NSAA 2019
Section 1

Model Solutions

PART A Mathematics

1 Evaluate

$$\left(\sqrt{7}+\sqrt{3}\right)^2-\left(\sqrt{7}-\sqrt{3}\right)^2$$

A 0

B $2\sqrt{7}$

C $4\sqrt{7}$

D $2\sqrt{21}$

E 10

F $4\sqrt{21}$

G 20

Since $a^2-b^2 \equiv (a+b)(a-b)$

$(\sqrt{7}+\sqrt{3})^2-(\sqrt{7}-\sqrt{3})^2 = (\sqrt{7}+\sqrt{3}+\sqrt{7}-\sqrt{3}) \times (\sqrt{7}+\sqrt{3}-\sqrt{7}+\sqrt{3})$

$= 2\sqrt{7} \times 2\sqrt{3}$

$= 4\sqrt{21}$

2 Find the complete set of values of x which satisfy the inequality

$$\frac{1}{2}(3x-2)-\frac{2}{3}(x-4)<x$$

A $x<-22$

B $x>-22$

C $x<-2.5$

D $x>-2.5$

E $x<1.2$

F $x>1.2$

G $x<10$

H $x>10$

$\frac{3}{2}x-1-\frac{2}{3}x+\frac{8}{3} < x$

$\frac{5}{6}x+\frac{5}{3} < x$

$\frac{x}{6} > \frac{5}{3}$

$x > 10$

3 The equation gives y in terms of x:

$$y = 3 - 4\left(1 - \frac{x}{2}\right)^2$$

Which one of the following is a rearrangement for x in terms of y?

A $x = -2 \pm 2\sqrt{\frac{3-y}{4}}$

B $x = -2 \pm 2\sqrt{\frac{4-y}{3}}$

C $x = 1 \pm \sqrt{\frac{3-y}{4}}$

D $x = 1 \pm 2\sqrt{\frac{3-y}{4}}$

E $x = 2 \pm 2\sqrt{\frac{3-y}{4}}$

F $x = 2 \pm 2\sqrt{\frac{4-y}{3}}$

G $x = 2 \pm 2\sqrt{\frac{3+y}{4}}$

$$4\left(1 - \frac{x}{2}\right)^2 = 3 - y$$

$$\left(1 - \frac{x}{2}\right)^2 = \frac{3-y}{4}$$

$$1 - \frac{x}{2} = \pm\sqrt{\frac{3-y}{4}}$$

$$\frac{x}{2} = 1 \pm \sqrt{\frac{3-y}{4}}$$

$$x = 2 \pm 2\sqrt{\frac{3-y}{4}}$$

4 The resistance to the motion of a car is directly proportional to the square of the speed of the car.

$R \propto v^2 \Rightarrow R = kv^2$

The car increases its speed by 20%.

What is the percentage increase in the resistance to the motion of the car?

A 20%

B 24%

C 44%

D 120%

E 224%

F 240%

G 400%

When $v \mapsto 1.2v$

$R \mapsto k(1.2v)^2$

$= 1.44kv^2$

$= 1.44R$

44% increase in R

5 The equation of a curve is $y = px^2 + qx$ where p and q are constants.

The curve passes through the points $(2, 6)$ and $(4, -4)$.

What is the value of $q - p$?

A 1

B 2

C 5

D 6

E 9

F 16

At $(2,6)$:

$6 = 4p + 2q$

$\Rightarrow 2p + q = 3 \quad -(1)$

At $(4,-4)$:

$-4 = 16p + 4q$

$\Rightarrow 4p + q = -1 \quad -(2)$

$(2) - (1)$: $2p = -4$

$\Rightarrow p = -2$, $q = 3 - 2p$

$= 3 - 2(-2) = 7$

$\Rightarrow q - p = 7 + 2$

$= 9$

6 Which of the following is a simplification of

$$4 - \frac{x(3x+1)}{x^2(3x^2-2x-1)}$$

A $\frac{12x^3 - 8x^2 - 7x - 1}{x(3x-1)(x-1)}$

B $\frac{4x^2 + 4x - 1}{x(x+1)}$

C $\frac{4x^2 + 4x + 1}{x(x+1)}$

D $\frac{4x^2 - 4x - 1}{x(x-1)}$

E $\frac{4x^2 - 4x + 1}{x(x-1)}$

F $\frac{12x^3 - 8x^2 - x + 1}{x(3x-1)(x-1)}$

In the denominator,

$3x^2 - 2x - 1 \equiv (3x + 1)(x - 1)$

Considering the whole expression:

$$4 - \frac{x(3x+1)}{x^2(3x+1)(x-1)}$$

$$= 4 - \frac{1}{x(x-1)}$$

$$= \frac{4x(x-1) - 1}{x(x-1)}$$

$$= \frac{4x^2 - 4x - 1}{x(x-1)}$$

7 The ball for a garden game is a solid sphere of volume 192 cm^3.

For the children's version of the game the ball is a solid sphere made of the same material, but the radius is reduced by 25%.

What is the volume, in cm^3, of the children's ball?

A 48

B 81

C 96

D 108

E 144

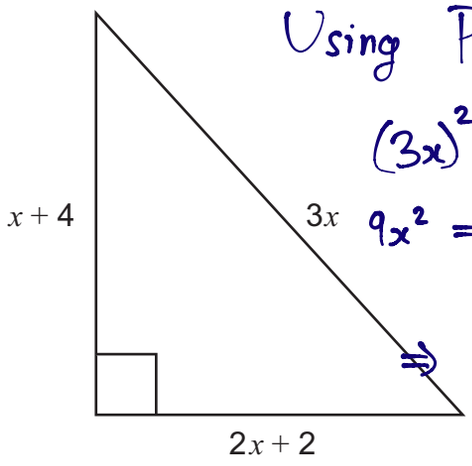
$V = \frac{4}{3}\pi r^3 \Rightarrow V \propto r^3$

$V_{child} = \frac{4}{3}\pi r_{child}^3$

$= \frac{4}{3}\pi\left(\frac{3}{4}r\right)^3 = \left(\frac{3}{4}\right)^3 V$

$= \frac{27}{64} \times 192 = 27 \times 3 = 81\ cm^3$

8 The diagram shows a right-angled triangle, with sides of length $x + 4$, $2x + 2$ and $3x$, all in cm.



[diagram not to scale]

Using Pythagoras Theorem,

$(3x)^2 = (x+4)^2 + (2x+2)^2$

$9x^2 = x^2 + 8x + 16 + 4x^2 + 8x + 4$

$\Rightarrow 4x^2 - 16x - 20 = 0$

$x^2 - 4x - 5 = 0$

$(x - 5)(x + 1) = 0$

$x = -1, 5$

$\hookrightarrow x > 0 \Rightarrow x = 5$

What is the area, in cm^2, of the triangle?

A 10

B 12

C 28

D 36

E 40

F 54

G 70

$\text{Area} = \frac{1}{2}(x+4)(2x+2)$

$= \frac{1}{2}(9)(12)$

$= 9 \times 6$

$= 54\ cm^2$

9 Given that

$$9^{2x-1} \times \frac{1}{27^x} = 81^x$$

what is the value of x?

Convert all bases to 3

$(3^2)^{2x-1} \times \frac{1}{(3^3)^x} = (3^4)^x$

$3^{4x-2} \times 3^{-3x} = 3^{4x}$

$3^{x-2} = 3^{4x}$

Equating exponents: $x - 2 = 4x$

$3x = -2$

$x = \frac{-2}{3}$

A $-\frac{2}{3}$

B $-\frac{2}{5}$

C $-\frac{1}{3}$

D $-\frac{1}{4}$

E $-\frac{1}{5}$

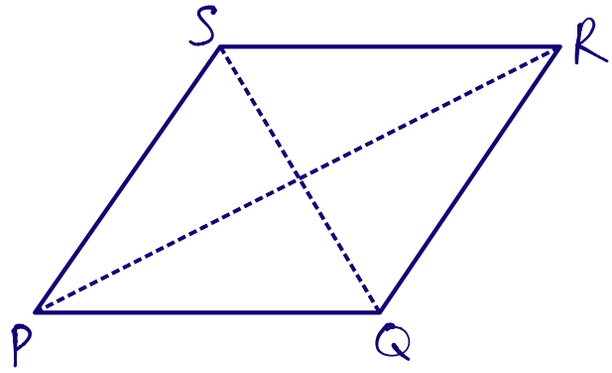
10 PR and QS are the diagonals of a rhombus $PQRS$.

$PR = (3x + 2)$ cm

$QS = (8 - 2x)$ cm

The area of $PQRS$ is 11 cm^2.

What is the difference, in cm, between the two possible lengths of PR?



A $2\frac{2}{3}$

B $4\frac{1}{2}$

C $5\frac{1}{3}$

D 8

E 14

Area of Rhombus $= \frac{\text{Product of diagonals}}{2}$

$\frac{(3x+2)(8-2x)}{2} = 11$

$12x - 3x^2 + 8 - 2x = 11$

$3x^2 - 10x + 3 = 0$

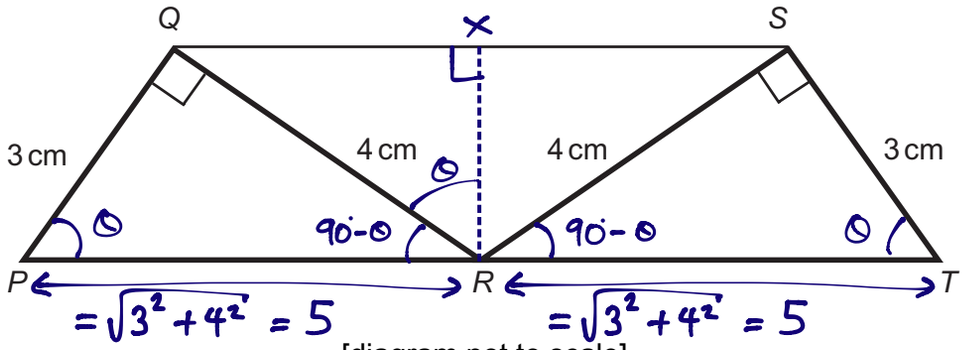
$(3x - 1)(x - 3) = 0 \Rightarrow x = 1/3, 3$

Possible PR lengths: $3(3) + 2 = 11$

$3(1/3) + 2 = 3$

$11 - 3 = 8$ cm difference

11



[diagram not to scale]

The diagram shows two congruent right-angled triangles *PQR* and *TSR* with right angles at *Q* and *S*, respectively.

PQ = *TS* = 3 cm

QR = *SR* = 4 cm

PRT is a straight line.

What is the length, in cm, of *QS*?

A 4

B $3\sqrt{2}$

C 5.2

D $4\sqrt{2}$

E 6.4

F 8.2

G 10

$\angle QRS = 180° - (90°-\theta) - (90°-\theta)$

$= 2\theta$

From $\triangle PQR$,

$\sin\theta = \frac{4}{5}$

Let X be the midpoint of QS

$QX = QR\sin\theta$

$= 4 \times \frac{4}{5}$

$= \frac{16}{5} = \frac{32}{10} = 3.2$

$QS = 2QX$

$= \underline{6.4}$

12 The total of three numbers p, q and r is 375 $\quad p+q+r=375$

The ratio $p:q$ is 5:7 $\equiv \quad 5\times4 : 7\times4 = 20:28$

The ratio $q:r$ is 4:11 $\equiv \quad 4\times7 : 11\times7 = 28:77$

What is the value of $p+r$? $\quad \therefore\ p:q:r = 20:28:77$

A 16

B 60

C 97

D 125

E 144

F 231

G 291

H 315

So if x is some number,

$p+q+r = 20x + 28x + 77x = 375$

Since the ratios tell us p, q and r will always be multiples of this ratio.

$\Rightarrow 125x = 375$

$x = 3$

$\Rightarrow \quad p = 20\times3 = 60$

$q = 28\times3 = 84$

$r = 77\times3 = 231$

$\therefore \quad p+r = 60+231 = 291$

13 The straight line P has equation $3y - 2x = 12$ and intercepts the y-axis at the point $(0, p)$.

The straight line Q is parallel to P, passes through the point $(6, -1)$ and intercepts the y-axis at the point $(0, q)$.

What is the value of $p - q$?

A −9

B −7

C 1

D 9

E 14

F 17

$P: \quad y = \frac{2}{3}x + 4$

↳ y-intercept = $p = 4$

$Q: \quad y = \frac{2}{3}x + q$

Sub in $(6, -1)$

$-1 = \frac{2}{3}\times6 + q \Rightarrow q = -5$

$p - q = 4 + 5 = 9$

14 The vertices of a rectangle have coordinates:

$P(4,5)$ $Q(4,8)$ $R(10,8)$ $S(10,5)$

$PQRS$ is transformed by a clockwise rotation of 90° about P followed by a reflection in the x-axis.

What are the coordinates of the final position of R?

A (–8, –10)

B (–7, –1)

C (–4, 1)

D (–1, 11)

E (1, –11)

F (4, –1)

G (7, 1)

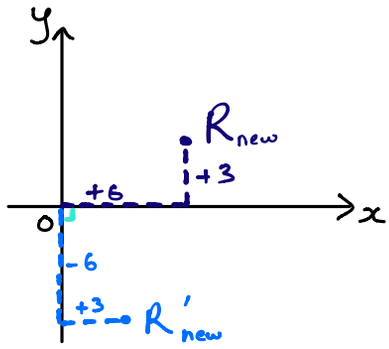
H (8, 10)

Move the rectangle so P is on the origin

$P_{new} = (0,0)$

$R_{new} = \begin{pmatrix}10\\8\end{pmatrix} - \begin{pmatrix}4\\5\end{pmatrix} = (6, 3)$

Rotate R_{new} 90° ↻



$\therefore R'_{new} = (3, -6)$

Translate R'_{new} by moving Rectangle so P is not origin,

$\begin{pmatrix}3\\-6\end{pmatrix} + \begin{pmatrix}4\\5\end{pmatrix} = (7,1)$

ALTERNATIVE METHOD if you are familiar with Linear Transforms

R_{final} = Move Rect. so P is origin THEN Rotate Rect THEN Move Rect so it looks like P was never at origin

$= \begin{pmatrix}4\\5\end{pmatrix} + \left(\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix} \left(\vec{R} - \begin{pmatrix}4\\5\end{pmatrix} \right) \right) = \begin{pmatrix}7\\1\end{pmatrix}$

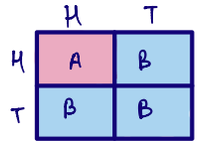
15 Box A contains exactly 10 balls, of which 6 are red and 4 are blue.

Box B contains exactly 15 balls, of which 3 are red and 12 are blue.

All the balls are identical in every respect, apart from colour.

One of the two boxes is chosen at random by tossing two fair coins, as follows:

"If **both** coins show heads, box A is selected. Otherwise box B is selected."



One ball is then randomly taken from the selected box.

What is the probability that a red ball is taken?

A $\frac{9}{400}$

B $\frac{3}{25}$

C $\frac{3}{10}$

D $\frac{2}{5}$

E $\frac{1}{2}$

F $\frac{4}{5}$

G $\frac{323}{400}$

$P(\text{red}) = P(\text{red} \mid A \text{ OR red} \mid B)$

$= P(\text{red} \mid A) + P(\text{red} \mid B)$

$= P(\text{red AND } A) + P(\text{red AND } B)$

$= P(A) \times P(\text{red from } A) + P(B) \times P(\text{red from } B)$

$= P(HH) \times P(\text{red from } A) + P(\overset{\text{NOT}}{HH}) \times P(\text{red from } B)$

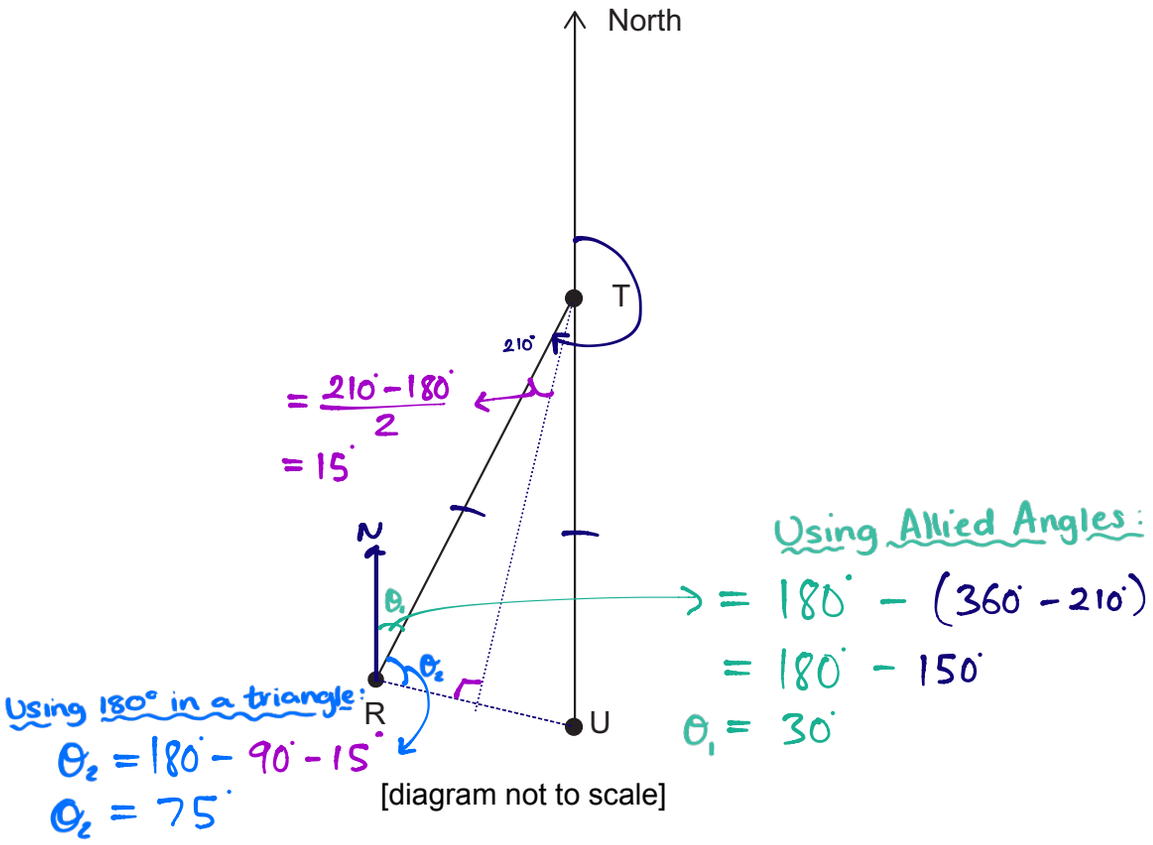
$= \frac{1}{4} \times \frac{6}{10} + \frac{3}{4} \times \frac{3}{15}$

$= \frac{6}{40} + \frac{9}{60}$

$= \frac{9+9}{60}$

$= \frac{18}{60} = \frac{3}{10}$

16 Three towns Ryeton, Tonbridge and Uphampton are represented on the diagram by the points labelled R, T and U, respectively.



