NSAA 2018
Section 1

Model Solutions

PART A Mathematics

1 A group of drivers, consisting of 200 women and 300 men, was asked if they passed their driving test at the first attempt.

Altogether 167 of the group said they passed at the first attempt.

Of the women, 143 said they did not pass at the first attempt.

How many of the men said they passed at the first attempt?

A 10

B 24

C 33

D 57

E 110

F 133

G 157

No. of women passing on 1st attempt $= 200 - 143$
$= 57$

No. of men passing on 1st attempt $= 167 - 57 = 110$

OR

Using a table:

	Women	Men	Total
Passed 1st time	57	110	167
Did not pass 1st time	143	190	333
Total	200	300	500

2 A cuboid has sides of length x, $\sqrt{2}x$ and $2x$, measured in cm.

The volume, in cm^3, of the cuboid is numerically equal to twice the total surface area, in cm^2, of the cuboid.

What is the value of x?

A 10

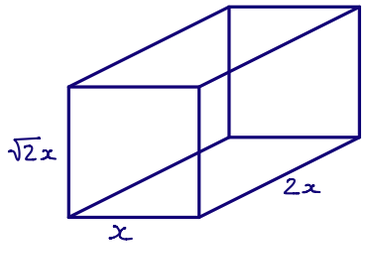
B $6 + 2\sqrt{2}$

C 5

D $3 + \sqrt{2}$

E $\frac{5}{2}$

F $\frac{3}{2} + \frac{1}{2}\sqrt{2}$



Volume of cuboid $= x \cdot \sqrt{2}x \cdot 2x$
$= 2\sqrt{2}x^3$

Total surface area of cuboid $= 2(x \cdot \sqrt{2}x) + 2(x \cdot 2x) + 2(\sqrt{2}x \cdot 2x)$
$= 2\sqrt{2}x^2 + 4x^2 + 4\sqrt{2}x^2$
$= (4 + 6\sqrt{2})x^2$

Since Volume $= 2 \times$ Total Surface Area

$\Rightarrow 2\sqrt{2}x^3 = 2(4 + 6\sqrt{2})x^2$

$2\sqrt{2}x = 8 + 12\sqrt{2}$

$x = \frac{8 + 12\sqrt{2}}{2\sqrt{2}} = \frac{4}{\sqrt{2}} + 6$

$= 6 + 2\sqrt{2}$

3 The line joining the points with coordinates $(p, p-1)$ and $(1-p, 2p)$ is parallel to the line with equation $2x + 3y + 1 = 0$

What is the value of p?

A -1

B $-\frac{1}{7}$

C $\frac{1}{9}$

D $\frac{1}{8}$

E 1

F $\frac{5}{4}$

G 2

H 5

To find the gradient, convert to $y = mx + c$ form

$2x + 3y + 1 = 0$

$\Rightarrow y = -\frac{2}{3}x - \frac{1}{3} \Rightarrow$ gradient $= -2/3$

Since both lines have the same gradient,

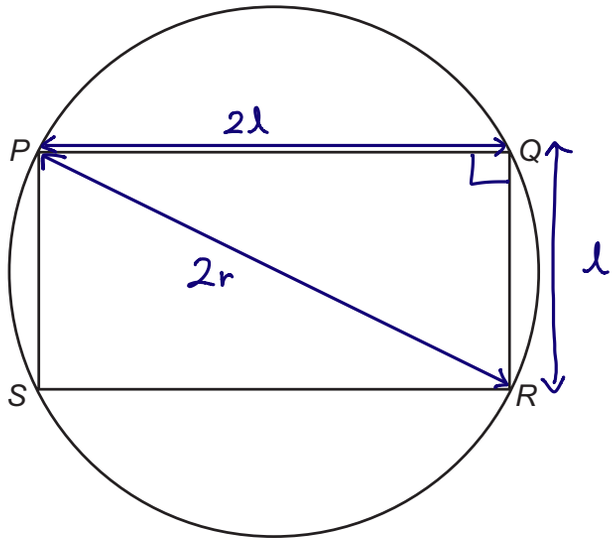
$$\frac{\Delta y}{\Delta x} = \frac{2p - (p-1)}{(1-p) - p} = -\frac{2}{3}$$

$$\frac{p+1}{1-2p} = -\frac{2}{3}$$

$$3p + 3 = 4p - 2$$

$$p = 2 + 3 = 5$$

4 A rectangle $PQRS$ is drawn inside a circle, with its vertices on the circumference of the circle.



[diagram not to scale]

The ratio of the length of PQ to the length of QR is $2:1$

The area of the rectangle $PQRS$ is $96\,\text{cm}^2$.

What is the radius, in cm, of the circle?

A $\sqrt{6}$

B 3

C $3\sqrt{2}$

D $2\sqrt{15}$

E $4\sqrt{6}$

F 12

G $12\sqrt{2}$

H $8\sqrt{15}$

To find the dimensions of rectangle PQRS,

area $= PQ \times QR = 2l \times l = 2l^2 = 96$ cm

$\Rightarrow l^2 = 48$ cm^2

$l = \sqrt{16 \times 3} = 4\sqrt{3}$ cm

By circle theorems, PR is a diameter of the circle since $\triangle PQR$ is right angled

$\Rightarrow$ If radius $= r$, $PR = 2r$

Using Pythagoras on $\triangle PQR$

$(2r)^2 = (2l)^2 + l^2$

$4r^2 = (2 \times 4\sqrt{3})^2 + (4\sqrt{3})^2$

$= 192 + 48 = 240$

$r^2 = 60$

$r = \sqrt{60} = \sqrt{4 \times 15} = 2\sqrt{15}$ cm

5 The expected number of bottles of water sold in a day at a sports ground is directly proportional to the square of the average outside temperature, in degrees Celsius, for that day.

On a day when the average outside temperature is 16 °C, 64 bottles of water, the expected number, are sold.

On a warmer day, when the average outside temperature is T °C, 256 bottles of water are sold, which is 31 bottles more than the expected number for that day.

What is the value of T?

A 7.5

B $\sqrt{450}$

C 30

D 32

E $\sqrt{1148}$

F 56.25

Let n be the expected number of bottles sold at temperature T

$n \propto T^2 \Rightarrow n = kT^2$, where k is constant

When $T = 16$, $n = 64$

$64 = (16)^2 k$

$k = \frac{64}{256} = \frac{1}{4}$

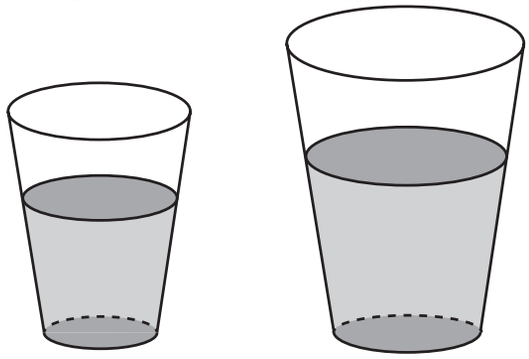
On the warmer day,

$n = 256 - 31 = 225$

$\Rightarrow 225 = \frac{1}{4} T^2$

$T = \sqrt{4 \times 225} = 30$

6



[diagram not to scale]

At a cinema, drinks are sold in regular and large sizes.

The cups for these are mathematically similar.

The ratio of the heights of the cups and the ratio of the depths of the drinks are both 4 : 5

The volume of drink in a regular size cup is 320 cm^3.

What is the volume, in cm^3, of drink in a large size cup?

A 384

B 400

C 500

D 576

E 625

F 640

Linear scale factor = $\frac{5}{4} = 1.25$

(large is 1.25 × taller than regular)

$\Rightarrow$ Volume scale factor = $(1.25)^3 = \frac{125}{64}$

Volume of large = $320 \times \frac{125}{64}$

$= 5 \times 125$

$= 625$

7 The mean of n numbers is p

The mean of two of these numbers is q

The mean of the remaining numbers is 10

Which of the following is a correct expression for n in terms of p and q?

A $\frac{2(q-10)}{(p-10)}$

B $\frac{2(q-10)}{(10-p)}$

C $\frac{2(q-10)}{(p+10)}$

D $\frac{2(10-q)}{(p+10)}$

E $\frac{2(10+q)}{(p-10)}$

F $\frac{2(10+q)}{(10-p)}$

Let the n numbers be $x_1, x_2, \dots x_n$

$p = \frac{x_1+x_2+x_3+\dots+x_n}{n} \Rightarrow x_1+\dots+x_n = np$

Let x_1 and x_2 be the 2 numbers with mean q

$\frac{x_1+x_2}{2} = q \Rightarrow x_1+x_2 = 2q$

Mean of the remaining is 10, $\frac{x_3+\dots+x_n}{n-2} = 10 \Rightarrow x_3+\dots+x_n = 10(n-2)$

$\therefore p = \frac{x_1+\dots+x_n}{n} = \frac{2q+10(n-2)}{n} \Rightarrow 2q+10n-20 = np$

$2q-20 = n(p-10)$

$n = \frac{2(q-10)}{(p-10)}$

8 The straight lines

$5x + 2y = 20$ $\Rightarrow$ $y = -\frac{5}{2}x + 10$

$y = 3x - 23$

$x = 0$

enclose a region with area K square units.

What is the value of K?

A 39

B 78

C 99

D 129

E 198

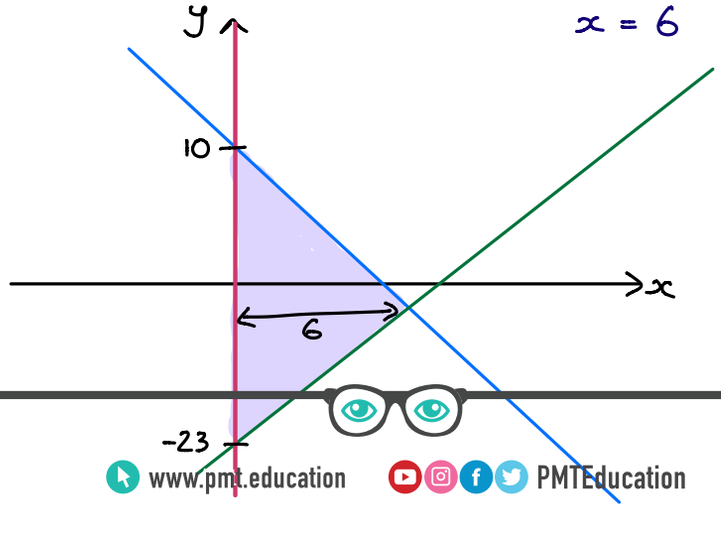
F 258

To find where the 2 diagonal lines intersect,

$-\frac{5}{2}x + 10 = 3x - 23$

$\frac{11}{2}x = 33$

$x = 6$



K = area of $\triangle$

$= \frac{1}{2} \times (10+23) \times 6$

$= \frac{1}{2} \times 33 \times 6$

$= 99$

9 A scale model of a cylindrical pillar is to be made.

The full-sized pillar has a volume of $12\pi\,\text{m}^3$.

