\,000}{30\,000} = 0.50\,\text{m s}^{-2}$

Now consider carriage 2. Carriage 2 is, in common with the rest of the train, accelerating at $0.50\,\text{m s}^{-2}$

The only horizontal force acting on carriage 2 is the tension T in its coupling with carriage 1.

Therefore, $T =$ mass $\times$ acceleration $= 5000 \times 0.50 = 2500$ N