[diagram not to scale]

The distance from Tonbridge to Ryeton is the same as the distance from Tonbridge to Uphampton.

Uphampton is south of Tonbridge.

Ryeton is on a bearing of 210° from Tonbridge.

What is the bearing of Uphampton from Ryeton?

A 030°

B 075°

C 105°

D 150°

E 300°

F 345°

Bearing required = $\theta_1 + \theta_2$
= 30° + 75°
= 105°

x_1, x_2, x_3, x_4, x_5

17 A list of five numbers has mean x, median y and range z.

x_6

A sixth number is added to the list. This sixth number is greater than x.

Which of the following statements **must** be true?

1 The median of the six numbers cannot be one of the numbers in the list.

2 The mean of the six numbers is greater than x.

3 The range of the six numbers is greater than z.

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

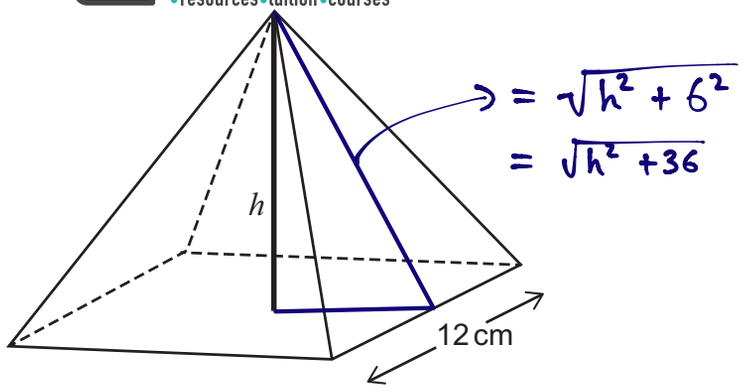
H 1, 2 and 3

1) FALSE : It can if x_6 is bigger than all the others

2) TRUE . $x_6 >$ old mean, so x_6 will definitely raise the mean

3) FALSE : The range would only increase if x_6 was bigger than the largest of the original 5

18



$= \sqrt{h^2 + 6^2}$

$= \sqrt{h^2 + 36}$

[diagram not to scale]

A solid pyramid has a square base of side length 12 cm and a vertical height of h cm.

The volume of the pyramid, in cm^3, is equal to the total surface area of the pyramid, in cm^2.

What is the value of h?

(volume of pyramid = $\frac{1}{3}$ × area of base × vertical height)

A $\frac{72}{35}$

B $2\sqrt{3}$

C 6

D $\frac{144}{23}$

E 8

F $2\sqrt{21}$

$V = S$

$\frac{1}{3} \times 12^2 \times h = 12^2 + 4(\text{Area of } \triangle \text{ face})$

$\frac{144}{3}h = 144 + 4\left(\frac{1}{2} \times 12 \times \sqrt{h^2+36}\right)$

$48h = 144 + 24\sqrt{h^2+36}$

$2h - \sqrt{h^2+36} = \frac{144}{24} = 6$

$h^2 + 36 = (2h-6)^2 = 4h^2 - 24h + 36$

$h^2 = 4h^2 - 24h$

$3h(h-8) = 0$

$h = 0$ ↓ No, since $h > 0$

$h - 8 = 0$

$h = 8$

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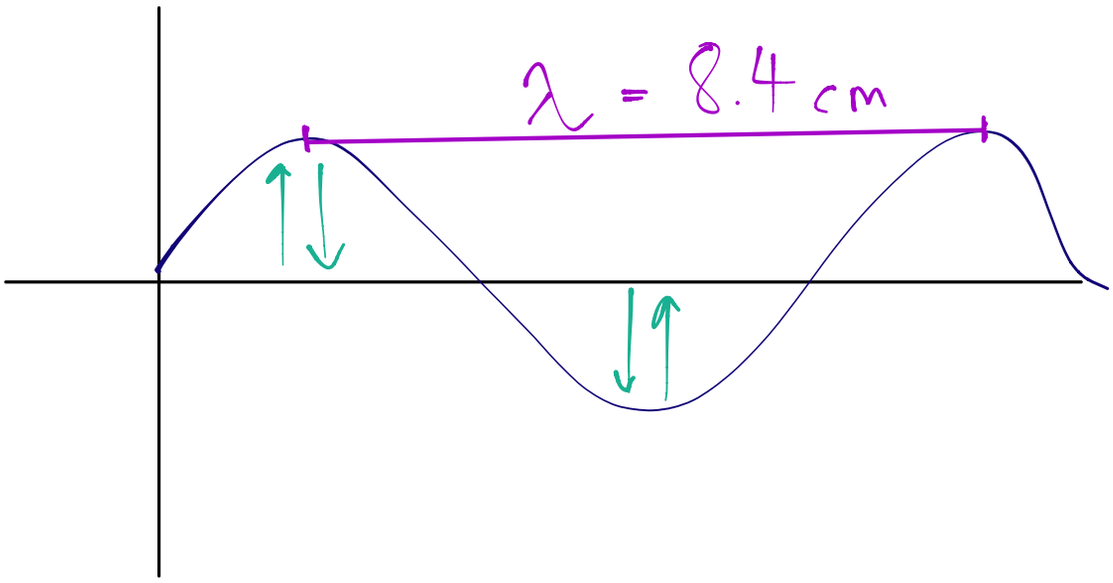
PART B Physics

19 A transverse wave is travelling through a medium. The distance between successive wave peaks is 8.4 cm and the total distance travelled by a particle during one complete oscillation is 5.6 cm.

What is the amplitude and wavelength of the wave?

	amplitude / cm	*wavelength* / cm
A	1.4	4.2
B	1.4	8.4
C	2.1	2.8
D	2.1	5.6
E	2.8	4.2
F	2.8	8.4
G	4.2	2.8
H	4.2	5.6

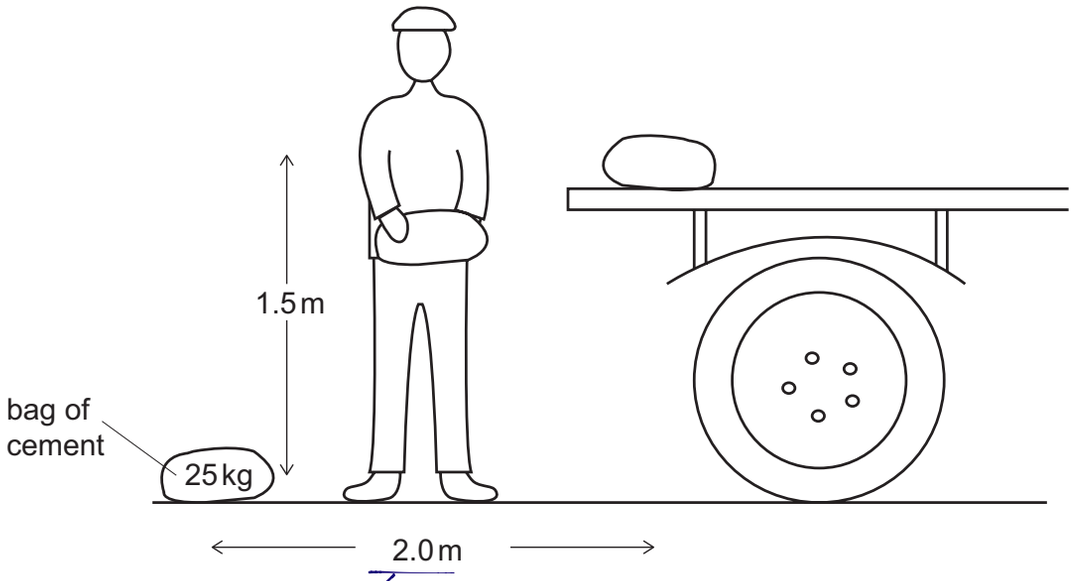


In 1 full oscillation,

$4A = 5.6$

$A = 1.4$ cm

20 A builder lifts bags of cement onto the back of a lorry. Each bag has a mass of 25 kg. It takes the builder 2.5 minutes to load ten bags.



[diagram not to scale]

Irrelevant as work is done against gravity

What are the total work done, T, on the ten bags and the average power required for T?

(gravitational field strength = 10 N kg^{-1})

	total work done T / J	*average power* / W
A	375	2.5
B	375	150
C	625	4.2
D	625	250
E	3750	25
F	3750	1500
G	6250	42
H	6250	2500

# of bags, Upwards Force on each bag, Upwards displacement of each

$T = n \times F \times d$

$= 10 \times 25g \times 1.5$

$= 10 \times 250 \times 1.5$

$= 2500 \times \frac{3}{2}$

$= 3750$ J

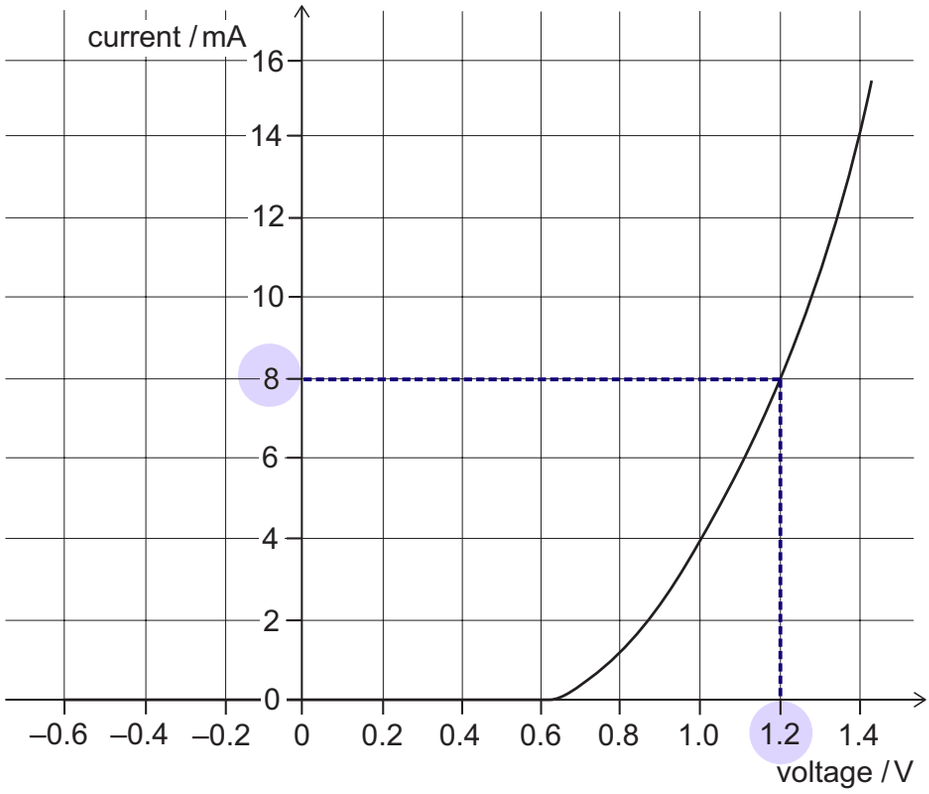
$W = \frac{T}{\Delta t} = \frac{3750}{2.5 \times 60}$

$= \frac{3750}{\frac{5}{2} \times 60}$

$= \frac{3750}{150} = 25$ W

$15 \overline{)375}$ → 25, 30, 75, 75, 0

21 The current–voltage graph for a diode is shown.



The diode is connected in series with a resistor and a 6.0 V battery. The current in the circuit is 8.0 mA.

What is the resistance of the resistor?

(Assume that the battery has negligible resistance.)

A 0.15 Ω

B 0.60 Ω

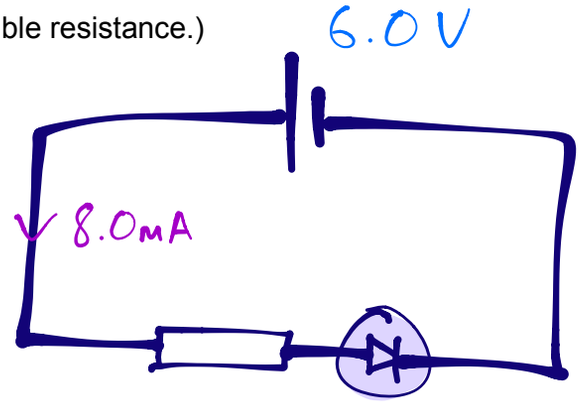
C 0.75 Ω

D 4.8 Ω

E 150 Ω

F 600 Ω

G 750 Ω



From the IV characteristic, there is a p.d. of 1.2V across diode

$\Rightarrow$ 6.0 − 1.2 = 4.8 V across resistor

Using Ohm's Law on Resistor,

$$R = \frac{V}{I} = \frac{4.8\text{V}}{8.0\text{mA}} = \frac{0.6}{0.001} = 600\,\Omega$$

22 Two electromagnetic waves P and Q travel in a vacuum and the ratio of their wavelengths is:

$c = f_P \lambda_P = f_Q \lambda_Q$

$\Rightarrow \frac{\lambda_P}{\lambda_Q} = \frac{f_Q}{f_P}$

$$\frac{\text{wavelength of P}}{\text{wavelength of Q}} = 1.0 \times 10^{8} = \frac{f_Q}{f_P} \Rightarrow \frac{f_P}{f_Q} = 1.0 \times 10^{-8}$$

Which row in the table shows the ratio of their speeds, the ratio of their frequencies, and identifies the possible natures of P and Q?

Both EM waves have same speed

Q has a higher frequency and must be MORE energetic

	$\frac{\text{speed of P}}{\text{speed of Q}}$	$\frac{\text{frequency of P}}{\text{frequency of Q}}$	*nature of* P	*nature of* Q
A	1.0	1.0×10^{-8}	microwave	X-ray
B	1.0	1.0×10^{-8}	microwave	radio wave
C	1.0	1.0×10^{8}	infrared	ultraviolet
D	1.0	1.0×10^{8}	visible light	infrared
E	1.0×10^{8}	1.0	gamma	X-ray
F	1.0×10^{8}	1.0	gamma	infrared
G	1.0×10^{8}	1.0×10^{16}	infrared	radio wave
H	1.0×10^{8}	1.0×10^{16}	visible light	ultraviolet

23 A block of aluminium of mass 0.80 kg, initially at a temperature of −21 °C, is supplied with 54 000 J of thermal energy.

The specific heat capacity of aluminium is 900 J kg^{-1} °C^{-1}.

What is the final temperature of the block?

(Assume that there is no other transfer of energy between the block and the surroundings.)

A 27 °C

B 39 °C

C 54 °C

D 75 °C

E 96 °C

$Q = mc\Delta T$

$54000 = 0.80 \times 900 \times \Delta T$

$\Delta T = \frac{54000}{720}$

$= \frac{9 \times 600 \times 10}{9 \times 8 \times 10}$

$= 75\ °C$

$T_{final} = T_{initial} + \Delta T$

$= -21 + 75 = 54\ °C$

24 A light spring is used to support a range of loads.

The spring obeys Hooke's law. The system is in equilibrium.

Which of the following statements is/are correct?