The model will use a length scale of $1:40$

The model is to be a solid cylinder made of a plastic which has a density of $\frac{4}{3}\,\text{g cm}^{-3}$.

What is the mass of the model in grams?

A $\frac{9}{640}\pi$

B $\frac{1}{40}\pi$

C 40π

D $\frac{1125}{8}\pi$

E 250π

F $10\,000\pi$

G $225\,000\pi$

H $400\,000\pi$

Volume scale factor $= 40^3$

(model will have $\left(\frac{1}{40}\right)^3$ vol. of full sized)

Model volume $= \frac{12\pi}{40^3}\,\text{m}^3 = \frac{12\pi}{64\times10^3}\times10^6\,\text{cm}^3$

$= \frac{3\pi}{16}\times10^3\,\text{cm}^3$

$10^2\,\text{cm} = 1\,\text{m}$

$\Rightarrow 10^6\,\text{cm}^3 = 1\,\text{m}^3$

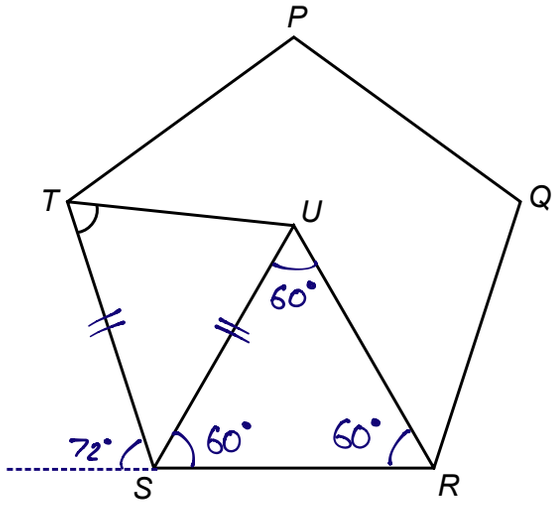
mass = density × volume

$= \frac{4}{3}\,\text{g cm}^{-3}\times\frac{3\pi}{16}\times10^3\,\text{cm}^3$

$= \frac{\pi}{4}\times1000\,\text{g}$

$= 250\pi\,\text{g}$

10



[diagram not to scale]

PQRST is a regular pentagon.

RSU is an equilateral triangle.

What is the size of angle *STU*?

A 48°

B 54°

C 60°

D 66°

E 84°

Sum of exterior angles of any polygon $= 360°$

Exterior angle of regular pentagon $= \frac{360°}{5} = 72°$

$\angle TSU = 180° - 60° - 72° = 48°$ [Angles on a straight line]

$\triangle STU$ is isosceles $\Rightarrow \angle STU = \angle SUT$

Sum of angles in $\triangle STU$:

$\angle TSU + 2\angle STU = 180°$

$\angle STU = \frac{180° - 48°}{2} = 66°$

11 The original price of an item is p

The price is **increased by** 125%

The increased price is then **decreased by** 40% to q

The relationship between p and q can be expressed as $mp = q$

What is the value of m?

A $\frac{7}{20}$

B $\frac{17}{20}$

C $\frac{27}{20}$

D $\frac{33}{20}$

E $\frac{37}{20}$

$\times(1 + 1.25)$

$\times (1 - 0.4)$

$q = 0.6 \times 2.25 \times p$

$= \frac{3}{5} \times \frac{9}{4} \times p$

$= \frac{27}{20} p$

$\Rightarrow m = \frac{27}{20}$

12 80% of a number is equal to two-thirds of a second number.

The whole number ratio of the first number to the second number in its lowest terms is $x : y$

What is the value of $x - y$?

A 7

B 2

C 1

D 0.2

E –0.2

F –1

G –2

H –7

Let the numbers be a and b such that:

$a = mx$

$b = my$

where m is a non-zero integer

[This ensures $a : b \equiv x : y$]

Question tells us:

$80\% \times a = \frac{2}{3} \times b$

$\Rightarrow \frac{4}{5} mx = \frac{2}{3} my$

$12x = 10y$

$\frac{x}{y} = \frac{5}{6} \Rightarrow x : y = 5 : 6$

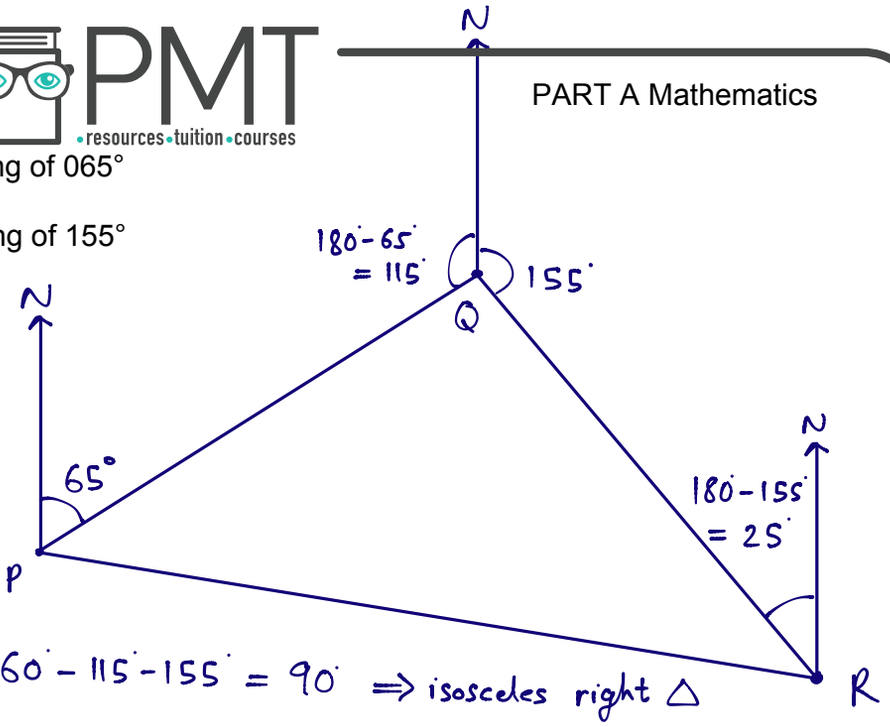
$x = 5, \ y = 6$

$x - y = -1$

13 Q is 5 km away from P on a bearing of 065°

R is 5 km away from Q on a bearing of 155°

What is the bearing of P from R?



A 070°

B 110°

C 225°

D 270°

E 290°

F 315°

G 335°

$\angle PQR = 360 - 115 - 155 = 90 \Rightarrow$ isosceles right $\triangle$

$\angle QRP = \frac{180-90}{2} = 45$ $\therefore$ Bearing of P from R $= 360 - 45 - 25$

$= 290$

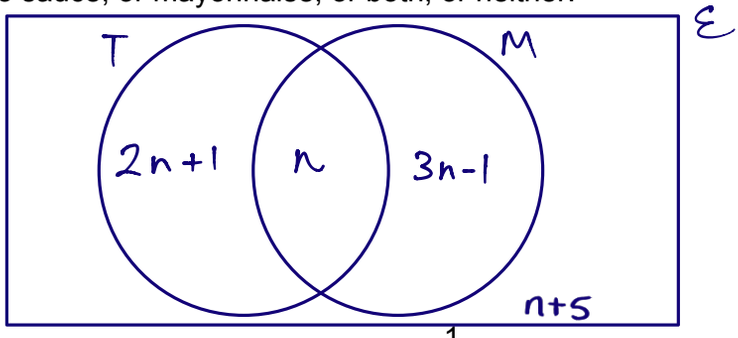
14 With school lunch, students can select tomato sauce, or mayonnaise, or both, or neither.

n students selected both.

$3n + 1$ students selected tomato sauce.

$3n - 1$ students selected **only** mayonnaise.

There were $7n + 5$ students in the group.



The probability of a student, chosen at random, selecting **only** mayonnaise is $\frac{1}{3}$

By finding n, what is the probability of a student, chosen at random, selecting **only** tomato sauce?

A $\frac{3}{11}$

B $\frac{7}{26}$

C $\frac{13}{33}$

D $\frac{3}{8}$

E $\frac{7}{13}$

No. of Tomato only $= n(T) - n(T \cap M)$

$= 3n+1 - n = 2n+1$

No. of neither $= 7n+5 - (2n+1 + n + 3n - 1)$

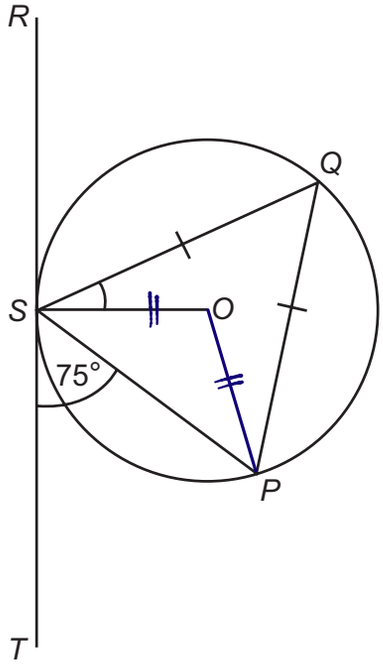
$= n + 5$

$P(\text{only Mayo}) = \frac{3n-1}{7n+5} = \frac{1}{3} \Rightarrow 9n - 3 = 7n + 5$

$n = 4$

$P(\text{only Tomato}) = \frac{2n+1}{7n+5} = \frac{9}{33} = \frac{3}{11}$

15



[diagram not to scale]

The line segment RT is a tangent at the point S to a circle with centre O

Q and P are points on the circumference of the circle such that $QS = QP$

Angle $PST = 75°$

What is the size of angle QSO?