1 The tension in the spring is directly proportional to the length of the spring.

2 The tension in the spring and the weight of the load it supports are a Newton's third law pair of forces.

3 When the extension of the spring is doubled, the energy stored in the spring increases by a factor of four.

A none of them

B 1 only

C 2 only

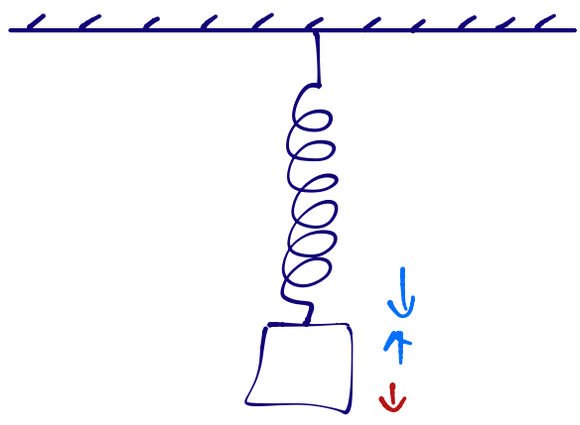
D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3



1) False : Hooke's Law says Tension $\propto$ EXTENSION

2) False : Both those forces act on the same object (the load). The actual 3rd law pairs are:

→ Tension from spring pulling Load up.
&
Downwards force from load pulling the spring down

→ Weight of Load attracted to Earth
&
→ Grav. attraction of Earth towards Load

3) True : Elastic Potential energy in spring = $U_E = \frac{1}{2}kx^2$

25 A water-tight cylinder with a thin, freely moving piston contains $2.0 \times 10^{-3}\,m^3$ of trapped air at atmospheric pressure of $1.0 \times 10^5\,Pa$.

When the cylinder is submerged in water of constant density $1000\,kg\,m^{-3}$, the volume of air in the cylinder decreases to $4.0 \times 10^{-4}\,m^3$.

The piston is at a depth h below the surface of the water and the water surface is open to the atmosphere.

What is the depth h?

(gravitational field strength = $10\,N\,kg^{-1}$; assume that the temperature of the air remains constant and that air is an ideal gas)

A 40 m

B 50 m

C 60 m

D 400 m

E 500 m

F 600 m

Using Boyle's Law:

$P_1V_1 = P_2V_2 \Rightarrow P_2 = \frac{P_1V_1}{V_2} = \frac{1.0\times10^5\,Pa \times 2.0\times10^{-3}\,m^3}{4.0\times10^{-4}\,m^3}$

$= \frac{2}{4} \times \frac{10^2}{10^{-4}} = 5\times10^5\,Pa$

pressure due to water = increase in pressure

$h\rho g = P_2 - P_1$

$h \times 1000 \times 10 = 5\times10^5 - 1\times10^5$

$h = \frac{4\times10^5}{10^4} = 40\,m$

26 The secondary coil of an ideal, 100% efficient transformer is connected to a resistor by cables of total resistance $1500\,\Omega$. The current in the primary coil is 4.0 A. There are 240 turns in the primary coil and 4800 turns in the secondary coil.

What is the power produced as heat in the cables?

A 60 W

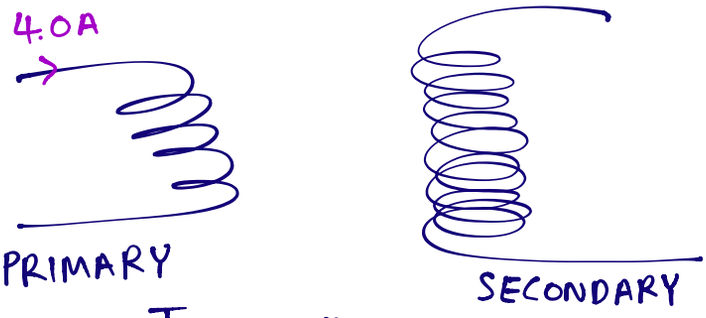
B 300 W

C 6000 W

D 24 000 W

E 120 000 W

F 9 600 000 W



$\frac{I_s}{I_p} = \frac{n_p}{n_s}$

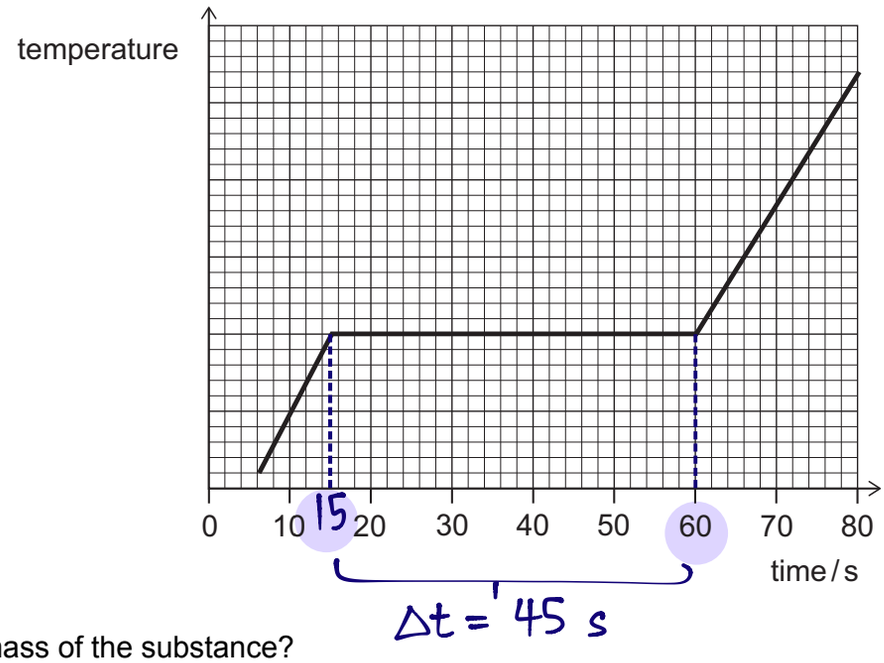
$I_s = \frac{I_p n_p}{n_s} = \frac{4 \times 240}{4800} = \frac{96}{480} = \frac{1}{5} = 0.2$

$P_{Secondary} = I^2R$

$= 0.2^2 \times 1500 = \frac{1500}{25} = \frac{300}{5} = 60\,W$

27 Heat is supplied to an initially solid substance at a rate of 60 W.

The graph shows the variation of the temperature of the substance with time.



What is the mass of the substance?

(specific latent heat of fusion of substance = 100 J g^{-1}; assume that there is no heat transferred to the surroundings)

L_f

A 0.013 g

B 0.60 g

C 3.0 g

D 9.0 g

E 27 g

F 36 g

$$Q = mL_f$$

$$P\Delta t = mL_f$$

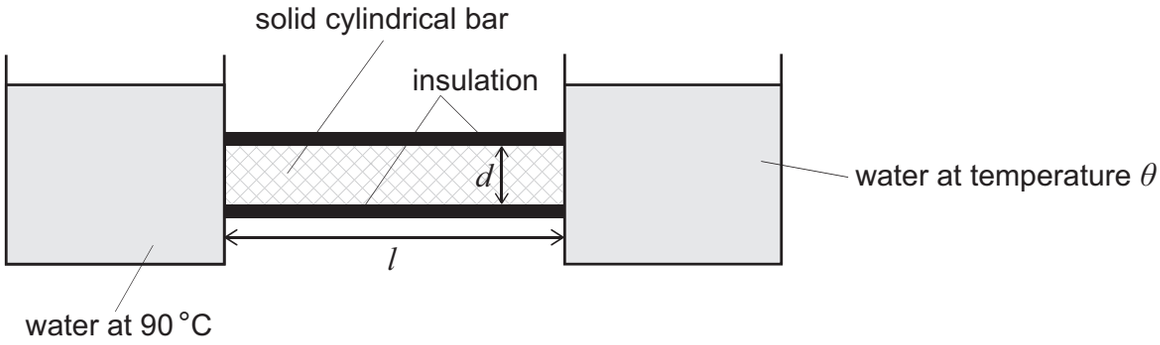
$$m = \frac{P\Delta t}{L_f} = \frac{60 \times 45}{100}$$

$$= 27\ g$$

28 Two tanks of water are connected by a solid cylindrical copper bar of length l and diameter d.

The bar is insulated.

One tank contains water at 90 °C and the other tank contains water at temperature θ.



For which of the following conditions is thermal energy conducted along the bar at the lowest rate?

	l / m	d / cm	θ / °C
A	0.40	4.0	20
B	0.40	4.0	40
C	0.40	8.0	20
D	0.40	8.0	40
E	0.80	4.0	20
F	0.80	4.0	40
G	0.80	8.0	20
H	0.80	8.0	40

Bigger l ⇒ longer distance for water to travel
Slower Heat transfer

Smaller d ⇒ Smaller cross section, less hot water can flow through the tube each second
Slower Heat transfer

Larger θ ⇒ Shallower Temperature gradient
Slower Heat transfer

29 A U-shaped permanent magnet rests on a balance.

A straight, horizontal wire of length 5.0 cm is fixed in position between the poles of the magnet, perpendicular to the horizontal magnetic field.

There is a current of 2.0 A in the wire and the reading on the balance is 202 g.

When the direction of the 2.0 A current is reversed, the reading changes to 198 g.

What is the strength of the magnetic field?

(gravitational field strength = 10 N kg^{-1})

A 0.020 T

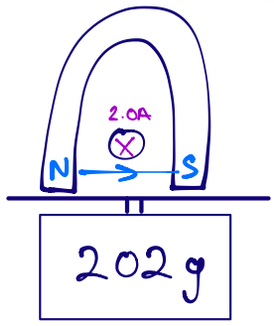
B 0.040 T

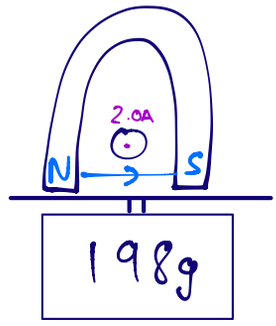
C 0.20 T

D 0.40 T

E 200 T

F 400 T





By Fleming's Left Hand Rule, wire experiences ↓ force when current goes out of page. By Newton's 3rd law, this means the magnet experiences an ↑ force and appears lighter

Since Force changes mass by ±2 g, the magnitude of the force = 0.002 kg × 10 N kg^{-1} = 0.02 N

$F = BlI \Rightarrow B = \frac{0.02N}{0.05m \times 2.0A} = \frac{0.01}{0.05} = 0.2T$

$B = \frac{F}{lI}$

30 The radioactive isotope X becomes the stable isotope Y after a succession of decays involving only the emission of alpha and beta (β^-) particles.

During the decay of one nucleus from X to Y, a total of seven particles are emitted. It is known that more of these particles are alpha particles than beta particles.

The atomic number of X is *Z* and the mass number of X is *A*. $^{A}_{Z}X$

Which row in the table could give the atomic number and the mass number of Y?

	atomic number of Y	*mass number of* Y
A	*Z* – 2	*A* – 12
B	*Z* – 5	*A* – 8
C	*Z* – 8	*A* – 20
D	*Z* – 10	*A* – 24
E	*Z* – 11	*A* – 16

Each α emission decreases A by 4 and Z by 2
Each β⁻ emission increases both Z by 1

After n α emissions and m β⁻ emissions,

$Z \longmapsto Z + m - 2n$

$A \longmapsto A - 4n$

Question says $m + n = 7$

$\Rightarrow m = 7 - n$

$\therefore Z \longmapsto Z + 7 - 3n$

$A \longmapsto A - 4n$

$n > m \Rightarrow n = 4, 5, 6$

when $n = 5$

$Z \longmapsto Z - 8$

$A \longmapsto A - 20$

(option C)

THE OTHER POSSIBILITIES AREN'T OPTIONS

31 The kinetic energy of an object of mass 4.0 kg, travelling in a straight line, increases from 32 J to 200 J in 3.0 seconds due to a constant resultant force.

What is the value of this resultant force?

A 2.0 N

B 4.0 N

C 8.0 N

D 24 N

E 28 N

F 56 N

Initially,
$E_K = \frac{1}{2} m v_i^2 = 32$
$v_i = \sqrt{\frac{32 \times 2}{4}} = 4 \ ms^{-1}$

After 3.0 s,
$E_K = \frac{1}{2} m v_f^2 = 200$
$v_f = \sqrt{\frac{200 \times 2}{4}}$
$= 10 \ ms^{-1}$

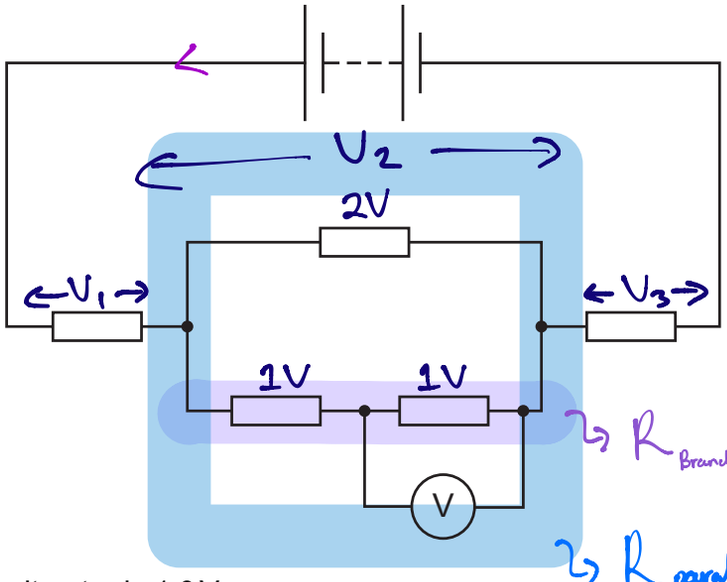
$\Delta v = 10 - 4 = 6$

Using Newton's 2nd Law,

$F = ma = m \frac{\Delta v}{\Delta t}$

$= \frac{4 \times 6}{3} = 8.0N$

32 In the following circuit, all five resistors have the same resistance.



$\rightarrow R_{Branch} = 2R$

$\rightarrow R_{parallel} = \frac{1}{\frac{1}{R} + \frac{1}{2R}}$

$= \frac{2}{3} R$

The reading on the voltmeter is 1.0 V.

What is the voltage across the battery?

A 4.0 V

B 5.0 V

C 6.0 V

D 7.0 V

E 8.0 V

F 9.0 V

G 10 V

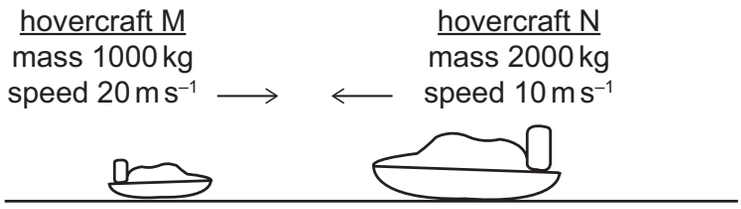
$V_{parallel} = 2.0V = V_2$

Since parallel section has $\frac{2}{3}$ x the resistance of its neighboring resistors, it must experience $\frac{3}{2}$ x the voltage as V_1 and V_2 to have the same current ($V = IR$)

$V_1 = V_3 = \frac{3}{2} V_2 = \frac{3}{2} \times 2 = 3$

$V_{tot} = 3 + 3 + 2 = 8.0V$

33 Two hovercraft travel horizontally in opposite directions along the same straight line. The mass and speed of each hovercraft are shown in the diagram. Horizontal resistive forces acting on each hovercraft are negligible.



The hovercraft collide and stick together. The collision lasts for 0.10 s.

Just before the collision, what is the total kinetic energy and the magnitude of the total momentum of the two hovercraft, and what is the magnitude of the average force acting horizontally on each hovercraft during the collision?

	total initial kinetic energy / kJ	*total initial momentum* / kg m s^{-1}	*average force on each hovercraft* / kN
A	100	0	2.0
B	100	0	200
C	100	4.0×10^4	2.0
D	100	4.0×10^4	200
E	300	0	2.0
F	300	0	200
G	300	4.0×10^4	2.0
H	300	4.0×10^4	200

Total KE before collision $= \frac{1}{2} m_M v_M^2 + \frac{1}{2} m_N v_N^2 = \frac{1}{2} \times 1000 \times 20^2 + \frac{1}{2} \times 2000 \times 10^2$

$= 200000 + 100000$

$= 300$ kJ

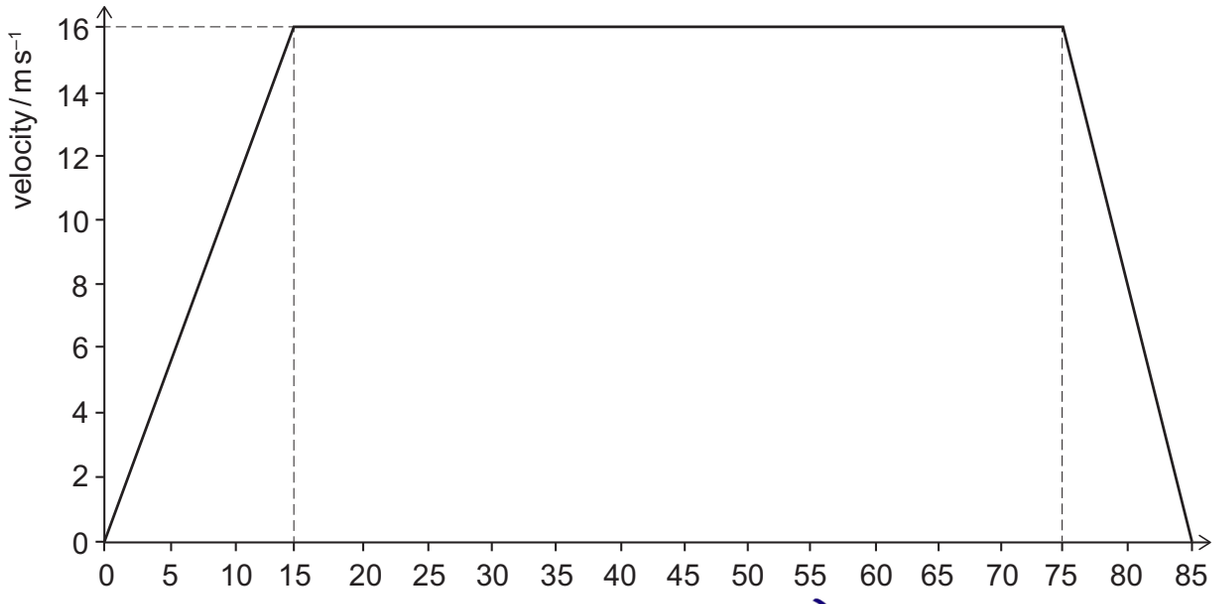
Total Initial momentum $= 1000 \times 20 + 2000 \times -10$

$= 0$ kg ms^{-1}

For a single Hovercraft M, $F = \frac{\Delta p}{\Delta t} = \frac{20000}{0.10} = 2.0 \times 10^5$ N

$= 200$ kN

34 The graph shows how a car's velocity changes in 85 seconds.



(when graph is flat / gradient = 0)

What proportion of the total distance is travelled at constant velocity?

A $\frac{5}{29}$

B $\frac{5}{17}$

C $\frac{12}{17}$

D $\frac{24}{29}$

E $\frac{8}{9}$

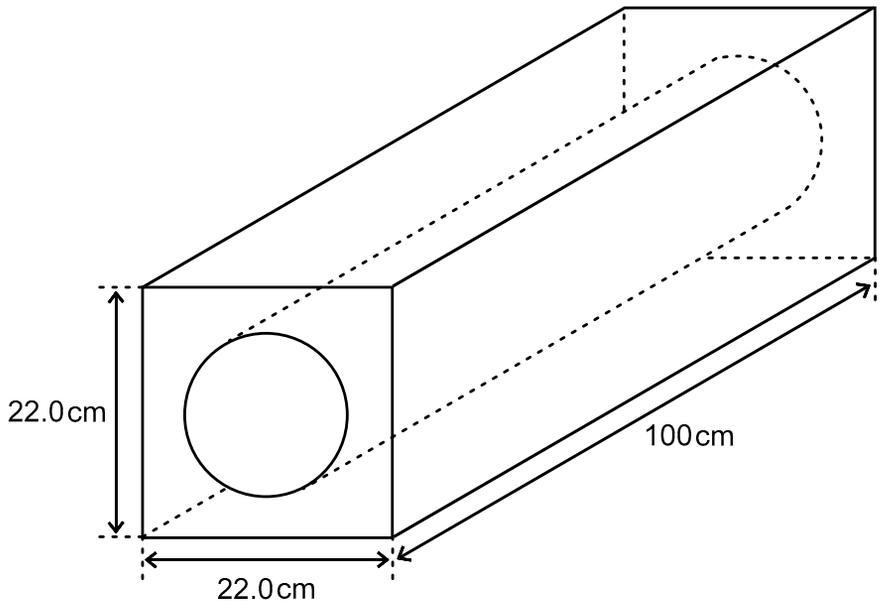
F $\frac{16}{17}$

Dist. travelled at const. velocity $= 16 \times 60$
$= 960\ m$

Total Dist. travelled $= \frac{1}{2}(60 + 85) \times 16$
$= 145 \times 8$
$= 1160$

Proportion $= \frac{960}{1160} = \frac{96}{116} = \frac{48}{58}$
$= \frac{24}{29}$

35 A block is designed with a cylindrical channel to accommodate a hot-water pipe. The block is 100 cm long and it has a square cross-section of side 22.0 cm with a cylindrical hole in the middle, as shown in the diagram:



[diagram not to scale]

The diameter of the cylindrical hole is 14.0 cm and the density of the material from which the block is made is 0.100 g cm^{-3}.

What is the mass of the block?

(take π to be $\frac{22}{7}$)

A 1.32 kg

B 3.30 kg

C 13.2 kg

D 33.0 kg

E 132 kg

F 330 kg

G 1320 kg

H 3300 kg

Volume of block = Volume of cuboid − Volume of Cylinder

$= 22 \times 22 \times 100 - \pi \times 7^2 \times 100$

$= 100\left(22 \times 22 - \frac{22}{7} \times 7^2\right)$

$= 100\,(22 \times 22 - 22 \times 7)$

$= 2200\,(22 - 7)$

$= 2200 \times 15$

$= 22000 + 11000$

$= 33000\ cm^3$

Density x volume = Mass

Mass of block $= 33000\ cm^3 \times 0.1\ g cm^{-3}$

$= 3300\ g$

$= 3.3\ kg$

36 A sample initially contains equal numbers of atoms of a radioactive isotope X and a stable isotope Y.

Isotope X has a half-life of 3 years and decays in a single stage to the stable isotope Y.

What is the ratio

number of atoms of X : *number of atoms of* Y

in the sample 6 years later?

A The sample contains only isotope Y.

B 1 : 7

C 1 : 4

D 1 : 3

E 7 : 4

Initially,

$X : Y \equiv X : X = 1 : 1$

After 3 years (One half life)

$\frac{X}{2} : Y + \frac{X}{2} \equiv \frac{X}{2} : \frac{3}{2}X = 1 : 3$

After 3 more years, (The 2nd half life)

$\frac{X}{4} : Y + \frac{X}{2} + \frac{X}{4} \equiv \frac{X}{4} : \frac{7}{4}X = \underline{1 : 7}$

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PART C Chemistry

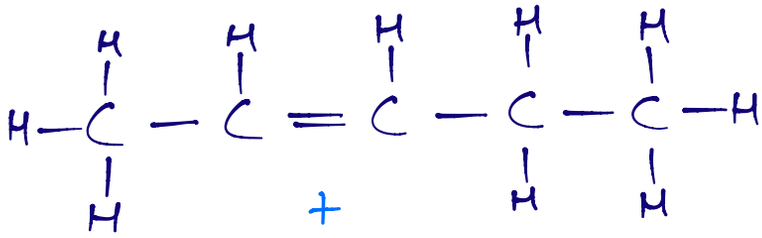
37 HBr reacts with pent-2-ene in an addition reaction.

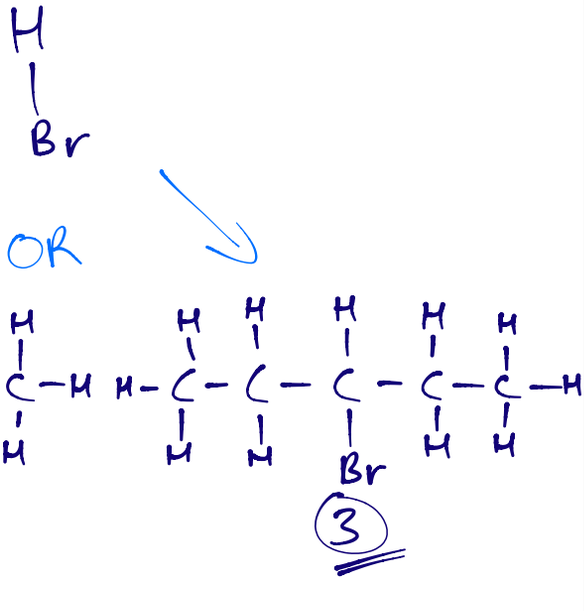
Which of the following products is/are formed in the reaction?

✓ **1** $CH_3CHBrCH_2CH_2CH_3$

✗ **2** $CH_2BrCH_2CH_2CH_2CH_3$

✓ **3** $CH_3CH_2CHBrCH_2CH_3$





A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

38 In which of the following solids does the bonding consist of single covalent bonds **only**?

✗ **1** graphite → Giant Covalent with weak intermolecular force between layers

✓ **2** SiO_2 → Giant Covalent

✗ **3** Al_2O_3 → Ionic

A none of them

B 1 only

C 2 only

D 3 only

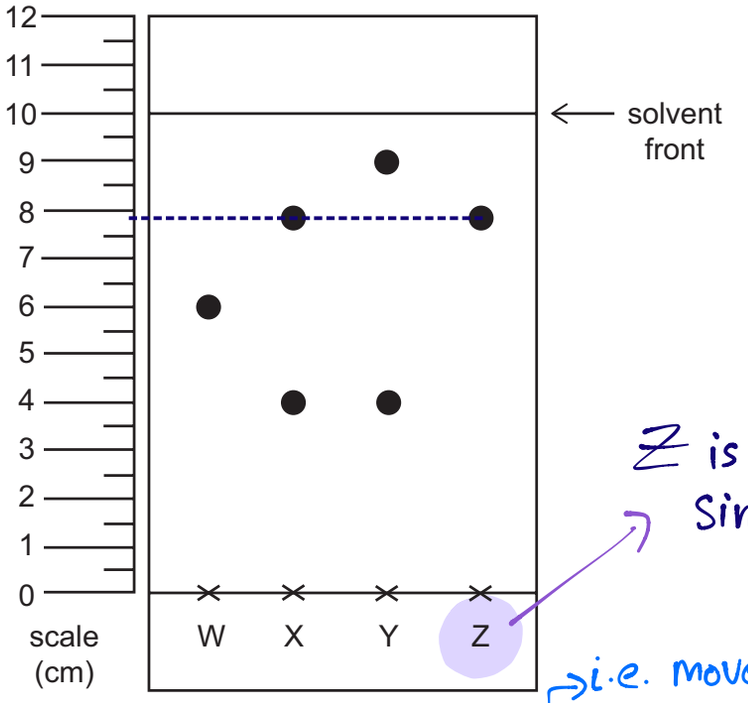
E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

39 Four samples, labelled W, X, Y and Z, were investigated using paper chromatography with a solvent that caused any mixtures present to be fully separated. The results are shown in the chromatogram.



Z is the highest single spot

i.e. moved up the furthest

What is the R_f value of the spot with the strongest attraction to the mobile phase relative to the stationary phase **and** that is from a sample containing only one substance?

produced only one spot

A 0.50

B 0.60

C 0.67

D 0.75

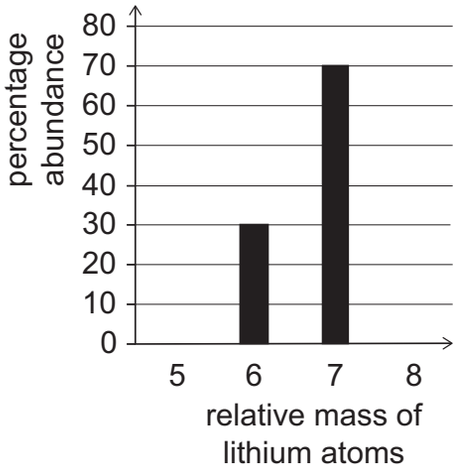
E 0.80

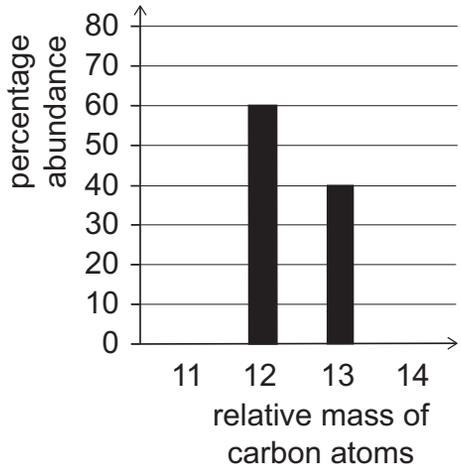
F 0.90

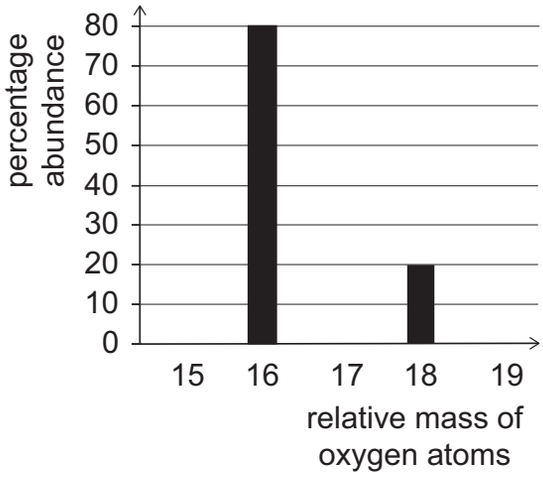
$R_f(Z) \approx \frac{8}{10} = 0.8$

40 A mass spectrometer is a device that can measure the mass of isotopes. It shows this data as a spectrum, giving both the relative mass and the percentage abundance of each isotope.

The charts indicate the relative mass and percentage abundance for lithium atoms, carbon atoms and oxygen atoms found in a sample taken from a nuclear reactor.







Using this data, what is the largest possible relative molar mass of lithium carbonate?

A 35

B 38

C 45

D 67

E 74

F 75

G 81

Li^+ CO_3^{2-}

Formula of Ion: Li_2CO_3

Max Mass (Li) = 7

Max Mass (C) = 13

Max Mass (O) = 18

Max Mass $(Li_2CO_3) = (7 \times 2) + 13 + (18 \times 3)$

$= 14 + 13 + 54 = 81$

41 The following information about metals labelled P, Q, R and S is given.

- Metals P and S can be extracted by electrolysis, but not by reaction with carbon.
- Metals Q and R can be extracted by reaction with carbon.
- Metal S forms positive ions more readily than metal P.
- Metal R reacts with the oxide of metal Q.

What is the order of reactivity of these four metals, starting with the most reactive?

A P, S, Q, R

B P, S, R, Q

C Q, R, P, S

D Q, R, S, P

E R, Q, P, S

F R, Q, S, P

G S, P, Q, R

H S, P, R, Q

42 A simple ion of an element with atomic number x has a mass number of $(2x + 2)$.

The ion has a charge of -2.

How many protons, neutrons and electrons are present in this ion?

	protons	*neutrons*	*electrons*
A	$x - 2$	$x + 4$	$x - 2$
B	$x - 2$	$x + 4$	x
C	$x - 2$	$x + 4$	$x + 2$
D	x	$x + 2$	$x - 2$
E	x	$x + 2$	x
F	x	$x + 2$	$x + 2$

43 A 116 g sample of an oxide of iron contains 84 g of iron.

Which of the following is the empirical formula of this oxide of iron?

(A_r values: O = 16; Fe = 56)

Amount of Fe = $\frac{84\text{ g}}{56\text{ g mol}^{-1}} = \frac{42}{28}$

$= \frac{6}{4} = 1.5$ mol

A FeO

B Fe_2O_2

C Fe_3O_2

D Fe_2O_3

E Fe_3O_4

Amount of O = $\frac{116-84}{16} = \frac{32}{16} = 2$

Molar Ratio = Fe : O :: 1.5 : 2

≡ 3 : 4 ∴ Fe_3O_4

44 Which of the following chemical equations represent(s) a redox reaction?

✓ **1** $2Cu^{2+} + 4I^- \rightarrow 2CuI + I_2$

✗ **2** $Al^{3+} + 3e^- \rightarrow Al$

✗ **3** $2Ag^+ + 2OH^- \rightarrow Ag_2O + H_2O$

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

1) REDOX as $e^- + Cu^{2+} \rightarrow Cu^+$

$2I^- \rightarrow I_2 + 2e^-$

2) No, as only Al is being reduced (gaining e^-)

3) No as oxidation numbers are unchanged

Ag stays +1

O stays −2

H stays +1

45 X is an anhydrous salt of iron containing one type of cation and one type of anion.

Iron Cation

An aqueous solution of X gives a white precipitate when aqueous barium chloride is added in the presence of hydrochloric acid. → +ve Sulphate Test

On adding aqueous sodium hydroxide to an aqueous solution of X, a brown precipitate formed immediately.

+ve Fe^{3+} Test

The relative atomic mass of iron is 56, and its atomic number is 26.

What is the relative molar mass of X?

(A_r values: C = 12; N = 14; O = 16; S = 32; Cl = 35.5; Br = 80)

A 127

B 152

C 162.5

D 208

E 264

F 272

G 360

H 400

Fe^{3+} SO_4^{2-}

$Fe_2(SO_4)_3$

$Mr(Fe_2(SO_4)_3) = 2 \times 56 + 3 \times (32 + 16 \times 4)$

$= 112 + 3 \times 96$

$= 112 + 288$

$= 400$

46 In the electrolysis of dilute sulfuric acid, hydrogen gas is formed at the negative electrode (cathode) and oxygen gas is formed at the positive electrode (anode).