A 15°

B 30°

C 37.5°

D 45°

E 52.5°

F 60°

G 67.5°

H 75°

By Circle Theorems,

$\angle OST = 90°$

$\Rightarrow \angle OSP = 90° - 75° = 15°$

$\triangle SOP$ is isosceles ($SO = OP =$ radius)

$\Rightarrow \angle OPS = \angle OSP = 15°$

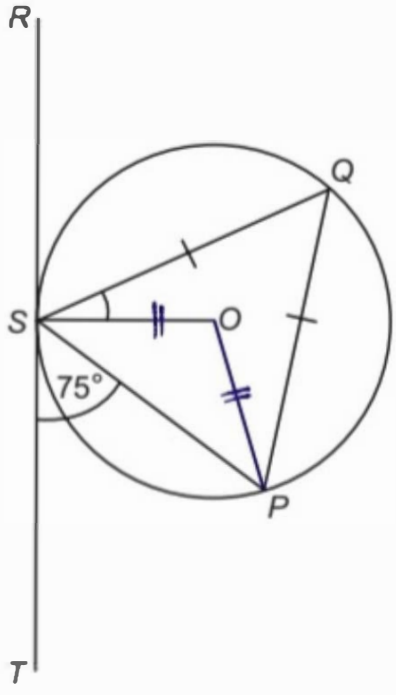
$\angle SOP = 180° - 15° - 15° = 150°$

By Circle Theorems, $\angle SQP = \frac{1}{2}\angle SOP = 75°$

$\triangle SQP$ is isosceles $\Rightarrow \angle QSP = \frac{180° - 75°}{2} = 52.5°$

$\angle QSO = \angle QSP - \angle OSP = 52.5° - 15° = 37.5°$

15



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A 15°

B 30°

C 37.5°

D 45°

E 52.5°

F 60°

G 67.5°

H 75°

By Circle Theorems:

$\angle SQP = 75°$

As $\triangle QSP$ is an isosceles,

$\angle QSP = \angle QPS$

Sum angle in a triangle $= 180°$

$180 - 75 = 105 = 2 \times \angle QSP$

$\frac{105}{2} = \angle QSP$

By Circle Theorems,

$\angle OST = 90°$

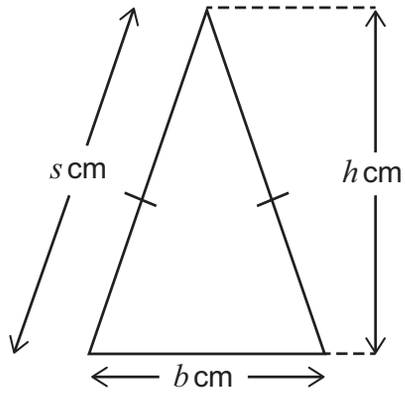
$\angle OST = 75° + (\angle QSP - \angle QSO)$

$\angle QSO = 75° + \angle QSP - \angle OST$

$\angle QSO = 75° + \frac{105}{2}° - 90°$

$= 37.5°$

16



[diagram not to scale]

(1) The vertical height h cm of an isosceles triangle is 3 cm longer than the base length of b cm.

The sloping side is of length s cm.

(2) The area of the triangle is 14 cm^2.

There is one value of s which satisfies these conditions.

Within which range does this value of s lie?

A $5 < s < 6$

B $6 < s < 7$

C $7 < s < 8$

D $8 < s < 9$

E $9 < s < 10$

F $10 < s < 11$

From (1),

$h = b + 3$

From (2),

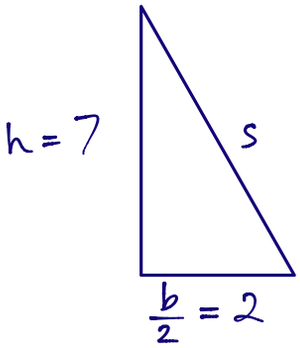
$\text{area} = \frac{1}{2} \times b \times h = \frac{1}{2} \times b(b+3) = 14$

$b^2 + 3b - 28 = 0$

$(b+7)(b-4) = 0$

$\Rightarrow b = 4$

$\Rightarrow h = 7$



Considering the right half of the △

By Pythagoras Theorem,

$s^2 = 7^2 + 2^2$

$s = \sqrt{53}$

$\Rightarrow \sqrt{49} < s < \sqrt{64}$

$\Rightarrow 7 < s < 8$

17 The first five terms of a sequence in order are:

2 17 42 77 122

The n^{th} term of this sequence is $pn^2 + q$ where p and q are integers.

What is the value of $\frac{p-q}{p+q}$

A $\frac{1}{4}$

B $\frac{1}{2}$

C 1

D $\frac{23}{17}$

E $\frac{13}{7}$

F 2

G 4

H 14

When $n = 1$

$p(1)^2 + q = p + q = 2$ ⟶ (1)

When $n = 2$

$p(2)^2 + q = 4p + q = 17$ ⟶ (2)

(2) − (1) : $3p = 15 \Rightarrow p = 5$

$q = 2 - 5 = -3$

$\therefore \frac{p-q}{p+q} = \frac{8}{2} = 4$

18 A bag contains 6 red and 6 green sweets. The sweets are identical apart from their colour.

A child takes a sweet at random from the bag.

If the sweet is red, the child stops taking sweets.

If the sweet is green, it is not replaced and the child takes another sweet.

This continues until a red sweet is taken at which point the child stops taking sweets.

What is the probability that the child takes **more** green sweets than red sweets?

A $\frac{3}{22}$

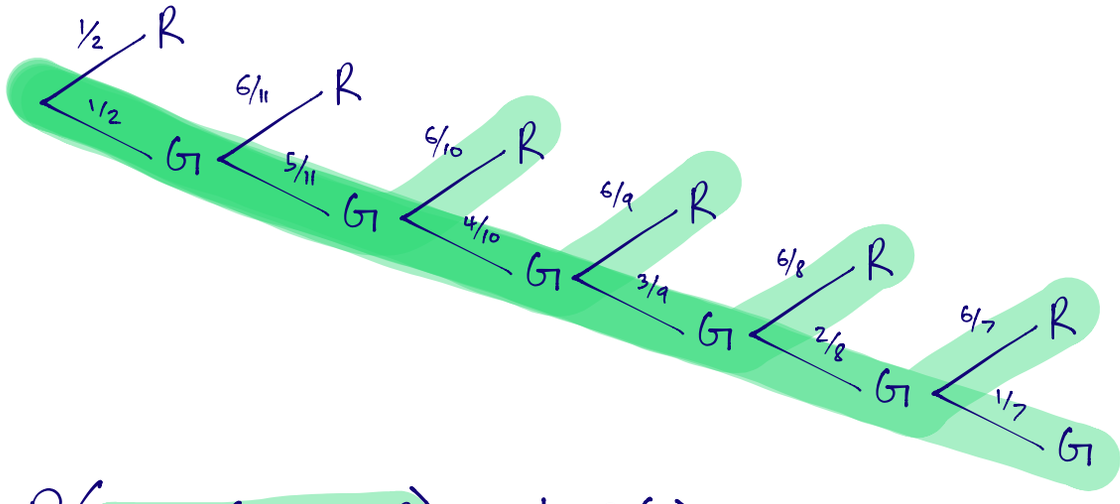
B $\frac{5}{22}$

C $\frac{3}{11}$

D $\frac{1}{2}$

E $\frac{8}{11}$

F $\frac{17}{22}$



$$P(\text{more G than R}) = 1 - P(R) - P(GR)$$

$$= 1 - \frac{1}{2} - \left(\frac{1}{2} \times \frac{6}{11}\right)$$

$$= \frac{1}{2} - \frac{6}{22}$$

$$= \frac{11-6}{22} = \frac{5}{22}$$

PART B Physics

19 An unstable nucleus X becomes a stable nucleus Y after a succession of decays, during which a total of 5 alpha particles and 2 beta (β^-) particles are emitted.

How many fewer protons does nucleus Y contain than nucleus X?

A 6

B 8

C 10

D 12

E 14

F 16

G 18

H 20

Each α emission takes away 2 protons from nucleus X

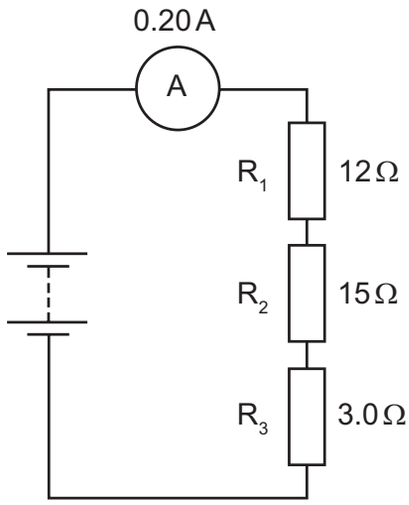
$5 \times 2 = 10$ protons lost

Each β^- emission adds 1 proton to the nucleus

$2 \times 1 = 2$ protons gained

$10 - 2 = 8$ protons less in Y compared to X

20 The diagram shows three resistors R_1, R_2 and R_3 connected in series with a battery of constant voltage. The resistance of each resistor and the corresponding current are also shown.



Resistor R_3 is now removed and the circuit is reconnected.

What is the new current in the circuit?

A 0.20 A

B 0.22 A

C 0.33 A

D 0.40 A

E 0.50 A

F 2.0 A

G 6.0 A

Before removing R_3, using Ohm's Law:

$$V = I R_{Total}$$
$$= 0.20\,A\ (12\,\Omega + 15\,\Omega + 3.0\,\Omega)$$
$$= 6.0\,V$$

After removing R_3, battery voltage is still 6.0 V

$$V = I R_{Total}$$
$$6.0\,V = I\,(12\,\Omega + 15\,\Omega)$$
$$I = \frac{6.0}{27}\,A$$
$$\approx 0.22\,A$$

21 When travelling in a vacuum, visible light has a wavelength between 400 nm and 700 nm.

The speed of light in a vacuum is $3.0 \times 10^8\,\text{m s}^{-1}$.

What can be concluded about **ultraviolet** radiation from this information?