If 100 g of hydrogen gas is formed in the electrolysis of dilute sulfuric acid, what mass of oxygen gas is also formed?

(A_r values: H = 1; O = 16)

A 50 g

B 100 g

C 200 g

D 800 g

E 1600 g

At cathode, $2H^+ + 2e^- \rightarrow H_2$ [REDUCTION]

At anode, $4OH^- \rightarrow 2H_2O + O_2 + 4e^-$ [OXIDATION]

Multiply cathode equn by 2 to balance e^-

Amount of $H_2 = \frac{100\ g}{2\ g\ mol^{-1}} = 50\ mol$

½ mol of O_2 produced for every mole of H_2

⇒ 50 mol of O_2 produced ≡ $50 \times 32 \times \frac{1}{2}$

$= 800\ g$

47 0.005 mol of a chloride of element X was dissolved in water and then reacted with excess silver nitrate solution to form a precipitate of silver chloride, AgCl. This precipitate is the only product of this reaction that contains chlorine.

After filtering, washing and drying, the mass of the precipitate was recorded to be 1.435 g.

Which of the following could be the formula of the chloride of X?

(M_r value: AgCl = 143.5)

amount of AgCl = $\frac{1.435}{143.5}$ = 0.01 mol

A X_5Cl

B X_2Cl

0.005 mol of the Chloride contains 0.01 mol of Cl^- ions

C XCl

D XCl_2 ⇒ Formula of the compound must contain Cl_2

E XCl_5

$\frac{0.01}{0.005} = 2$ XCl_2

48 A chemical equation that represents the reaction of phosphorus with concentrated nitric acid is:

$$P_4 + w\,HNO_3 + H_2O \rightarrow x\,H_3PO_4 + y\,NO + z\,NO_2$$

What is the value of the sum $w + x + y + z$?

A 24

Comparing P ratio ⇒ $x = 4$

This means there are 12 mol of H atoms on RHS

⇒ $w = 12 - 2 = 10$

B 28

C 30

Comparing N ratio ⇒ $w = y + z = 10$

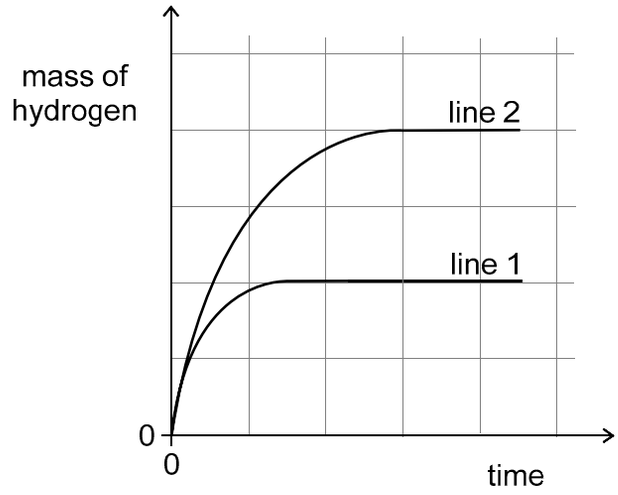
D 32

E 36

$\therefore x + y + w + z = 4 + 10 + 10 = 24$

49 A 2.40 g lump of magnesium was added to 500 cm^3 of a 2.00 mol dm^{-3} solution of HCl in a conical flask that was on an electronic balance. The neck of the flask was plugged with cotton wool, and the decrease in mass of the flask and its contents was recorded at regular intervals.

The mass of the hydrogen released (equal to the mass loss recorded) was plotted against time. The result is line 1 on the graph.



Which of the following experiments performed under the same conditions would give line 2?

(A_r value: Mg = 24.0)

A a 2.40 g lump of magnesium added to 500 cm^3 of 2.00 mol dm^{-3} H_2SO_4

B 2.40 g of magnesium powder added to 500 cm^3 of 2.00 mol dm^{-3} HCl

C a 2.40 g lump of magnesium added to 1000 cm^3 of 2.00 mol dm^{-3} HCl

D a 4.80 g lump of magnesium added to 500 cm^3 of 2.00 mol dm^{-3} HCl

E 4.80 g of magnesium powder added to 500 cm^3 of 2.00 mol dm^{-3} HCl

$$Mg + 2HCl \longrightarrow MgCl_2 + H_2$$

For line 1:

Amount of $Mg = \frac{2.4}{24} = 0.1$ mol Amount of $HCl = 0.5 \times 2 = 1.0$ mol

$\therefore$ Mg is the limiting reagent in this case.

For line 2:

The rate is not faster (reaction does NOT finish quicker), so Mg was NOT powdered.

More H_2 was produced (as line 2 finishes higher than 1)

So MORE Mg must have been used.

50 A technician needs to separate three liquids (X, Y and Z) which have been accidentally mixed together. None of the liquids react with each other.

liquid	X	Y	Z
boiling point / °C	65	51	100
density / g cm^{-3}	0.79	0.68	1.00

X and Z are miscible, but Y is immiscible with both X and Z.

The technician uses a separating funnel to separate the upper and lower layers of the mixture.

What should the technician do next to maximise separation of the three liquids?

A distil the lower layer at 51 °C

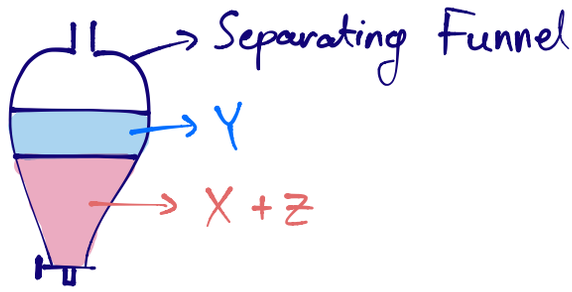
B distil the lower layer at 65 °C

C distil the lower layer at 100 °C

D distil the upper layer at 51 °C

E distil the upper layer at 65 °C

F distil the upper layer at 100 °C



Density (Y) < Density (X), Density (Z)

⇒ Upper layer is Y

∴ Must Distil the LOWER layer at 65°C to obtain X as distillate.

Since B.P. of X is lower, it boils off first.

51 In each of the following procedures an excess of the metal is added to 1.0 dm^3 of a 1.0 mol dm^{-3} solution of the acid.

= 1.0 mol

copper added to sulfuric acid

iron added to hydrochloric acid

magnesium added to sulfuric acid

zinc added to hydrochloric acid

Which row in the following table identifies combinations of metal and acid that will react and produce the largest, and the smallest, theoretical mass of anhydrous salt?

(M_r values: $CuSO_4$ = 160; $FeCl_2$ = 127; $MgSO_4$ = 120; $ZnCl_2$ = 136)

Largest mass of salt occurs when salt has a high M_r. Similarly for lowest

BUT

Cu and Zn are quite unreactive

∴ $FeCl_2$ → Largest M_r

$MgSO_4$ → Lowest M_r

	reaction that produces the largest mass of salt	*reaction that produces the smallest mass of salt*
A	Cu and H_2SO_4	Fe and HCl
B	Cu and H_2SO_4	Mg and H_2SO_4
C	Fe and HCl	Zn and HCl
D	Mg and H_2SO_4	Fe and HCl
E	Mg and H_2SO_4	Zn and HCl
F	Zn and HCl	Mg and H_2SO_4

52 3.4 g of an impure sample of silicon tetrachloride is reacted with water. The mixture is then filtered and the resulting solution made up to 250 cm^3.

$$SiCl_4(l) + 2H_2O(l) \rightarrow SiO_2(s) + 4HCl(aq)$$

12.5 cm^3 of this solution is neutralised exactly by 20.0 cm^3 of 0.100 mol dm^{-3} sodium hydroxide.

= 0.02 × 0.1 = 0.002 mol of NaOH

What is the percentage purity of the silicon tetrachloride?

(M_r value: $SiCl_4$ = 170. Assume that the impurity does not react.)

A 1.7%

B 2.5%

C 10%

D 32%

E 50%

0.002 mol of NaOH reacts with 0.002 mol HCl (1:1 ratio)

12.5 cm^3 contains 0.002 mol

↓ × 20

250 cm^3 contains 0.04 mol of HCl ⇒ $\frac{0.04}{4}$ = 0.01 mol of $SiCl_4$ reacted

Amount of "$SiCl_4$" used = $\frac{3.4}{170} = \frac{2 \times 1.7}{170}$ = 0.02 mol

% purity = $\frac{0.01}{0.02}$ = 50%

53 The Haber process is represented by the following chemical equation:

$$N_2 + 3H_2 \rightarrow 2NH_3$$

What is the overall enthalpy change for the reaction?

(Bond enthalpies: N ≡ N = 945 kJ mol^{-1}; H – H = 435 kJ mol^{-1}; N – H = 390 kJ mol^{-1})

A +90 kJ mol^{-1}

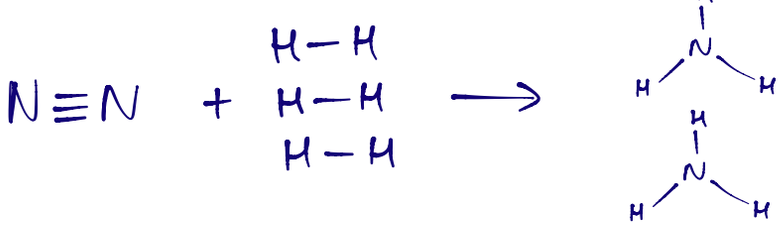
B –90 kJ mol^{-1}

C +990 kJ mol^{-1}

D –990 kJ mol^{-1}

E +1080 kJ mol^{-1}

F –1080 kJ mol^{-1}



Enthalpy Change = Energy taken in to break bonds − Energy released to form bonds

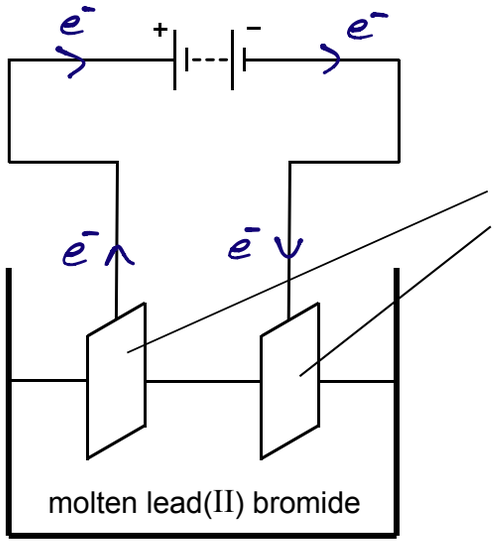
$= (1 \times 945 + 3 \times 435) - (6 \times 390)$

$= (945 + 1305) - (2340)$

$= 2250 - 2340$

$= -90 \text{ kJ mol}^{-1}$

54 The diagram shows the electrolysis of molten lead(II) bromide, $PbBr_2$, using graphite electrodes to separate the compound into its elements.



$Pb^{2+} + 2e^- \longrightarrow Pb$

$2Br^- \longrightarrow Br_2 + 2e^-$

Which of the following statements about this electrolysis is/are correct?

✓ **1** Lead is formed at the negative electrode. Pb^{2+} cations reduced at cathode

✓ **2** Electrons flow through the external circuit away from the positive electrode towards the negative electrode. e^- flow to the +ve battery terminal

✗ **3** Bromine molecules and lead are produced in a 2 : 1 molar ratio.

$Br : Pb = 2:1$, BUT $Br_2 : Pb = 1:1$

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

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PART D Biology

55 Which of the following statements is/are correct for both bacterial cells and sperm cells?

✗ **1** The cell can divide. Sperm cells are haploid and CAN'T DIVIDE FURTHER

✓ **2** The cell contains DNA.

✗ **3** The cell has a cell wall. Sperm cells DO NOT have a cell wall

✗ **4** The cell has a nucleus. Bacteria are prokaryotes

✓ **5** The cell can carry out respiration.

A 2 only

B 4 only

C 1 and 2 only

D 2 and 5 only

E 3 and 4 only

F 3 and 5 only

G 1, 2 and 5 only

H 1, 3 and 4 only

56 Which one of the following statements about cells or tissues is correct?

A Any adult stem cells can naturally give rise to all tissue. ✗ Embryonic

B Endocrine glands secrete enzymes into the bloodstream. ✗ Hormones

C Embryonic stem cells divide by meiosis to form all cell types. ✗ Mitosis

D Receptor cells send chemical impulses along neurones. ✗ Electrical impulses

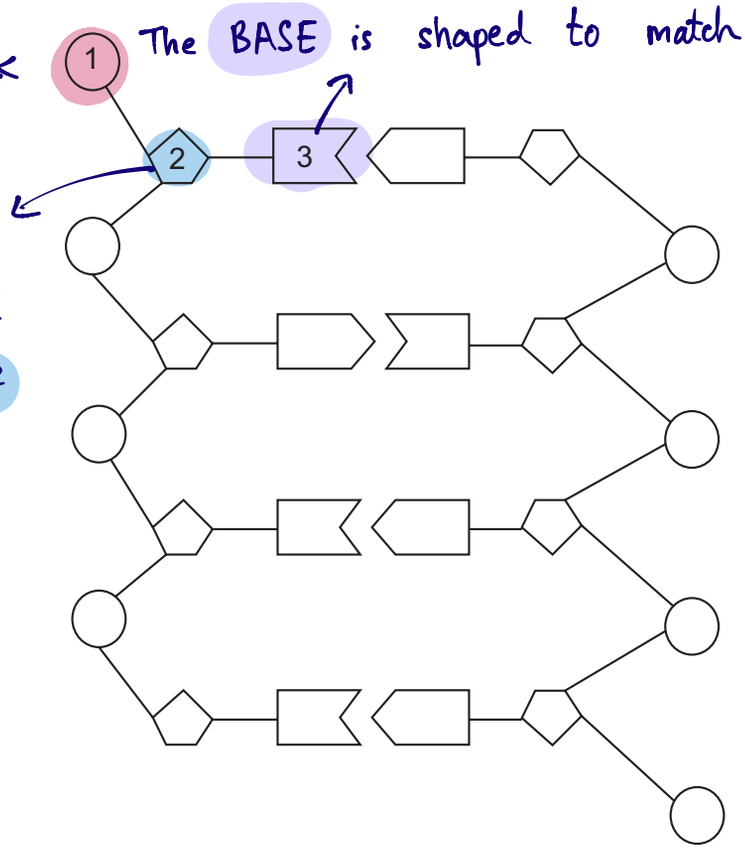
E The stomach wall contains a tissue that enables movement of food. ✓

The stomach does both mechanical AND chemical digestion

57 The diagram represents a section of DNA.

PHOSPHATES link sugars

The BASE is shaped to match its complementary pair



The base is always attatched to a deoxyribose SUGAR

The base, sugar & phosphate together form a NUCLEOTIDE

What do the parts labelled 1, 2 and 3 represent?

	1	*2*	*3*	*1 and 2 and 3*
A	base	phosphate	glycerol	gene
B	base	sugar	amino acid	gene
C	phosphate	sugar	base	nucleotide
D	phosphate	sugar	base	DNA strand
E	sugar	phosphate	base	nucleotide
F	sugar	phosphate	fatty acid	DNA strand

58 Using the information in the table, which animal (**A**-**F**) is most at risk of extinction?

animal	*geographic range*	*habitat tolerance*	*population size*
A	restricted	broad	large
B	extensive	broad	large
C	restricted	narrow	small
D	extensive	narrow	small
E	restricted	broad	small
F	extensive	broad	small

→ restricted geographic range means there are limited regions the species can travel to

→ A narrow Habitat Tolerance means the species is less likely to accept new habitats if needed

→ A small population size lowers the possibility of mutation / diversity within the species, hindering their ability to adapt to change.

All these factors INCREASE risk of extinction.

59 An ecologist used a 50 cm × 50 cm square quadrat to estimate the number of meadow buttercups present in a field with an area of 50 m^2. The quadrat was distributed randomly on ten occasions in the field and the number of buttercups counted in each quadrat.

quadrat	*number of buttercups*
1	3
2	10
3	0
4	4
5	21
6	19
7	6
8	11
9	15
10	3

How many buttercups were there estimated to be in the 50 m^2 field?

A 368

B 460

C 920

D 1840

E 4600

F 18400

G 45000

Average # of buttercups per quadrat $= \frac{3+10+0+4+21+19+6+11+15+3}{10}$

$= \frac{92}{10} = 9.2$

Area of a quadrat $= 0.5\,m \times 0.5\,m = 0.25\,m^2$

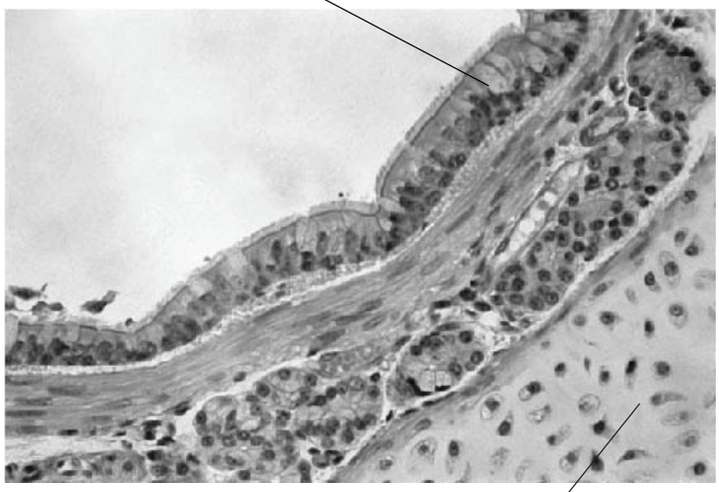
# of quadrats in field $= \frac{50\,m^2}{0.25\,m^2} = 200$

# of buttercups in field $= 200 \times 9.2 = 1840$

60 The photomicrograph shows a section through the wall of a human bronchus, one of the tubes which carries air towards the lungs.