A It has a **maximum** frequency of $2.7 \times 10^{14}\,\text{Hz}$

B It has a **maximum** frequency of $4.3 \times 10^{14}\,\text{Hz}$

C It has a **maximum** frequency of $7.5 \times 10^{14}\,\text{Hz}$

D It has a **maximum** frequency of $1.0 \times 10^{15}\,\text{Hz}$

E It has a **minimum** frequency of $2.7 \times 10^{14}\,\text{Hz}$

F It has a **minimum** frequency of $4.3 \times 10^{14}\,\text{Hz}$

G It has a **minimum** frequency of $7.5 \times 10^{14}\,\text{Hz}$

H It has a **minimum** frequency of $1.0 \times 10^{15}\,\text{Hz}$

UV has a higher frequency than visible light

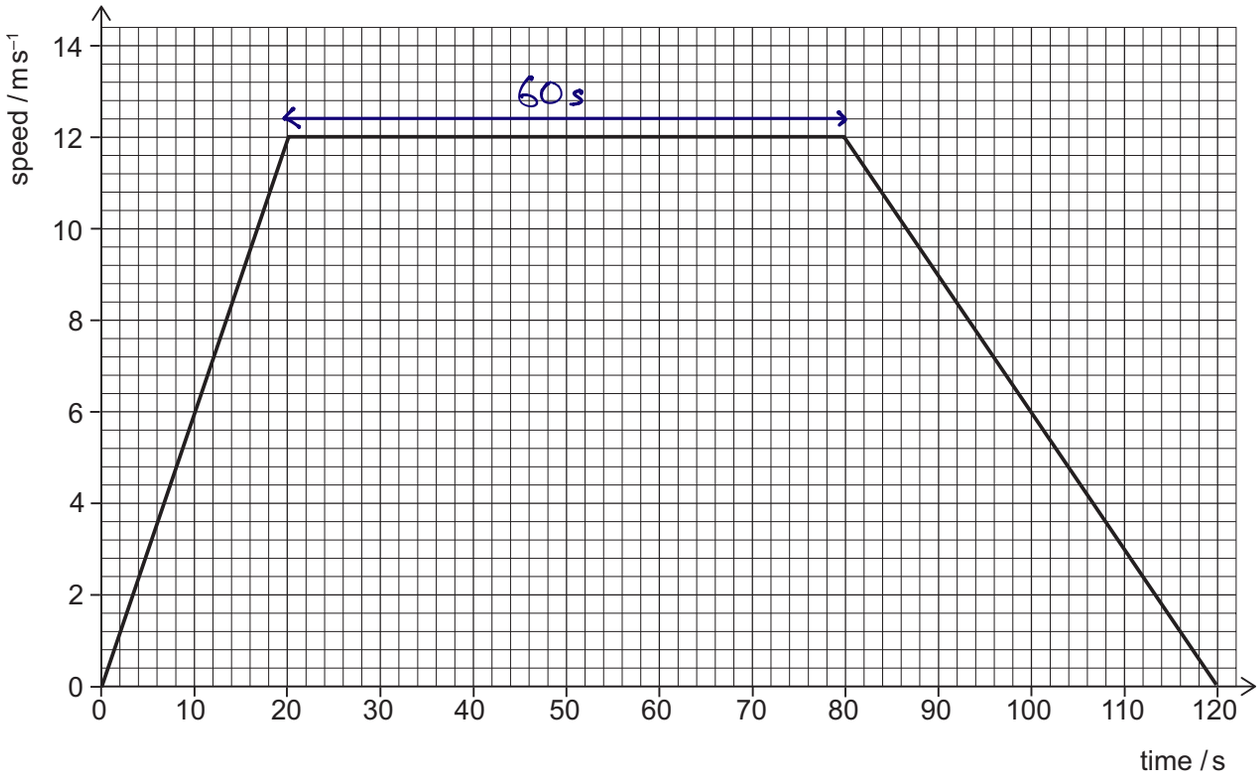
Max frequency of visible light occurs at min. wavelength ($v = f\lambda$)

$= 400\,\text{nm}$

$$f_{\text{max visible}} = \frac{c}{\lambda_{\text{min visible}}} = \frac{3.0 \times 10^8}{400 \times 10^{-9}} = 7.5 \times 10^{14}\ \text{Hz}$$

$$\Rightarrow f_{uv} > 7.5 \times 10^{14}\ \text{Hz}$$

22 The graph is the speed–time graph for a bus travelling in a straight line between two stops.



What is the average speed of the bus during this time?

A 3.0 ms^{-1}

B 4.5 ms^{-1}

C 6.0 ms^{-1}

D 8.0 ms^{-1}

E 9.0 ms^{-1}

F 11 ms^{-1}

G 12 ms^{-1}

Total distance travelled = Area under graph

= Area of trapezium

$= \frac{1}{2}(60 + 120) \times 12$

$= 1080$ m

average speed $= \frac{\text{total distance travelled}}{\text{total time taken}}$

$= \frac{1080\text{ m}}{120\text{ s}} = 9.0\text{ ms}^{-1}$

23 A filament lamp working at its operating voltage converts electrical energy at a rate of 100 W.

The lamp has an efficiency of 5.0%.

How much energy is wasted by the lamp in 10 minutes?

A 50 J

B 950 J

C 1000 J

D 3000 J

E 57 000 J

F 60 000 J

Power lost = 95% × 100 W
= 95 W

Energy wasted = P × Δt
= 95 W × 10 × 60 s
= 57000 J

24 A student is investigating heat flow along a solid uniform metal bar.

The bar has length l, cross-sectional area A, and has its ends maintained at temperatures T_1 and T_2 (where $T_1 > T_2$).

Which relationship represents the rate of heat flow P along the bar?

(Assume that there is no heat transfer through the sides of the bar.)

A $P \propto \frac{(T_1 - T_2)}{Al}$

B $P \propto \frac{(T_1 + T_2)}{Al}$

C $P \propto \frac{A(T_1 - T_2)}{l}$

D $P \propto \frac{A(T_1 + T_2)}{l}$

E $P \propto \frac{l}{A(T_1 - T_2)}$

F $P \propto \frac{l}{A(T_1 + T_2)}$

G $P \propto \frac{Al}{(T_1 - T_2)}$

H $P \propto \frac{Al}{(T_1 + T_2)}$

Larger temperature difference between ends of bar
⇓
Quicker rate of heat flow along bar
⇓
$P \propto (T_1 - T_2)$

Longer bar ⇒ shallower temperature gradient along bar
⇒ slower heat flow
⇒ $P \propto \frac{1}{l}$

Larger cross-section of bar ⇒ More heat can flow along bar
⇒ quicker heat flow
⇒ $P \propto A$

Similar logic to current flow through a wire

$\therefore P \propto \frac{A(T_1 - T_2)}{l}$

25 The potential difference across the motor in an electric car is 400 V and the current in the motor is 1250 A.

The car accelerates along a horizontal road from rest for 4.0 s.

The efficiency of the overall system is 45%.

What is the kinetic energy of the car at the end of the 4.0 s?

(Ignore energy losses due to air resistance and due to friction between the tyres and the road.)

A 225 000 J

B 500 000 J

C 900 000 J

D 1 250 000 J

E 2 000 000 J

$P_{input} = VI = 400V \times 1250\ A$
$= 5.0 \times 10^5\ W$

$P_{useful} = \text{eff.} \times P_{input}$
$= 45\% \times 5.0 \times 10^5\ W$
$= 2.25 \times 10^5\ W$

$KE = P_{useful} \times \Delta t$
$= 2.25 \times 10^5\ W \times 4.0\ s = 900\ 000\ J$

26 The momentum of a small object moving in a straight line is 24 kg m s^{-1} and its kinetic energy is 96 J.

What is the mass of the object?

A 3.0 kg

B 4.0 kg

C 6.0 kg

D 8.0 kg

E 12 kg

Working in SI units

$p = 24$
$mv = 24$

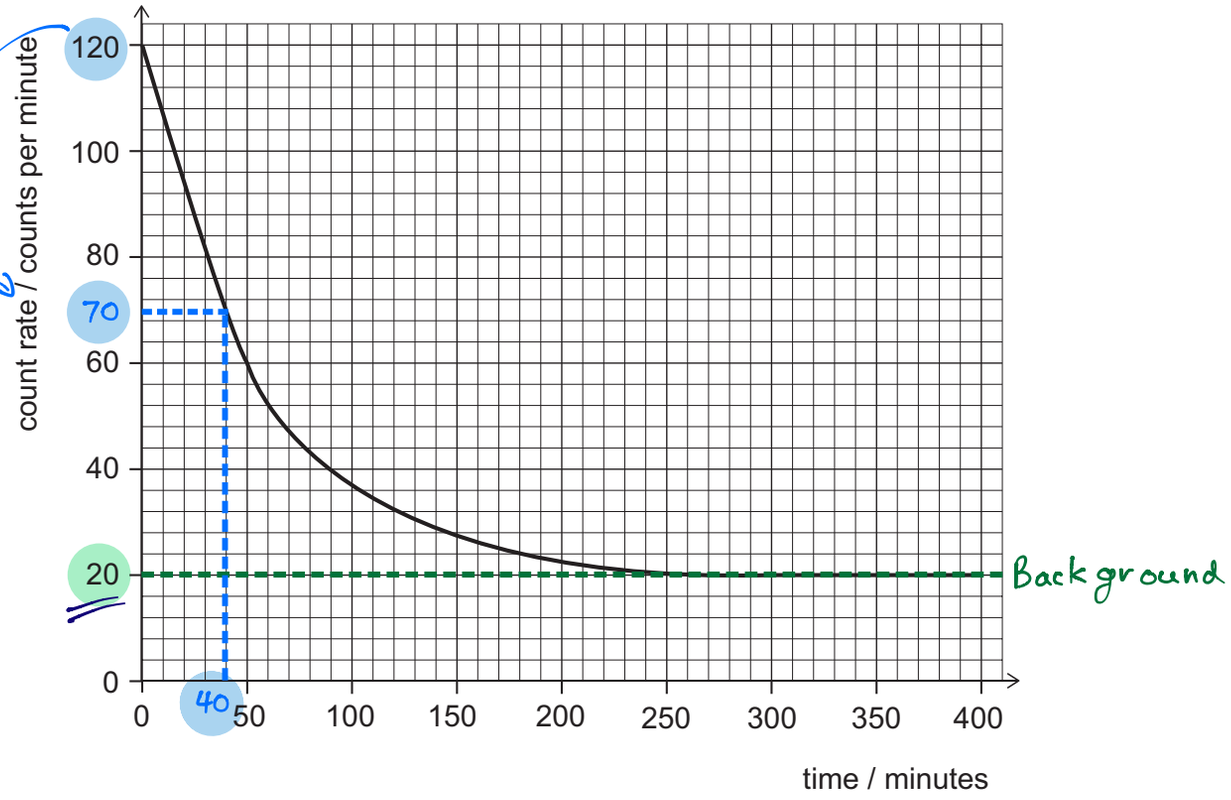
$E_k = 96$
$\frac{1}{2}mv^2 = 96$
$mv \times v = 192$
$24v = 192$
$v = 8$

$\Rightarrow p = m \times 8 = 24$
$m = 3$
$\therefore 3.0\ kg$

27 A radioactive isotope decays in a single step to a stable isotope.

A radiation detector is placed very near to a sample of the radioactive isotope in a laboratory. The count rate on the detector changes as time elapses. The graph shows how the measured count rate changes with time.

120 halves to 70 AFTER background correction $\left(\frac{120-20}{2} + 20\right)$



What is the background count rate and what is the half-life of the isotope?

	background count rate / counts per minute	*half-life of isotope* / minutes
A	20	40
B	20	50
C	20	60
D	20	65
E	120	40
F	120	50
G	120	60
H	120	65

Subtracting Background Radiation,

Initial Count rate = 100

After 1 half-life ↓

50

Add Background to know corresponding measurement on graph ↓

70

On graph it takes 40s for measurement to fall from 120 to 70

28 A rock falling vertically experiences an air resistance force of 12 N at an instant when its acceleration is 2.0 m s^{-2} downwards.

What is the mass of the rock?

(gravitational field strength = 10 N kg^{-1})

A 1.0 kg

B 1.2 kg

C 1.5 kg

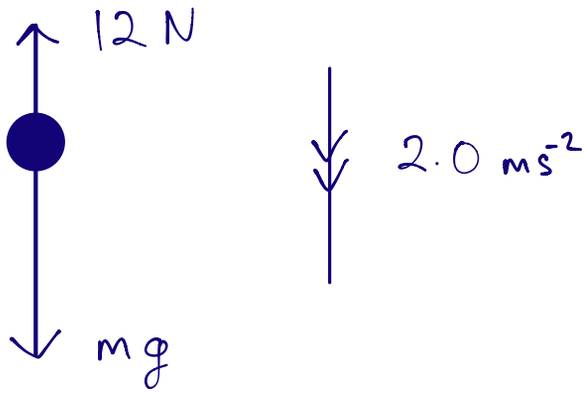
D 6.0 kg

E 10 kg

F 12 kg

G 15 kg

H 60 kg



Using Newton's 2nd Law:

$F_{net} = ma$ [Taking ↓ as +ve]

$10m - 12 = 2m$

$m = 1.5$ kg

29 A transverse wave with an amplitude of 4.0 cm and a frequency of 10 Hz travels along a rope at a speed of 2.4 m s^{-1}.

What is the total distance travelled by a particle in the rope in a time of 20 s?

A 2.4 m

B 4.8 m

C 8.0 m

D 16 m

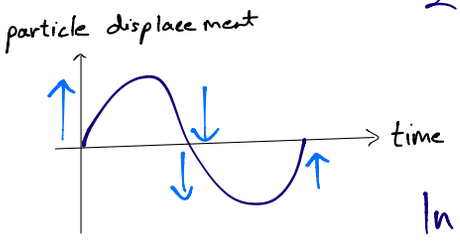
E 32 m

F 48 m

Particle only oscillates ⊥ to wave's direction of travel

$f = 10$ Hz ⟹ In 20 s,

$20 \times 10 = 200$ full wave cycles occur



In 1 cycle, particle travels 4×4.0 cm

$= 0.16$ m

In 200 cycles,

$= 200 \times 0.16$ m

$= 32$ m

30 A student places a measuring cylinder on a balance. She pours a volume V of water into the measuring cylinder, and finds that the mass of the measuring cylinder and water together is 290 g.

She then empties the measuring cylinder and dries it before putting it back on the balance.

She now pours the same volume V of olive oil into the measuring cylinder, and finds that the mass of the measuring cylinder and olive oil together is 270 g.

What is the mass of the measuring cylinder?

(densities: olive oil = 0.90 g cm^{-3}; water = 1.0 g cm^{-3})

A 18 g

B 20 g

C 90 g

D 180 g

E 200 g

Mass of water used = $\rho_w \times V = V$

Mass of oil used = $\rho_o \times V = 0.9V$

Let m = mass of measuring cyl.

From 1st measurement: $m + V = 290$

From 2nd measurement: $m + 0.9V = 270$

$\Rightarrow 0.1V = 20$

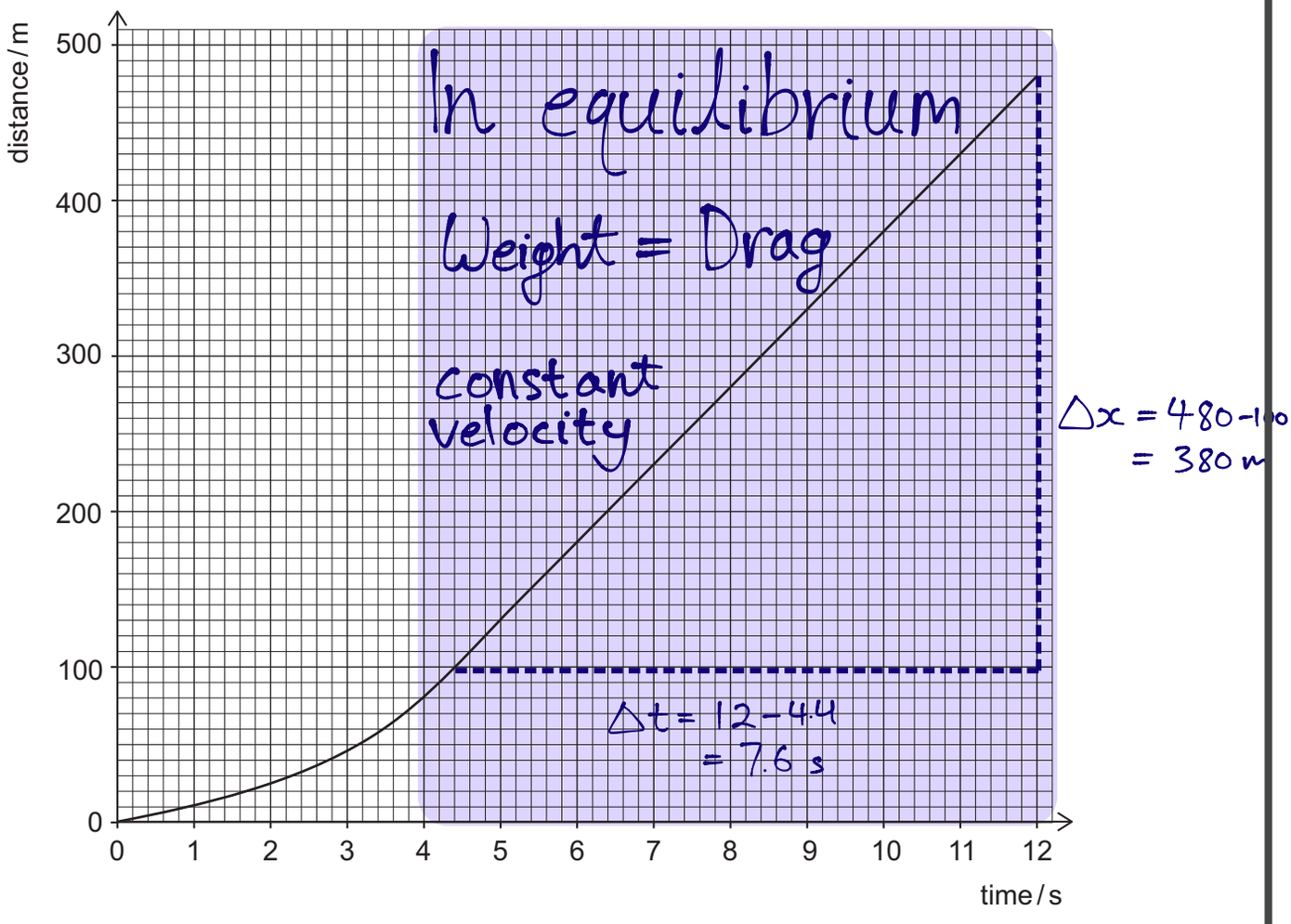
$V = 200$

$m = 290 - 200$

$= 90$ g

31 A skydiver of weight 1000 N falls vertically.

The distance–time graph for the skydiver is shown below.



The air resistance F (in N) acting on the skydiver travelling at velocity v (in $\mathrm{m\,s^{-1}}$) is given by the equation

$$F = kv^2$$

where k (in $\mathrm{N\,m^{-2}\,s^2}$) is a constant.

What is the numerical value of k for the skydiver?

A 0.050

B 0.40

C 0.63

D 2.5

E 20

$v = \frac{380}{7.6} = 50\ \mathrm{ms^{-1}}$

In equilibrium,

$kv^2 = 1000$

$\Rightarrow k = \frac{1000}{50^2} = 0.4$

32 A source generates water waves of fixed frequency that have a wavelength of 1.5 cm.

As they cross a boundary into shallower water their frequency does not change, but their speed is reduced by 18 cm s^{-1}.

The new wavelength is 1.2 cm.

What is the speed of the waves in the shallower water?

A 42 cm s^{-1}

B 50 cm s^{-1}

C 54 cm s^{-1}

D 60 cm s^{-1}

E 72 cm s^{-1}

F 90 cm s^{-1}

Initially, $v_1 = f\lambda_1$

After boundary, $v_2 = f\lambda_2$

$\lambda_1 = 1.5$, $\lambda_2 = 1.2$, $v_1 = v_2 + 18$

Equating frequencies,

$$\frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2} \Rightarrow \frac{v_2+18}{1.5} = \frac{v_2}{1.2}$$

$$1.2v_2 + 21.6 = 1.5v_2$$

$$0.3v_2 = 21.6$$

$$v_2 = \frac{216}{3} = 72 \text{ cms}^{-1}$$

33 A neutron is absorbed by a uranium-235 ($^{235}_{92}U$) nuclide.

The resulting nuclide undergoes fission to produce a bromine-88 ($^{88}_{35}Br$) nuclide, a lanthanum-145 nuclide and some neutrons.

The lanthanum-145 nuclide is radioactive and emits a beta (β^-) particle.

How many neutrons are emitted in the fission reaction and how many protons are there in the nuclide formed by the decay of lanthanum-145?

	neutrons	*protons*
A	2	55
B	2	56
C	2	57
D	2	58
E	3	55
F	3	56
G	3	57
H	3	58

$^{235}_{92}U$ → $^{236}_{92}U$ FISSION

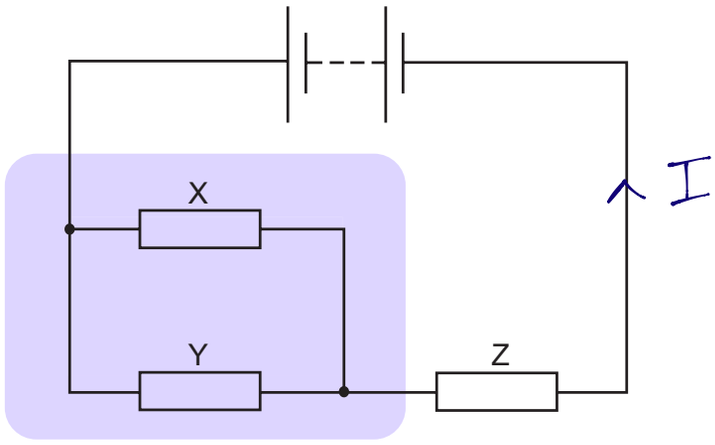
# of neutrons = 236 − 88 − 145 = 3

$^{88}_{35}Br + {}^{145}_{57}La + 3\,{}^{1}_{0}n$

92 − 35 = 57, # protons conserved in fission

$^{145}_{57}La$ → $^{145}_{58}?$ + $^{0}_{-1}\beta$ (β^- DECAY)

34 The diagram shows a circuit containing a battery and three identical resistors X, Y and Z.



The total power supplied by the battery is 18 W.