A student studying this structure annotated the photomicrograph by describing the cells found in two different layers in the wall of the bronchus.

Two different types of cells are found in this single layer. Goblet cells, which synthesise and secrete the protein in mucus, and cells with cilia, which sweep mucus that has trapped dust and dirt away from the lungs.



A layer consisting of a group of similar cells. Each cell synthesises and secretes a protein that then surrounds the cells.

The student used these observations to write some conclusions.

Which of the following conclusions is/are correct?

✓ **1** The two different layers are both tissues.

✓ **2** The bronchus can be described as an organ.

✓ **3** Amino acids are found in the cytoplasm of cells in each layer.

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 1, 2 and 3

1) TRUE: Tissues are a group of specialised cells with the same function

2) TRUE: Organs are groups of tissues (like layers 1 and 2) working together to do a specific job

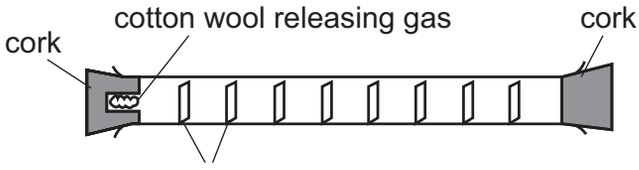
3) TRUE: Animal cell cytoplasms contain amino acids.

61 A student carried out an investigation using the diffusion of a gas along a tube to model movement of gases in the air space of a leaf. An alkaline gas was released from a solution that evaporated from the cotton wool.

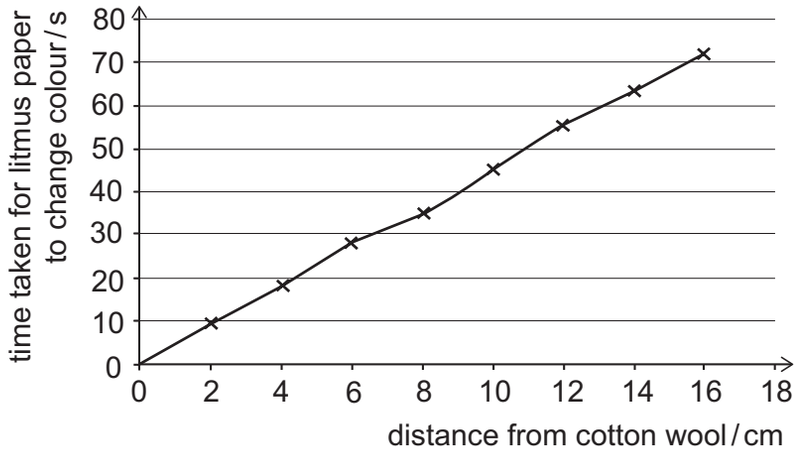
As the gas diffused, it caused damp litmus paper to change colour.

The time was recorded when each piece of litmus paper changed colour.

The apparatus and a graph of the results are shown below.



pieces of damp litmus paper at 2 cm intervals



Which of the following statements about the investigation is/are correct?

1 The dependent variable has been plotted on the x-axis. ✗ The independent variable (i.e. variable we can change) has been plotted on x

2 If diffusion of the gas was slower, the graph line would become steeper. ✓ colour changes would have taken longer

3 If a more concentrated solution was used in an identical set of apparatus, the data collected would result in a line on the graph below the points plotted for the original gas. ✓ More conc. ⇒ Quicker diffusion ⇒ colour changes would have taken LESS time

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

62 A bacterium was genetically modified by inserting a human gene into a plasmid. A plasmid is a small circle of DNA that can be used to transfer genes into bacterial cells. These modified bacteria then secrete the protein that the human gene codes for. The gene contains a sequence of bases that is not repeated anywhere else in its DNA and is not found in the DNA of other organisms. This sequence of bases is found in every allele of this gene.

Assuming that no mutations occur, how many copies of this base sequence would you expect to find at each of the stages X, Y and Z?

Liver cell is a diploid cell, so a homologous pair of chromosomes would each have an identical copy of this gene
⇒ 2 @ X

X one human liver cell containing required gene

↓

one copy of gene removed from human cell

↓

A SINGLE gene is inserted ↙ Y
⇒ 1 @ Y

gene inserted into one bacterial plasmid

↓

one modified plasmid moved into bacterial cell and human protein released

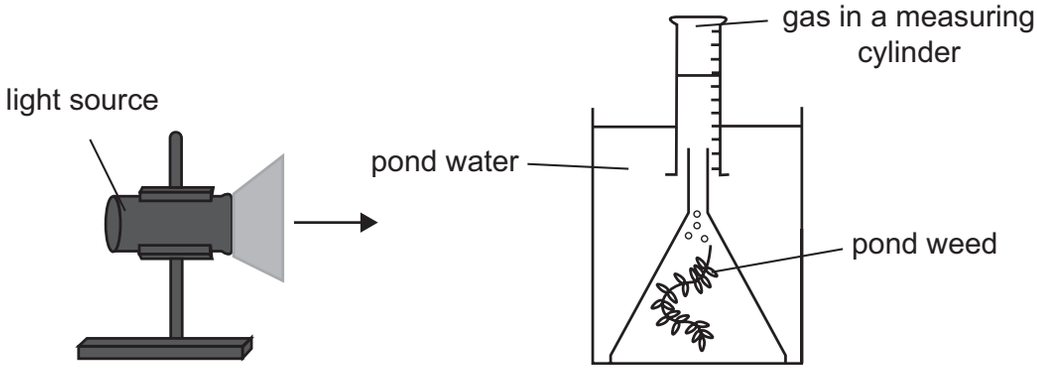
↓

Z purified human protein

The protein would not contain or carry any of the genetic information that made it. The gene stays in the plasmid
⇒ 0 @ Z

	number of copies of DNA sequence found		
	stage X	*stage* Y	*stage* Z
A	1	1	0
B	1	2	1
C	1	1	1
D	2	1	0
E	2	1	1
F	2	2	0

63 A student set up an experiment to measure the rate of photosynthesis, as shown in the diagram.



Data was collected and plotted on a graph.

If plotted, which of the following variables would give a gradient that is directly proportional to the rate of photosynthesis?

(All other variables were kept constant.)

	x-axis	*y*-axis
1	time	volume of CO_2 released
2	time	number of gas bubbles released per minute
3	volume of oxygen released	time

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

1) NO. gradient = CO_2 produced per second $\propto$ Rate of Respiration

↓

CO_2 is TAKEN IN during photosynthesis

2) NO. gradient = Bubbles produced per (minute)^2 → This includes bubbles of O_2 which is a product of photosynthesis

↓

It is the y-coordinate that is $\propto$ rate of photosynthesis

3) NO. gradient = $\frac{1}{O_2 \text{ per second}} \propto \frac{1}{\text{rate of photosyn.}}$

↓

gradient would be inversely proportional to the rate of photosynthesis.

64 Bacteria reproduce asexually by dividing into two by binary fission.

Which of the following statements is/are correct about binary fission in bacteria?

✗ **1** As it is a form of asexual reproduction, there cannot be any variation in the offspring.

✓ **2** It can lead to a repeated doubling in population size if there are no limiting factors.

✗ **3** If binary fission occurs every 20 minutes, one bacterium would become 72^2 bacteria in 24 hours.

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

1) FALSE. Variation can occur through random mutations.

2) TRUE. The population grows exponentially ($y = 2^x$) without limiting factors

3) FALSE.

# of 20 mins in 24h = $\frac{24h}{1/3h} = 72$

After 72 binary fission periods

$1 \longmapsto 2^{72}$, NOT 72^2

65 Which of the following may stay the same when a mutation occurs in a human gene that codes for a protein?

✓ **1** the genotype of the organism's offspring

✓ **2** the phenotype of the organism

✓ **3** the sequence of amino acids in the protein

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

1. CAN HAPPEN — No guarantee the offspring will inherit the mutated gene

2. CAN HAPPEN — Depends on the role of the protein

3. CAN HAPPEN — Silent mutations don't affect the sequence

66 A child is affected by a dominant genetic condition. All of his cells have the same genotype. The allele associated with this condition is not present in either of his parents.

Which of the following statements could explain this?

✓ **1** A mutation occurred during meiosis in his father.

✗ **2** A random mutation occurred in his DNA after he was born.

✗ **3** Both of his grandmothers had the condition.

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

1 TRUE. The zygote would have received this from the father's chromosomes. And all of the child's cells would have reflected this genotype by mitosis

2. FALSE. He would have had some cells of the original genotype.

3. FALSE. The parents would have also shown the condition if it were dominant

67 Phosphatase enzymes break the bond between a phosphate group and the rest of a molecule.

Phenolphthalein phosphate (PPP) is colourless and is a substrate for phosphatase enzymes.

- The phosphate group of PPP can be removed by the phosphatase enzyme to produce the indicator phenolphthalein.
- Phenolphthalein goes pink in alkaline conditions.

The SAME enzyme – substrate reaction occurs at a different pH in each test tube

Five tubes were set up with equal concentrations of PPP and a plant phosphatase enzyme.

Each tube was at a different pH. The tubes were incubated at the same temperature. After 10 minutes Na_2CO_3 was then added to each tube until a pH of 9.5 was reached. The colour was observed immediately and then again after another 5 minutes.

tube	1	2	3	4	5
pH of the initial reaction	3.2	4.2	5.2	8.2	9.2
colour immediately after Na_2CO_3 addition	pale pink	pink	dark pink	colourless	colourless
colour after another 5 minutes	pale pink	pink	dark pink	colourless	colourless

A stronger pink means more indicator was produced from the initial enzyme – substrate reaction

Which of the following statements is/are correct?

✓ **1** Na_2CO_3 solution inhibited the activity of the phosphatase enzyme.

✗ **2** More substrate was produced in conditions of high acidity compared to low acidity.

✗ **3** The optimum pH for this plant phosphatase must be 5.2.

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

1) TRUE as the results show no phenolphthalein was produced in alkaline conditions

2) FALSE. More phenolphthalein was produced in low acidity (as shown from indicator colour)

3) FALSE. The optimum pH is certainly NEAR 5.2, but more measurements would be needed to determine its exact value.

A a

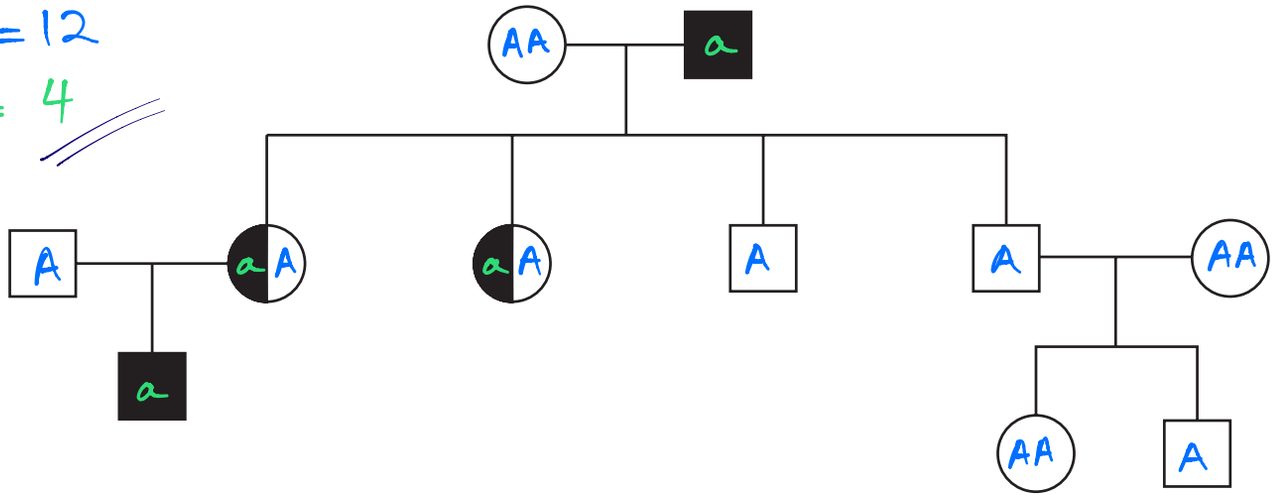
68 A gene found on the X chromosome in humans has two alleles, dominant and recessive. Individuals who have only recessive alleles have a condition that affects their eyesight.

The inheritance of the condition in one family is shown in the family tree.

Key
- female without the condition
- female carrier
- male without the condition
- male with the condition

Genotype
AA
Aa
A
a
} only one since it's on the X chromosome

A = 12
a = 4



If one living skin cell from each member of this family were analysed, how many of the following alleles would be found?

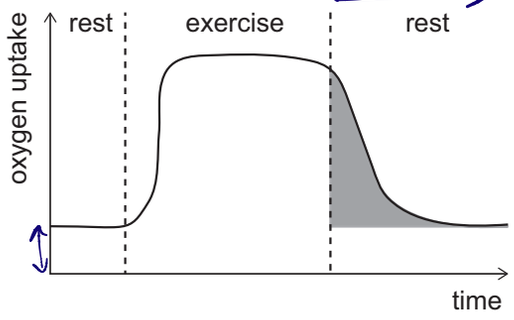
	number of copies of the recessive allele	*number of copies of the dominant allele*
A	2	7
B	2	12
C	2	14
D	4	7
E	4	12
F	4	14
G	6	7
H	6	12

69 The graphs below show the oxygen uptake before, during and after a period of strenuous physical exercise.

Which shaded area correctly represents the additional oxygen taken in to repay the oxygen debt acquired through anaerobic respiration?

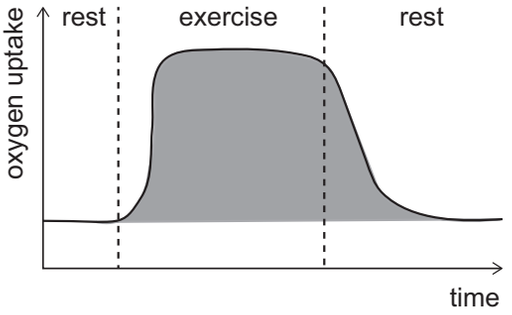
O_2 debt repaid AFTER EXERCISE

A

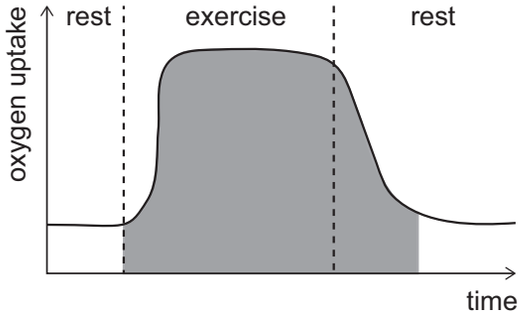


Resting O_2 intake

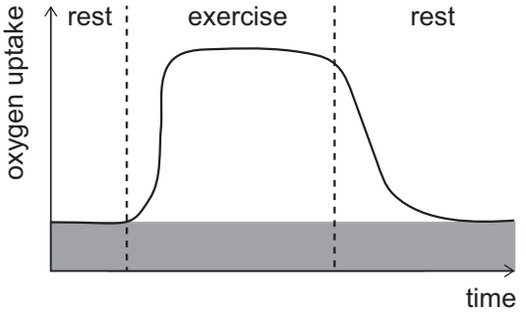
B



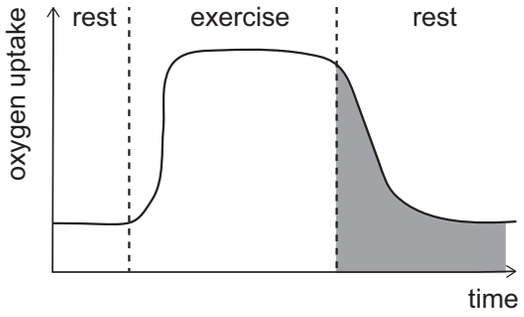
C



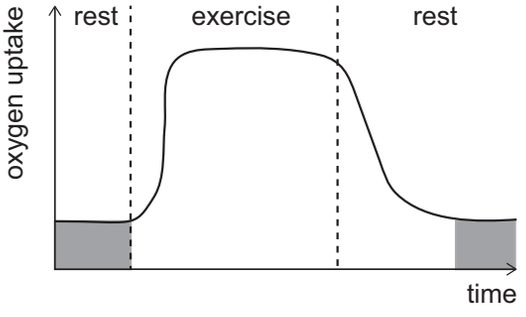
D



E



F

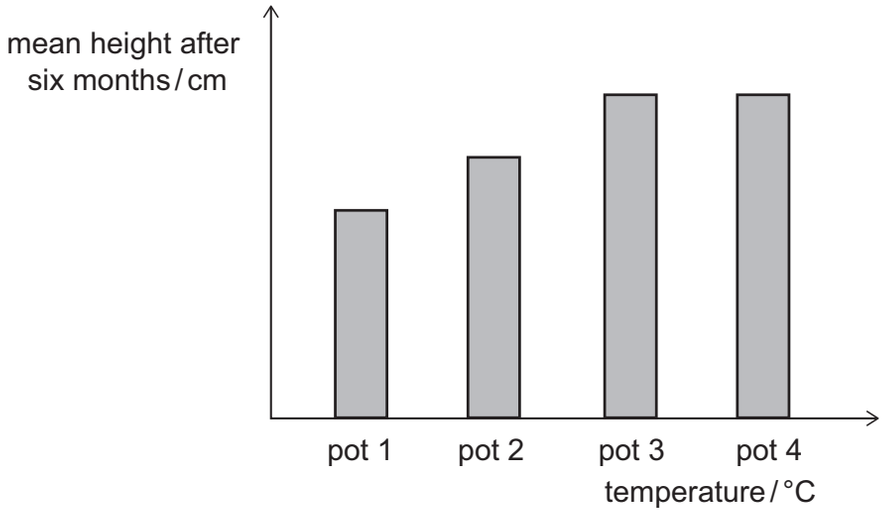


70 An investigation was carried out using clones of one plant.

Twenty plants of the same initial height were selected and divided into four equal groups. Each group was grown for six months and their environments were controlled as follows:

- kept at a different temperature to each other
- grown in a pot with an equal mass of soil with the same nutrients
- watered with an equal volume of water
- kept in the same light intensity

After six months, the height of the plants was measured. The mean height of the plants in each pot was calculated. The results are shown in the chart.