What is the power dissipated as heat in resistor X?

A 1.5 W

B 2.0 W

C 3.0 W

D 4.5 W

E 6.0 W

F 8.0 W

G 12 W

Let each resistor have resistance R

Resistance of Parallel section:

$$= \frac{1}{\frac{1}{R}+\frac{1}{R}} = \frac{R}{2} = R_{parallel}$$

Total Power dissipated = 18 W

$$\Rightarrow P_{parallel} + P_Z = 18$$

$$I^2 R_{parallel} + I^2 R = 18$$

$$\frac{I^2 R}{2} + I^2 R = 18$$

$$\Rightarrow P_{parallel} : P_Z = 1:2$$

$\therefore \frac{1}{3}$ of total power is dissipated by parallel section

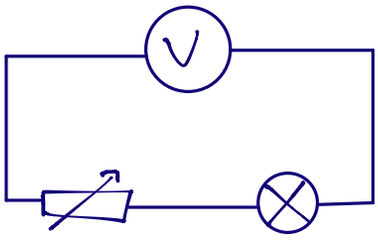
$= 6\,W$

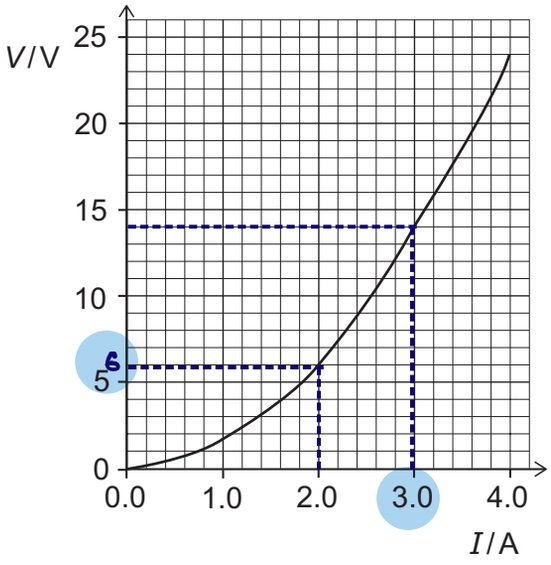
Since each branch of the parallel section has the same current $\left(\frac{I}{2}\right)$ and resistance (R), X and Y dissipate the same power

$$= \frac{6}{2} = \underline{3.0\,W}$$

35 A filament lamp and a 0-10 Ω variable resistor are connected in series with a power supply of constant voltage.

The graph shows the voltage–current (*V–I*) characteristic of the filament lamp.





When the resistance of the variable resistor is 4.0 Ω, the current in the lamp is 2.0 A.

What is the power dissipated in the lamp when the resistance of the variable resistor is zero?

A 12 W

B 14 W

C 16 W

D 28 W

E 42 W

F 96 W

From the V-I characteristic,
when $I = 2.0A$, $V = 6.0V$

$$\Rightarrow R_{lamp} = \frac{V}{I} = 3.0\Omega$$

$$\Rightarrow V_{Power\ Supply} = I R_{Total}$$

$$= 2.0 \times (4 + 3)$$
$$= 14\,V$$

When var. res. is switched off,
V-I graph indicates 3.0 A flows through lamp

$$\therefore P_{lamp} = VI = 14 \times 3 = 42\ W$$

36 Three detectors X, Y and Z are separated by large distances.

Each of the detectors records a seismic wave from the same earthquake whose epicentre (source) is very close to the surface of the Earth.

The wave travels out from the epicentre at $4.0\,km\,s^{-1}$.

Detectors X and Y start to detect the wave at the same time, but detector Z starts to detect it one minute later.

Which of the following statements **must** be correct?

1 The epicentre is at the midpoint of the line XY.

2 Z is equidistant from X and Y.

3 Z is no more than 240 km away from X and from Y.

A none of them

B 1 only

C 2 only

D 3 only

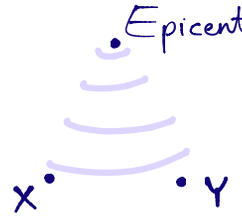
E 1 and 2 only

F 1 and 3 only

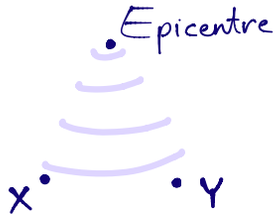
G 2 and 3 only

H 1, 2 and 3

1) False. Counter example:



2) False. Just because Z is further away from the epicentre than X and Y does not imply it is equidistant from X and Y. Counter Example:



3) False. Z is at least $60s \times 4.0\,km\,s^{-1} = 240\,km$ away From either X or Y. It is not equidistant for the same reasons as 2

PART C Chemistry

37 Which row in the following table gives the numbers of protons, neutrons and electrons in $^{64}_{29}Cu^{2+}$?

Atomic Number 29
⇓
29 protons

	number of protons	*number of neutrons*	*number of electrons*
A	27	33	27
B	27	35	29
C	29	35	27
D	29	35	29
E	31	33	29
F	31	35	29

2^+ ion
⇓
lost 2 e^-
29 − 2 = 27 e^-

38 The following exothermic reaction reaches equilibrium at room temperature.

$$C_2H_5OH(l) + CH_3COOH(l) \rightleftharpoons CH_3COOC_2H_5(l) + H_2O(l)$$

Which of the following changes, when applied independently, will alter the position of the equilibrium?

1 increasing the temperature by 25 °C →

2 adding 20 cm^3 of water to the equilibrium mixture

3 adding a catalyst

4 adding an extra 0.5 mol of ethanol (C_2H_5OH)

A 1 only

B 1 and 3 only

C 1, 2 and 4 only

D 2 and 4 only

E 1, 2, 3 and 4

1) Shifts equ. LEFT as reverse reaction is endothermic
2) Shifts equ. LEFT to get rid of water
3) Rate of Forward and Reverse reactions equally raised
⇒ NO NET SHIFT
4) Shifts equ. RIGHT to get rid of ethanol

39 What is the overall process that takes place at the cathode (negative electrode) in the electrolysis of dilute aqueous sodium sulfate?

A $2H^+ + 2e^- \rightarrow H_2$

B $2O^{2-} \rightarrow O_2 + 4e^-$

C $4OH^- \rightarrow O_2 + 2H_2O + 4e^-$

D $Na^+ + e^- \rightarrow Na$

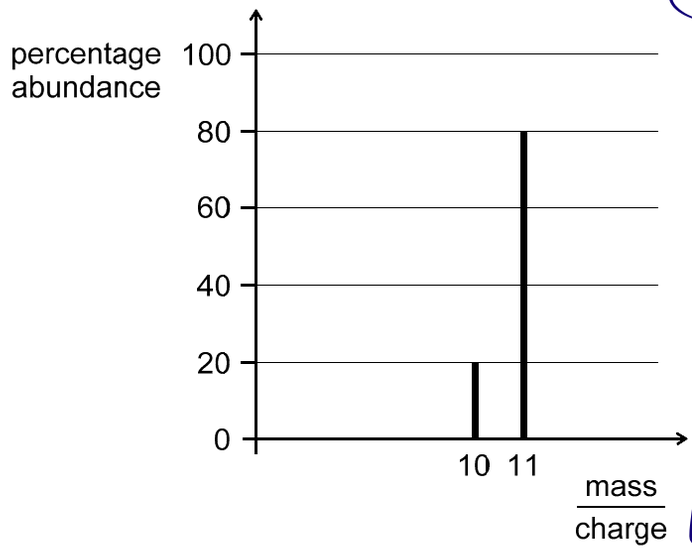
E $SO_4^{2-} \rightarrow SO_2 + O_2 + 2e^-$

Reduction ALWAYS occurs at cathode

According to the Reactivity Series, Na is MORE reactive than H

⇒ Na would rather remain an ion, H^+ is preferentially reduced at cathode

40 A mass spectrum of a sample of element X with atomic number 5 is shown.



⇒ 5 electrons in atom

configuration: 2, 3

3 outer e^- ⇒ GROUP 13

$A_r = 10 \times 0.2 + 11 \times 0.8$

$= 10.8$

Or, you could just pick the A_r closer to 11 than 10

Using the data, which row in the following table best describes the position of X in the Periodic Table and the relative atomic mass of this sample of X?

	Period	*Group*	*relative atomic mass*
A	1	15	10.2
B	1	15	10.8
C	2	13	10.2
D	2	13	10.8
E	3	2	10.2
F	3	2	10.8

41 The gases nitrogen, oxygen and argon can be separated from liquefied air by fractional distillation.

Given the data in the table, in which order would the gases be collected?

	melting point / °C	*boiling point* / °C
nitrogen	–210	–196
oxygen	–218	–183
argon	–189	–186

← LOWEST BP

← HIGHEST BP

A nitrogen, oxygen, argon

B nitrogen, argon, oxygen

C oxygen, nitrogen, argon

D oxygen, argon, nitrogen

E argon, nitrogen, oxygen

F argon, oxygen, nitrogen

Gases are collected in the order they boil off

As the temperature rises,
1st Nitrogen boils off, (−196 °C)
then Argon, (−186 °C)
lastly Oxygen (−183 °C)

42 Concentrated aqueous sodium chloride was electrolysed. After a few minutes, the remaining electrolyte solution was tested with a pH probe at 25 °C.

The gases produced at the electrodes were collected and tested with a colourless aqueous solution of sodium bromide.

Which row in the following table best describes the observations in these tests?

	pH of the remaining solution	*test of gas from anode (positive electrode)*	*test of gas from cathode (negative electrode)*
A	2	no observable change	no observable change
B	2	no observable change	orange solution forms
C	7	orange solution forms	no observable change
D	7	orange solution forms	orange solution forms
E	12	orange solution forms	no observable change
F	12	no observable change	orange solution forms

At cathode : $2H^+ + 2e^- \longrightarrow H_2$ {As Na is MORE reactive than H}

At anode : $2Cl^- \longrightarrow Cl_2 + 2e^-$ {Cl_2 preferred over O_2 because of redox potentials}

Electrolyte is left with NaOH (aq) {BASIC}

Cl_2 displaces Br^- in NaBr, producing orange Br_2

43 The following equations show the main reactions that take place in a blast furnace during the extraction of iron and the removal of the impurities:

$$C + O_2 \rightarrow CO_2$$

I $\underline{CO_2} + C \rightarrow 2CO$

$$Fe_2O_3 + 3CO \rightarrow 2Fe + 3CO_2$$

II $\underline{CaCO_3} \rightarrow CaO + CO_2$

III $\underline{CaO} + SiO_2 \rightarrow CaSiO_3$

NOT REDOX
SEE TABLES BELOW

Which row in the following table correctly identifies whether the underlined substance is oxidised, or reduced, or neither?