Which of the following statements could explain the results?

✓ **1** The difference in mean height between plants in pot 1 and plants in pot 2 could be due to the environment.

✓ **2** The mean height of plants in pot 4 equals that of plants in pot 3 because of another environmental factor in addition to temperature.

✓ **3** The mean height of plants in pot 3 equals that of plants in pot 4 because of the genotype of the plants.

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

1) TRUE: The independent variable (Temperature) is a part of the environment

2) TRUE: Another variable may be acting as a limiting factor preventing additional growth

3) TRUE: They are identically affected by the same limiting factor as they're clones. (eg. one does not tend to grow taller with the same nutrients due to variation)

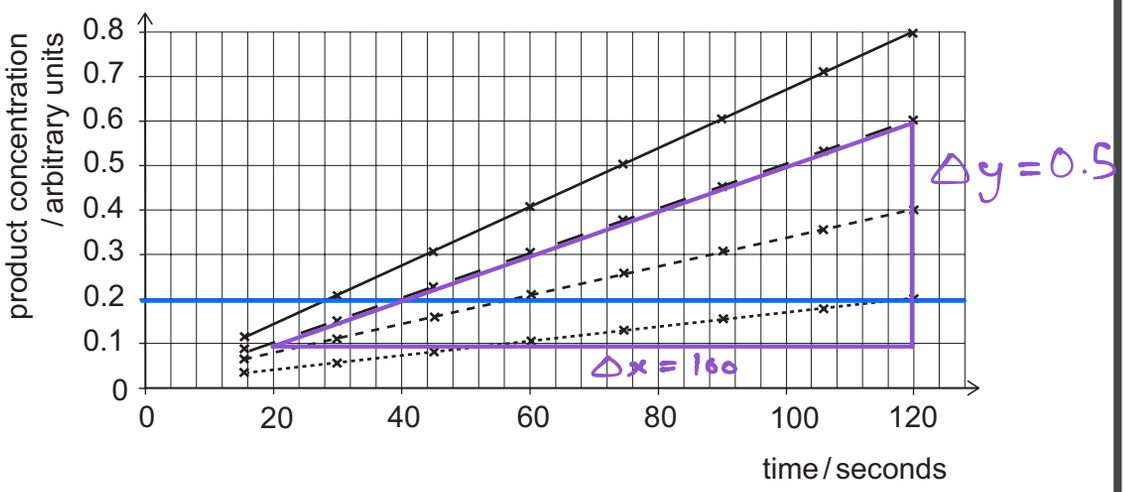
71 An investigation was carried out on the effect of substrate concentration on an enzyme-controlled reaction.

Four different concentrations of substrate were tested. In each case, the concentration of product was measured at regular intervals following the introduction of the substrate.

All the other variables were kept constant.

The results obtained are shown on the graph.

Key
——— 5.0 mmol dm^{-3}
– – 1.0 mmol dm^{-3}
- - - - 0.50 mmol dm^{-3}
····· 0.25 mmol dm^{-3}



Which of the following statements is/are correct?

✗ **1** As substrate concentration increases, the time taken to produce 0.2 arbitrary units of product increases.

✗ **2** Doubling the substrate concentration always doubles the rate of the reaction.

✓ **3** The average rate of reaction for a substrate concentration of 1.0 mmol dm^{-3} is 0.005 arbitrary units per second.

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

1) False. Time taken decreases

2) False. going from 0.25 mmol dm^{-3} to 0.50 mmol dm^{-3} decreases time to produce 0.2 units from 120s to 56s (more than doubling the rate of reaction).

3) True: Average rate = gradient = $\frac{\Delta y}{\Delta x} = \frac{0.5 \text{ units}}{100 \text{ s}}$ = 0.005 units/s

72 The sequence of a coding section of DNA is shown below.

1st 2nd 3rd 5th 6th last

CTAGTCTGGTGGCTACGTCCTCCTCTACTATGGCTAGTCGTCTGGTGGCTA

The number of each type of amino acid coded for by this sequence of DNA bases is shown in the table.

Each amino-acid is made from 3 base pairs

amino acid coded for	*number of this type of amino acid present*
Arg	1
Leu	6
Pro	2
Trp	5
Val	3

What is the correct sequence of amino acids coded for by this sequence of DNA?

✗ **A** Leu-Trp-Trp-Leu-Arg-Pro-Pro-Leu-Leu-Val-Leu-Val-Val-Trp-Trp-Trp

✗ **B** Leu-Trp-Trp-Leu-Arg-Pro-Pro-Leu-Leu-Trp-Leu-Trp-Val-Val-Val-Val

✗ **C** Leu-Val-Trp-Trp-Leu-Arg-Pro-Pro-Leu-Leu-Trp-Leu-Val-Val-Trp-Trp

✗ **D** Leu-Trp-Trp-Leu-Arg-Pro-Pro-Leu-Leu-Val-Leu-Val-Val-Trp-Trp-Trp-Leu

✗ **E** Leu-Trp-Trp-Leu-Arg-Pro-Pro-Leu-Leu-Trp-Leu-Trp-Val-Val-Val-Val-Leu

✗ **F** Leu-Leu-Trp-Trp-Leu-Arg-Pro-Pro-Leu-Leu-Trp-Leu-Val-Val-Trp-Trp-Val

✗ **G** Leu-Val-Trp-Trp-Leu-Leu-Pro-Pro-Leu-Leu-Trp-Leu-Val-Val-Trp-Trp-Leu

✓ **H** Leu-Val-Trp-Trp-Leu-Arg-Pro-Pro-Leu-Leu-Trp-Leu-Val-Val-Trp-Trp-Leu

To solve this at a GCSE level, eliminate the wrong options

→ The 2nd and 3rd triplets are NOT identical. So the 2nd and 3rd amino acid CAN'T be the same.

→ The 1st and last triplets ARE identical. So the 1st and last aminoacids MUST be the same.

→ The 5th and 6th triplets are NOT identical. So the 5th and 6th amino acid CAN'T be the same.

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PART E Advanced Mathematics and Advanced Physics

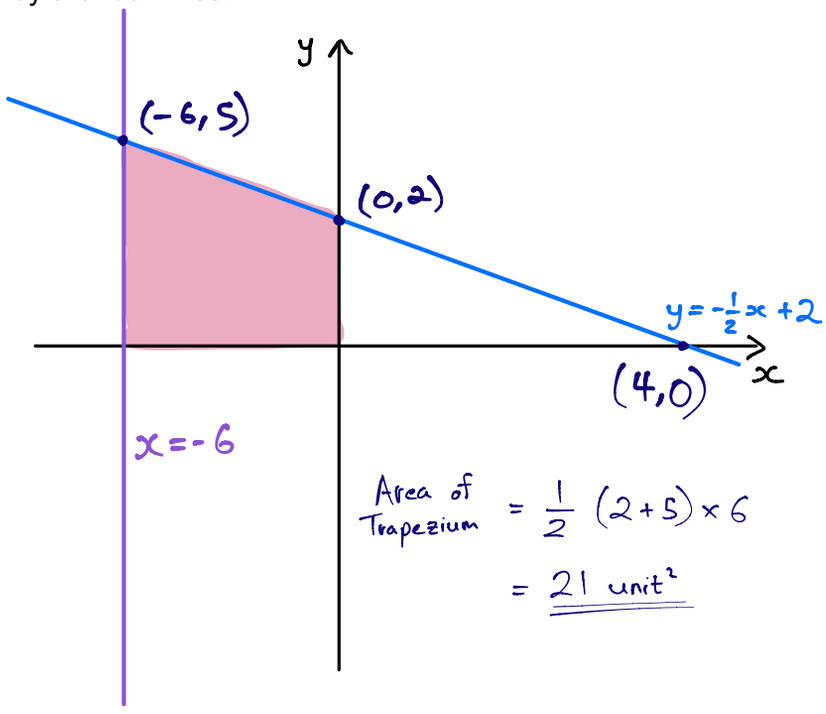
73 Find the area of the shape bounded by the four lines:

$y = -\frac{1}{2}x + 2 \iff 2y + x = 4$

$x = -6$

$x = 0$

$y = 0$



Area of Trapezium $= \frac{1}{2}(2+5) \times 6$

$= 21 \text{ unit}^2$

A 4

B 12

C 21

D 25

E 27

F 30

74 A hydroelectric power station uses the water in a reservoir to power the generators. The water falls through a vertical height of 150 m to the turbines which power the generators. gpe → ke

The efficiency of the power station is 90% and the output power of the power station is 1800 MW.

The gravitational field strength is $10\,\text{N kg}^{-1}$ and the density of water is $1000\,\text{kg m}^{-3}$.

What volume of water passes through the turbines in one minute?

A $6.48 \times 10^4\,\text{m}^3$

B $7.20 \times 10^4\,\text{m}^3$

C $8.00 \times 10^4\,\text{m}^3$

D $6.48 \times 10^7\,\text{m}^3$

E $7.20 \times 10^7\,\text{m}^3$

F $8.00 \times 10^7\,\text{m}^3$

Input Power $= \frac{\text{Output Power}}{\text{Efficiency}} = \frac{1800\,\text{MW}}{0.9} = 2000\,\text{MW}$

$= \frac{1800\,\text{MW}}{0.1}$

$= $ KE gathered per second

$\therefore$ In 1 second,

$\frac{1}{2}mv^2 = 2000 \times 10^6\ \text{J} \longrightarrow (1)$

To find v, use conservation of energy over the 150m drop

$\frac{1}{2}mv^2 = mgh \Rightarrow v = \sqrt{2gh}$

$= \sqrt{2 \times 10 \times 150} = \sqrt{3000}$,

Plug into (1),

$m = \frac{2000 \times 10^6}{v^2} \times 2 = \frac{4000 \times 10^6}{3000} = \frac{4}{3} \times 10^6 = \rho V \Rightarrow V = \frac{\frac{4}{3} \times 10^6}{1000} = \frac{4}{3} \times 10^3$

In 1 min,

$V = 60 \times \frac{4}{3} \times 1000 = 80 \times 10^4\ \text{m}^3$

75 The curve

$$y = x^3 + px^2 + qx + r \Rightarrow \frac{dy}{dx} = 3x^2 + 2px + q$$

has a local maximum when $x = -1$ and a local minimum when $x = 3$

What is the value of p?

$\hookrightarrow \left.\frac{dy}{dx}\right|_{x=-1} = 0$ $\quad$ $\hookrightarrow \left.\frac{dy}{dx}\right|_{x=3} = 0$

A −9

B −3

C −1

D 1

E 3

F 9

$3(-1)^2 - 2p + q = 0$

$-2p + q = -3 \quad \hookrightarrow (1)$

$3(3)^2 + 2p(3) + q = 0$

$6p + q = -27 \quad \hookrightarrow (2)$

$(2) - (1): \; 8p = -24$

$p = -3$

76 A car P of mass 1000 kg is travelling north at 30 m s^{-1} along a straight, horizontal road when it hits another car Q which is directly ahead of P and travelling in the same direction. Car Q has a mass of 500 kg and is travelling at 20 m s^{-1}.

The collision lasts for 0.20 s and immediately after the collision car Q is moving north at 30 m s^{-1}.

What is the speed of P immediately after the collision and what is the size of the average resultant force that acts on Q during the collision?

(Assume that no external forces act on the cars during the collision.)

	speed of P / m s^{-1}	*average force on* Q / N
A	20	25 000
B	20	50 000
C	20	100 000
D	20	125 000
E	25	25 000
F	25	50 000
G	25	100 000
H	25	125 000

Before:

Q: $p_q = 500 \times 20 = 10000$ kgms^{-1}

P: $p_p = 1000 \times 30 = 30000$ kgms^{-1}

Total Initial momentum = 40 000 kgms^{-1}

After:

Q: $p_Q = 500 \times 30 = 15000$ kgms^{-1}

P: By conservation of momentum,

$p_p = 40000 - 15000 = 25000$

$= 1000\text{kg} \times 25\text{ms}^{-1}$

Change in momentum of Q $= \Delta p_Q = 15000 - 10000$

$= 5000$ kgms^{-1}

$= F\Delta t$

$\Rightarrow F = \frac{5000}{0.20} = 25\,000\text{N}$

77 It is given that

$$7\cos x + \tan x \sin x = 5$$

where $0° < x < 90°$

What are the possible values of $\tan x$?

A $\frac{1}{2}$ or $\frac{1}{3}$

B $\frac{1}{\sqrt{3}}$ or $\frac{1}{2\sqrt{2}}$

C $\frac{\sqrt{3}}{2}$ or $\frac{2\sqrt{2}}{3}$

D $\sqrt{3}$ or $2\sqrt{2}$

E 3 or 2

$\sin^2 x + \cos^2 x = 1$

$\Rightarrow \tan^2 x + 1 = \sec^2 x$

$\Rightarrow 7\cos x + \frac{\sin x}{\cos x}\sin x = 5$

$7\cos x + \frac{\sin^2 x}{\cos x} = 5$

$7\cos x + \frac{1-\cos^2 x}{\cos x} = 5$

$6\cos x + \frac{1}{\cos x} = 5$

$6\cos^2 x - 5\cos x + 1 = 0$

$\cos x = \frac{5 \pm \sqrt{25-24}}{12} = \frac{1}{2}, \frac{1}{3}$

$\Rightarrow \sec x = 2, 3 \Rightarrow \sec^2 x = 4, 9$

$\tan^2 x = \sec^2 x - 1 = 3, 8$

$\tan x = \sqrt{3}, 2\sqrt{2}$

78 A metal wire of length 0.50 m has a uniform cross-sectional area of $4.0 \times 10^{-7}\,\text{m}^2$.

There is a current of 4.0 A in the wire.

What is the potential difference across the ends of the wire?

(resistivity of the metal = $1.6 \times 10^{-7}\,\Omega\text{m}$)

A 0.05 V

B 0.20 V

C 0.80 V

D 3.2 V

E 5.0 V

F 20 V

$R = \frac{\rho L}{A} = \frac{1.6 \times 10^{-7} \times 0.50}{4.0 \times 10^{-7}}$

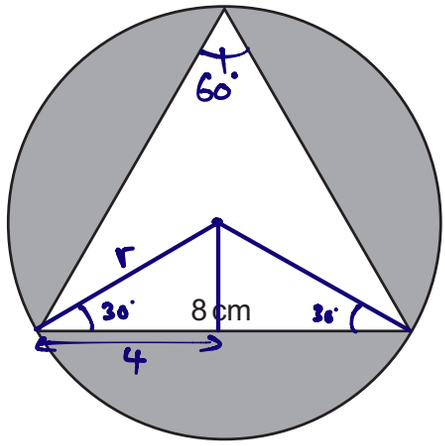
$= 0.4 \times 0.5 = 0.20$

$V = IR$

$= 4.0 \times 0.20$

$= \frac{4}{5} = 0.8\text{V}$

79 An equilateral triangle of side 8 cm is drawn so that its vertices lie on the circumference of a circle, as shown in the diagram.



all angles of an equilateral △ are 60°

$r\cos 30° = 4$

$\frac{\sqrt{3}}{2} r = 4$

$r = \frac{8}{\sqrt{3}}$

What is the total of the three areas shaded in the diagram, in cm^2?

A $8(2\pi - 3)$

B $24(\pi - \sqrt{3})$

C $48(4\pi - \sqrt{3})$

D $\frac{16}{3}(4\pi - 6 - 3\sqrt{3})$

E $\frac{16}{3}(4\pi - 3\sqrt{3})$

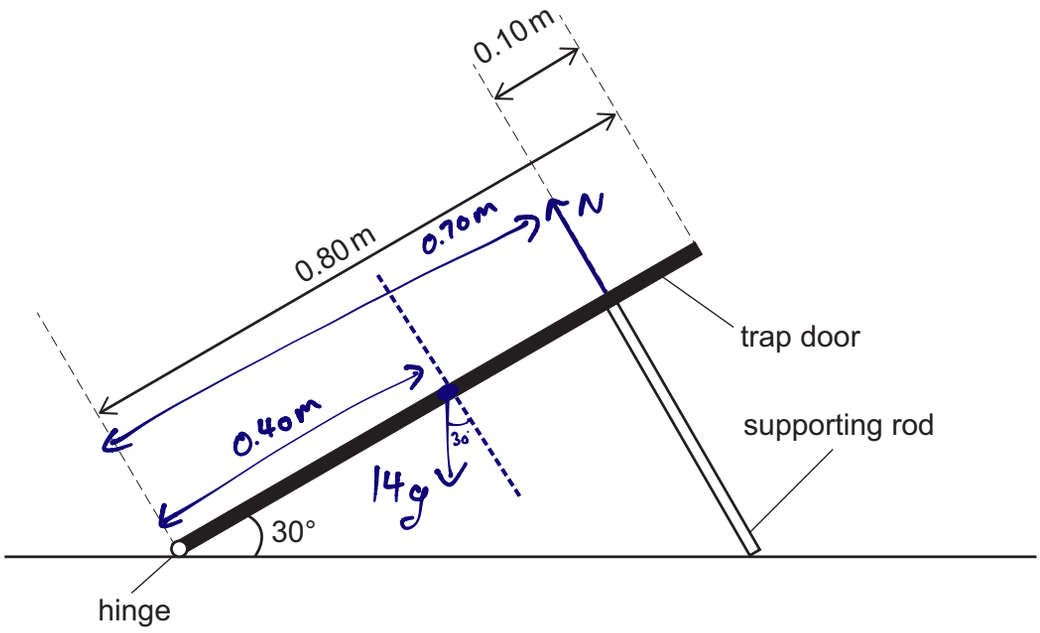
= Area of circle − Area of △

$= \pi\left(\frac{8}{\sqrt{3}}\right)^2 - \frac{1}{2} \times 8 \times 8 \times \sin 60°$

$= \frac{64}{3}\pi - 16\sqrt{3}$

$= \frac{16}{3}(4\pi - 3\sqrt{3})$

80 A uniform square trap door of side 0.80 m and mass 14 kg has a smooth hinge at one edge and is held open at an angle of 30° to the horizontal. It is supported by a single rigid rod placed so that it meets the surface of the trap door at 90° at a distance 0.10 m from the top edge of the trap door, as shown.



What is the normal contact force exerted on the trap door by the rod?

(gravitational field strength = 10 N kg^{-1})

A 40 N

B $35\sqrt{3}$ N

C $40\sqrt{3}$ N

D 80 N

E $80\sqrt{3}$ N

F $280\frac{\sqrt{3}}{3}$ N

Taking moments about the Hinge,

Sum of $\circlearrowleft$ = Sum of $\circlearrowright$

$0.70\,N = 0.40 \times 14g \cos 30°$

$N = \frac{4}{7} \times 14 \times 10 \times \frac{\sqrt{3}}{2}$

$= 40\sqrt{3}$

81 Which one of the following is the real solution of the equation

$$3\times5^{2x+1}-5^{x}-2=0$$

A $x=\log_5\left(\frac{1}{3}\right)$

B $x=\log_5\left(\frac{2}{5}\right)$

C $x=\log_5\left(\frac{3}{5}\right)$

D $x=\log_5\left(\frac{2}{3}\right)$

E $x=\log_5\left(\frac{5}{3}\right)$

F $x=\log_5\left(\frac{5}{2}\right)$

Let $y = 5^x$

$\Rightarrow 3\times5y^2 - y - 2 = 0$

$15y^2 - y - 2 = 0$

$y = \frac{1 \pm \sqrt{1+120}}{30}$

$y = \frac{12}{30}, \frac{-10}{30} = \frac{2}{5}, \frac{-1}{3}$

Since $y = 5^x > 0$ for all real x,

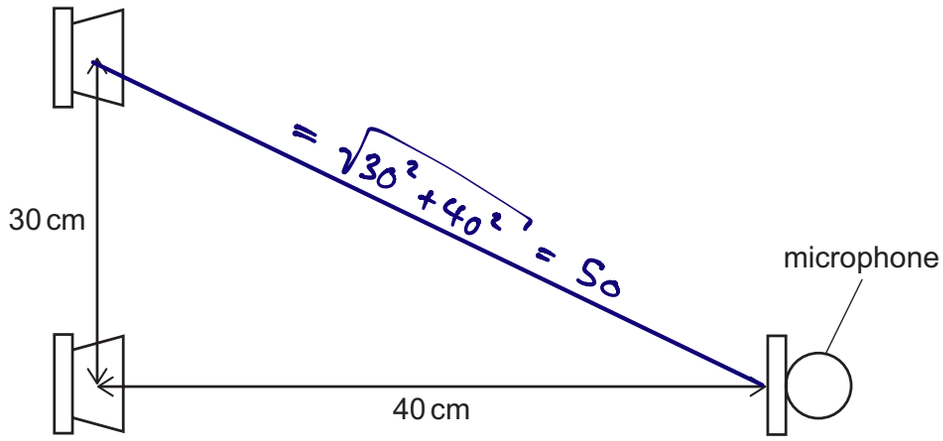
$y = 5^x = \frac{2}{5}$

$x = \log_5 (2/5)$

82 Two small loudspeakers are placed side by side 30 cm apart.

They are connected to the same signal generator so that they emit sound of frequency 400 Hz in phase with one another.

The sounds both reach a microphone placed 40 cm directly in front of one of the two loudspeakers as shown.



What is the phase difference between waves from the loudspeakers as they arrive at the microphone?

(speed of sound = 320 m s^{-1})

A 30°

B 36°

C 45°

D 60°

E 72°

F 90°

G 120°

$\lambda = \frac{v}{f} = \frac{320}{400} = 0.8\text{ m} = 80\text{ cm}$

Path difference between the 2 waves $= 50 - 40 = 10\text{ cm} = \frac{1}{8}^{th}$ of a wavelength

$\therefore$ Phase difference is $\frac{1}{8}$ of a cycle out of sync,

As phase difference is an angular measure:

$= \frac{1}{8} \times 360°$

$= 45°$

83 For a particular function $f(x)$, it is given that:

$$\int_{-2}^{2} 2f(x)\,dx + \int_{2}^{4} f(x)\,dx = 4 \quad (1)$$

and also:

$$\int_{-2}^{2} 5f(x)\,dx - \int_{-2}^{4} f(x)\,dx = 7 \quad (2)$$

$$= 5\int_{-2}^{2} f(x)\,dx - \left(\int_{-2}^{2} f(x)\,dx + \int_{2}^{4} f(x)\,dx\right)$$

$$= 4\int_{-2}^{2} f(x)\,dx - \int_{2}^{4} f(x)\,dx$$

Find the value of $\int_{2}^{4} f(x)\,dx$

Let $a = \int_{-2}^{2} f(x)\,dx$

$b = \int_{2}^{4} f(x)\,dx$

So (1) and (2) can be expressed as:

$2a + b = 4$

$4a - b = 7$

$\Rightarrow 6a = 11$

$a = \frac{11}{6} \Rightarrow b = 4 - 2a$

$= 4 - \frac{22}{6}$

$= \frac{2}{6} = \frac{1}{3}$

A $\frac{1}{3}$

B $\frac{11}{7}$

C $\frac{11}{6}$

D $\frac{13}{6}$

E $\frac{13}{3}$

84 An astronaut on the Moon throws a ball vertically upwards. The ball has a mass of 2.0 g and is thrown upwards at 80 m s^{-1}.

What is the maximum height gained by the ball?

(gravitational field strength close to the Moon's surface = 1.6 N kg^{-1})

A 25 m

B 50 m

C 320 m

D 2000 m

E 3200 m

F 4000 m

$v^2 = u^2 + 2as$

$0 = 80^2 + 2 \times 1.6 \times s$

$s = \frac{6400}{3.2}$

$= \frac{2 \times 32 \times 100}{32 \times 0.1}$

$= 2000$ m

$s = ?$

$u = 80$

$v = 0$

$a = 1.6$

85 Given that

$$f(x) = \int_0^x (3+2t)^7 \, \mathrm{d}t$$

what is the coefficient of x^4 in the expansion of $f(x)$ in powers of x?

A 70

B 162

C $\frac{2835}{4}$

D 3024

E 5670

F 15 120

G 22 680

The x^4 term in $f(x)$ only comes from the t^3 term in $(3+2t)^7$

$$\binom{7}{3}(3)^4(2t)^3$$

$$= \frac{7!}{(7-3)!\,3!} \times 81 \times 8t^3$$

$$= \frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times 81 \times 8t^3$$

$$= 35 \times 81 \times 8t^3$$

When integrated,

$$= \frac{35 \times 81 \times 8}{4} t^4 = 70 \times 81 t^4$$

$$= 5670\,t^4$$

Plug in bounds

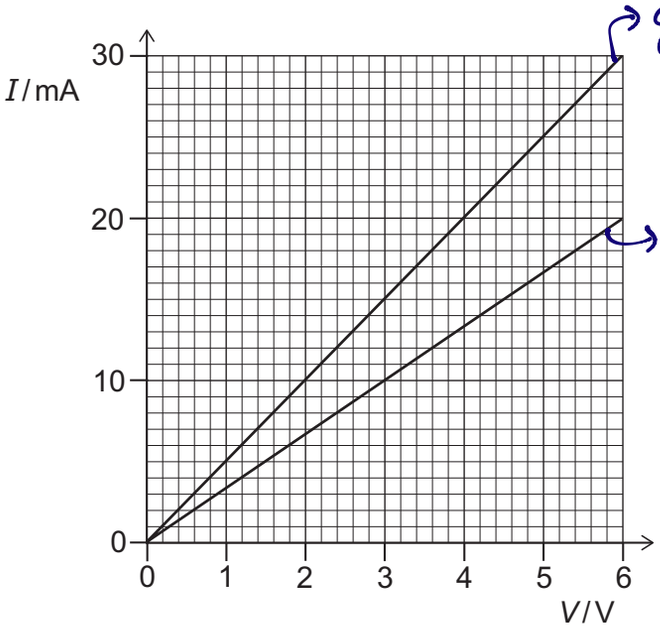
$$= 5670x^7 - 5670(0)^4$$

$$= 5670x^7$$

86 A student has one 300 Ω resistor and another resistor of resistance R.

The student plots a graph of current I against potential difference V for the 300 Ω resistor and then for both resistors connected in parallel.

$R = \frac{V}{I} = \frac{1}{\text{gradient}}$



$\text{grad} = \frac{6}{0.03} = 200\,\Omega = R_{\text{parallel}}$

$\text{grad} = \frac{6}{0.02} = 300\,\Omega$

What is the resistance R?

A 3.3 Ω

B 5.0 Ω

C 10 Ω

D 100 Ω

E 200 Ω

F 600 Ω

G 1000 Ω

$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R} + \frac{1}{300}$$

$$\Rightarrow R = \frac{1}{\frac{1}{200} - \frac{1}{300}}$$

$$= \frac{1}{\frac{300 - 200}{60000}}$$

$$= 600\,\Omega$$

87 The three internal angles in a triangle are α, β and θ, and

$$3\tan\alpha - 2\sin\beta = 2 \longrightarrow (1)$$
$$5\tan\alpha + 6\sin\beta = 8 \longrightarrow (2)$$

What is the value of θ in degrees?

$3\times(1) + (2)$: $14\tan\alpha = 14$

$\tan\alpha = 1$

$\Rightarrow \alpha = \tan^{-1}(1)$

$= 45^\circ, 135^\circ$

A 15

B 45

C 75

D 105

E 135

Plug $\tan\alpha = 1$ into (1),

$3 - 2\sin\beta = 2$

$\sin\beta = 1/2 \Rightarrow \beta = 30^\circ, 150^\circ$

Since $\alpha + \beta + \theta = 180^\circ$,

$\theta = 180^\circ - 45^\circ - 30^\circ = 105^\circ$

88 A light, vertical, copper wire of length 2.4 m and uniform cross-sectional area $2.0 \times 10^{-6}\,m^2$ supports a load of mass 4.0 kg.

The Young modulus of copper is 1.2×10^{11} Pa. $= \frac{\sigma}{\varepsilon}$

What is the strain energy in the wire? $= \frac{1}{2}kx^2$

(gravitational field strength = $10\,N\,kg^{-1}$; assume that the wire obeys Hooke's law and that the cross-sectional area remains constant)

A 8.0×10^{-5} J

B 1.7×10^{-4} J

C 4.0×10^{-4} J

D 8.0×10^{-3} J

E 4.0×10^{-2} J

F 1.6×10^{-2} J

Strain $= \varepsilon = \frac{\text{Stress}}{\text{Young. M.}}$

$= \frac{\sigma}{E} = \frac{F}{EA} = \frac{4\times10}{1.2\times10^{11}\times2\times10^{-6}}$

$= \frac{40}{2.4\times10^{5}}$

$= \frac{10^{-3}}{6}$

Extension = Strain × Original Length

$x = \frac{10^{-3}}{6} \times 2.4$

$= 4\times10^{-4}$ m

Strain Energy $= \frac{1}{2}kx^2$

$= \frac{1}{2}kx\cdot x = \frac{1}{2}Fx = \frac{1}{2}\times4\times10\times4\times10^{-4}$

$= 8\times10^{-3}$ J

89 Find the complete set of values of x for which

$$x^3 - 2x^2 - 7x - 4 > 0$$

A $x < -1$

B $x > -1$

C $-1 < x < 4$

D $x < -1$ or $x > 4$

E $x < 4$

F $x > 4$

Let $f(x) = x^3 - 2x^2 - 7x - 4$

By trial and error with Factor Theorem: $f(-1) = 0$

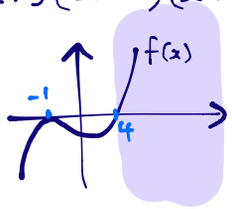
$\Rightarrow (x+1)$ is a factor of $f(x)$

By algebraic long division:

$$\begin{array}{r} x^2 - 3x - 4 \\ x+1 \overline{)\,x^3 - 2x^2 - 7x - 4} \\ x^3 + x^2 \\ \hline -3x^2 - 7x \\ -3x^2 - 3x \\ \hline -4x - 4 \\ -4x - 4 \\ \hline 0 \end{array}$$

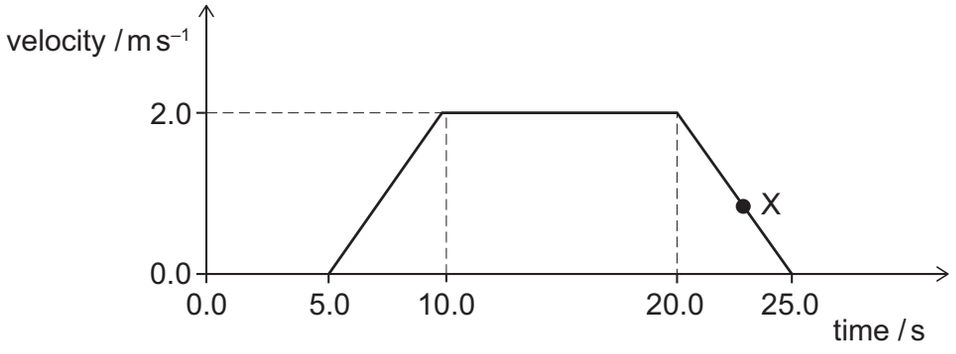
$(x+1)(x^2 - 3x - 4) > 0$

$\Rightarrow (x+1)(x-4)(x+1) > 0$



$\Rightarrow x > 4$

90 The velocity–time graph is for an 80 kg person in a lift that is moving vertically upwards.



What is the magnitude of the contact force between the person and the lift floor at the time corresponding to X?

(gravitational field strength = 10 N kg^{-1})

A 640 N

B 768 N

C 800 N

D 832 N

E 960 N

Acceleration @ X = gradient

$$= \frac{\Delta v}{\Delta t} = \frac{-2.0}{5.0}$$

$$= -0.4 \text{ m s}^{-2}$$

Using Newton's 2nd Law on the person,

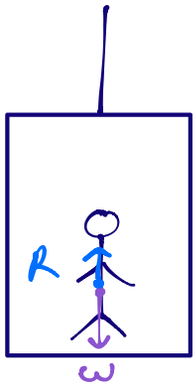
$$F_{net} = ma$$

$$R - W = ma$$

$$R = 80\text{ kg} \times -0.4\text{ m s}^{-2} + 80 \times 10\text{ N}$$

$$= -32 + 800$$

$$= 768\text{ N}$$



R: Upwards Normal contact from floor on person

W: Downwards weight on person