	CO_2	$CaCO_3$	CaO
A	oxidised	reduced	neither
B	oxidised	neither	neither
C	oxidised	reduced	oxidised
D	oxidised	neither	oxidised
E	reduced	reduced	neither
F	reduced	neither	neither
G	reduced	reduced	oxidised
H	reduced	neither	oxidised

I in terms of oxidation nos:

CO_2	CO
C : +4	C : +2
O : −2×2	O : −2

∴ CO_2 is reduced by C

II in oxidation nos:

$CaCO_3$	CaO	CO_2
Ca : +2	Ca : +2	
C : +4		C : +4
O : −2×6	O : −2	O : −2×2

III in oxidation nos:

CaO	SiO_2	$CaSiO_3$
Ca : +2		Ca : +2
	Si : +4	Si : +4
O : −2	O : −2×2	O : −2×3

44 X, Y and Z have the **same** electron configuration.

X is an atom, Y is a monatomic anion and Z is a monatomic cation.

Which of the following statements is **always** correct?

A Anion Y has fewer protons than atom X.

B Cation Z has more electrons than protons.

C X, Y and Z are in the same group of the Periodic Table.

D X, Y and Z have consecutive atomic numbers.

E X, Y and Z have the same mass number.

Y had fewer electrons when it was an atom (Smaller atomic no. than X)

Z had more electrons when it was an atom (Larger atomic no. than X)

45 In the Contact process, sulfur dioxide reacts with oxygen to make sulfur trioxide in a reversible reaction.

$$SO_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons SO_3(g)$$

When 5.00 mol of SO_2 and 11.0 mol of O_2 are allowed to reach equilibrium at 450 °C, 80.0% of the SO_2 is converted to SO_3.

What is the volume of the resulting mixture?

(Assume that temperature and pressure are constant, and that at this temperature the volume of one mole of gas is 60.0 dm^3.)

A 240 dm^3

B 336 dm^3

C 600 dm^3

D 720 dm^3

E 840 dm^3

F 960 dm^3

SO_2 remaining = 20% × 5.00 mol
= 1.00 mol
≡ 60.0 dm^3 of SO_2

4.0 mol of SO_2 reacted ⇒ 2.0 mol of O_2 was used
⇒ 11 − 2 = 9.0 mol of O_2 remaining
≡ 9 × 60 = 540 dm^3 of O_2

4.0 mol of SO_3 produced
≡ 4 × 60 dm^3
= 240 dm^3 of SO_3

Total volume at equilibrium = 60 + 540 + 240
= 840 dm^3

46 The non-metallic element phosphorus forms two stable chlorides: PCl_3 (boiling point 76 °C) and PCl_5 (boiling point 161 °C).

Which of the following statements explain(s) the difference in boiling points?

1 There are more covalent bonds in PCl_5 so more energy is required to break them.

2 The forces between the molecules in liquid PCl_5 are stronger.

3 The covalent bonds in PCl_3 are weaker so less energy is required to break them.

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

1) False: Boiling point depends on intermolecular forces (IMF), not covalent bonds

2) True: Higher temperatures needed to overcome stronger IMF

3) False: same reason as 1)

47 A student calculated the mass of anhydrous copper(II) sulfate ($CuSO_4$) required to make 250 cm^3 of an aqueous solution of concentration 0.200 mol dm^{-3}.

However, the student mistakenly made the solution using the same mass of hydrated copper(II) sulfate ($CuSO_4 \cdot 5H_2O$) instead.

What is the concentration, in mol dm^{-3}, of the solution made with the hydrated copper(II) sulfate?

(A_r values: Cu = 64; S = 32; O = 16; H = 1.0)

A 0.128 mol dm^{-3}

B 0.160 mol dm^{-3}

C 0.180 mol dm^{-3}

D 0.200 mol dm^{-3}

E 0.223 mol dm^{-3}

F 0.313 mol dm^{-3}

$M_r(CuSO_4) = 159.6$

$M_r(CuSO_4 \cdot 5H_2O) = 249.6$

amount of $CUSO_4$ needed $= 0.2 \times 0.25$
$= 0.05$ mol

mass of $CUSO_4$ needed $= 0.05 \times 159.6$
$= 7.98$ g

amount of $CUSO_4 \cdot 5H_2O$ used $= \frac{7.98}{249.6} = 0.0319...$

conc. of solution $= \frac{0.0319...}{0.25} = 0.12788...$

≈ 0.128 mol dm^{-3}

48 Bromine is an element in Group 17 of the Periodic Table.

Which of the following statements is/are correct about the element bromine?

1 Bromine will oxidise chloride ions in aqueous solution to form chlorine.

2 Bromine has a lower boiling point than chlorine.

3 Bromine reacts with calcium (Group 2) to form a compound containing 80% bromine by mass.

(A_r values: Cl = 35.5; Ca = 40; Br = 80)

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

1) False : Chlorine is more reactive than Bromine
$\Rightarrow Br_2$ cannot displace Cl^-

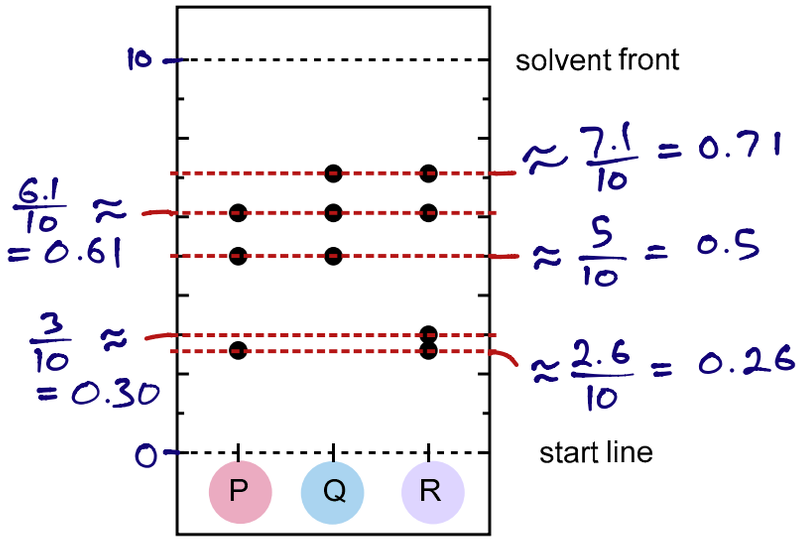
2) False : Bromine has a higher boiling point. Br_2 has more electrons than Cl_2, and hence stronger intermolecular forces

3) True

$$\% Br = \frac{80 \times 2}{80 \times 2 + 40} = 80\%$$

$80 \times 2 + 40$ = $Mr(CaBr_2)$

49 Paper chromatography was used to separate three mixtures of amino acids. The mixtures were labelled P, Q and R.



Each mixture contains some of the five amino acids in the following table. The R_f values were measured for each amino acid with the solvent used to produce the chromatogram.

amino acid	R_f *value*	
asparagine	0.50	P Q
glutamic acid	0.30	
glycine	0.26	
leucine	0.71	Q R
valine	0.61	P Q

Which of the following statements is/are correct?

1 Mixture P contains valine and glycine. ✓

2 Leucine is found in all three mixtures. ✗

3 Glutamic acid is the least mobile amino acid with this solvent. ✗ [2nd lowest R_f]

4 Mixtures P and Q both contain asparagine. ✓

A 1 and 2 only

B 1 and 4 only

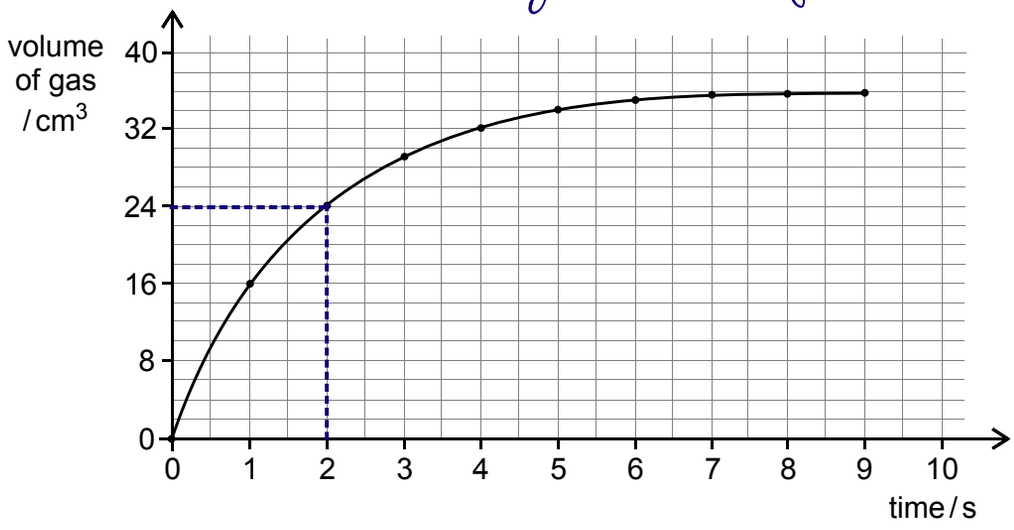
C 2 and 3 only

D 3 only

E 4 only

50 Dilute hydrochloric acid and magnesium were mixed and the total volume of gas released was measured over time.

$2HCl + Mg \longrightarrow MgCl_2 + H_2$



What is the average rate of reaction, in $g\,s^{-1}$, with respect to the magnesium over the first **two** seconds?

(A_r value: Mg = 24. Assume that the volume of one mole of gas is $24\,dm^3$.)

A $0.012\,g\,s^{-1}$

B $0.024\,g\,s^{-1}$

C $0.048\,g\,s^{-1}$

D $12\,g\,s^{-1}$

E $24\,g\,s^{-1}$

F $48\,g\,s^{-1}$

average rate of H_2 production $= \frac{24}{2} = 12\,cm^3\,s^{-1}$

$= 0.012\,dm^3\,s^{-1}$

$= 0.0005\,mol\,s^{-1}$

each mole of H_2 is produced from 1 mol of Mg

$\Rightarrow$ Rate of reaction wrt Mg $= 0.0005\,mol\,s^{-1}$

$\downarrow \times 24$

$= 0.012\,g\,s^{-1}$

51 Copper can react with concentrated nitric acid to form the gas nitrogen monoxide.

$$3Cu + a\,HNO_3 \rightarrow b\,Cu(NO_3)_2 + c\,H_2O + d\,NO$$

What is the value of a when the equation is balanced?

A 6

B 7

C 8

D 9

E 10

F 11

G 12

Balancing Cu : $b = 3$ (1)

Balancing N : $a = 2b + d = d + 6$ (2)

Balancing O : $3a = 6b + c + d \Rightarrow 3a = c + d + 18$ (3)

Balancing H : $a = 2c$ (4)

From (2) : $2c = d + 6$

From (3) : $5c = d + 18$

$\Rightarrow 3c = 12$

$c = 4$

$a = 8$

52 A small amount of a solid mixture, containing calcium carbonate and an inert substance, was added to 50.00 cm^3 dilute hydrochloric acid of concentration 0.1000 mol dm^{-3}.

$$CaCO_3 + 2HCl \rightarrow CaCl_2 + CO_2 + H_2O$$

After all of the calcium carbonate had reacted, the solution was heated to drive off the carbon dioxide.

$HCl_{(aq)}$ was in excess

The resulting solution was neutralised by 12.50 cm^3 of 0.1000 mol dm^{-3} sodium hydroxide solution.

What was the mass of calcium carbonate in the mixture added to the hydrochloric acid?

(M_r value: $CaCO_3$ = 100.0)

A 0.06250 g

B 0.1250 g

C 0.1875 g

D 0.3750 g

E 0.6250 g

F 0.7500 g

Total amount of acid used = 0.05×0.1

$= 0.005$ mol

Amount of acid neutralized by NaOH = 0.0125×0.1

$= 0.00125$ mol

Amount of acid reacting with $CaCO_3$ = $0.005 - 0.00125$

$= 0.00375$ mol

Amount of $CaCO_3 = \frac{1}{2} \times 0.00375$

$= 0.001875$ mol

Mass of $CaCO_3 = 100 \times 0.001875$

$= 0.1875$ g

53 2.80 g of lithium metal is placed in a closed system with 1.20 dm^3 of pure oxygen gas (volume measured at room temperature and pressure).

If a complete reaction occurs between the lithium and the oxygen, what is the maximum mass of lithium oxide that can be formed?

(A_r values: Li = 7; O = 16. Assume that one mole of gas occupies 24.0 dm^3 at room temperature and pressure.)

A 1.50 g

B 3.00 g

C 3.90 g

D 4.60 g

E 6.00 g

F 12.0 g

G 15.6 g

$4Li + O_2 \longrightarrow 2Li_2O$

amount of $O_2 = \frac{1.20}{24} = 0.05$ mol

amount of Li needed $= 0.05 \times 4$
$= 0.2$ mol
$= 0.2 \times 7 = 1.4$ g

$\Rightarrow$ Li is in excess, O_2 is the limiting reactant

Max amount of $LiO_2 = 0.05 \times 2 = 0.1$ mol
$\equiv 0.1(2 \times 7 + 16) = 3.00$ g

54 The following reaction between nitrogen oxide and oxygen releases 116 kJ of energy as heat for each mole of oxygen that reacts.

$$2NO(g) + O_2(g) \rightleftharpoons 2NO_2(g)$$

An excess of NO and y moles of oxygen are mixed in a sealed container. The reaction reaches equilibrium in one hour.

At equilibrium, there are z moles of NO_2.

Assume that the pressure is constant throughout the experiment.

How much heat will be released over this hour?

A 0 kJ

B 58y kJ

C 116y kJ

D 232y kJ

E 58z kJ

F 116z kJ

G 232z kJ

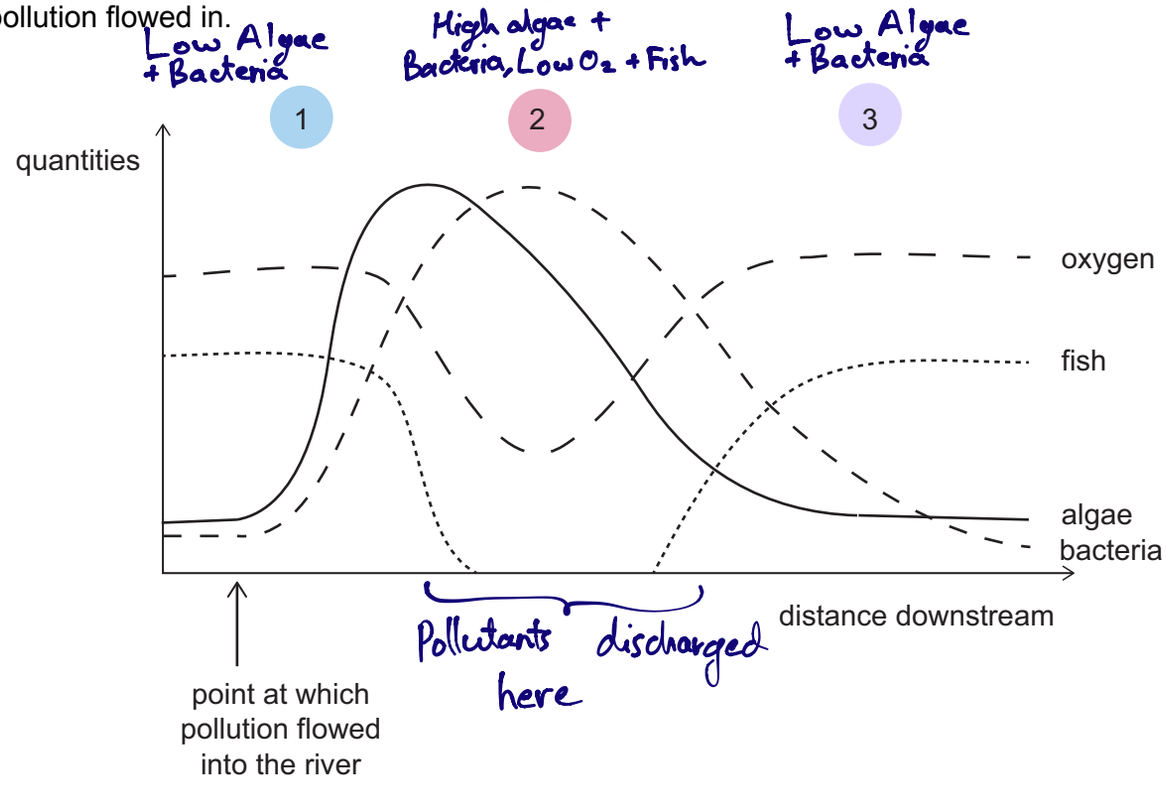
z mol of NO_2 are produced from $\frac{z}{2}$ mol of O_2

1 mol of O_2 produces 116 kJ

$\frac{z}{2}$ mol of O_2 produces $116 \times \frac{z}{2}$
$= 58z$ kJ

PART D Biology

55 The graph shows how four variables change with the distance down a river, after a source of pollution flowed in.



Which indicator species could be expected to be found in large numbers at 1, 2 and 3?

	1	*2*	*3*
A	bloodworm	bloodworm	bloodworm
B	bloodworm	bloodworm	stonefly
C	bloodworm	stonefly	bloodworm
D	bloodworm	stonefly	stonefly
E	stonefly	bloodworm	bloodworm
F	stonefly	bloodworm	stonefly
G	stonefly	stonefly	bloodworm
H	stonefly	stonefly	stonefly

Pollution by sewage / fertilizer causes EUTROPHICATION → Algal bloom, lots of bacteria → Low O_2, No Fish

Bloodworms indicate polluted water

Stonefly indicates clean water

56 Sickle cell anaemia is a recessive genetic condition that results in abnormally-shaped red blood cells due to the production of a faulty type of haemoglobin. Children born with sickle cell anaemia rarely live to adulthood without significant medical intervention. Carriers, who only have one copy of the sickle cell allele, have greater resistance to the disease malaria than people with two copies of the allele for normal functional haemoglobin.

Using this information, which of the following statements is/are correct?

1 People with sickle cell anaemia would have reduced anaerobic respiration in their muscle cells.

2 In areas with malaria the percentage of people surviving with sickle cell anaemia increases.

3 In parts of Africa where malaria is more common you would expect to find more people with a sickle cell allele.

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

1) False. An aerobic respiration would be used more often if the blood has a lower oxygen-carrying capacity

2) False. The risk of dying from SCA is the same

3) True as there is a natural selection pressure that favours individuals with a sickle cell allele

57 A student studied this photograph of part of an organ.

The student drew the following conclusions about the two cells labelled X and Y.

1 Both cells X and Y are found in the same tissue.

2 Both cells X and Y were produced by mitosis.

3 Both cells X and Y have a cell wall.

Which of these conclusions is/are correct?

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

1) True. They are both part of the same protective layer of cells

2) True. They are diploid cells formed by cell division for replacing worn out cells

3) False. Animal cells do not have a cell wall

58 A student investigated the tadpole population in a large pond.

A net with a rectangular opening measuring 0.1 m × 0.2 m was swept through the water for a fixed distance of 1 m. This was repeated 10 times.

All the sweeps were made at the edge of the pond as the student had no waders or boat.

The number of tadpoles in each sweep was recorded in the table.

sweep number	1	2	3	4	5	6	7	8	9	10
number of tadpoles	20	12	32	0	4	8	4	8	12	20

The student made the following statements.

1 Each sweep sampled 0.02 m^3 of water.

2 The frequency of occurrence of the tadpoles was 90%.

3 An accurate estimate of the population size of tadpoles in the pond could be calculated using this data if the total volume of water was known.

Which of the statements about the investigation is/are correct?

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

1) True. Volume = 0.1m × 0.2m × 1m
= 0.02 m^3

2) True. 9 out of the 10 sweeps caught At least 1 tadpole

3) False. All sweeps were made at the edge of the pond. Tadpole population density is likely to be different at other regions of the pond
NOT enough information to ACCURATELY estimate

59 A breeding experiment was carried out using rats.

A pair of rats has eight offspring per litter. The offspring breed freely amongst each other within the same generation. Each female is only allowed to have one litter of eight.

The expected ratio of male to female offspring in this breeding population is the same in rats as in humans, and is seen in every generation.

In the 4th generation of offspring, how many individuals would be expected to have the XY genotype?

A 16

B 32

C 64

D 128

E 256

F 512

G 1024

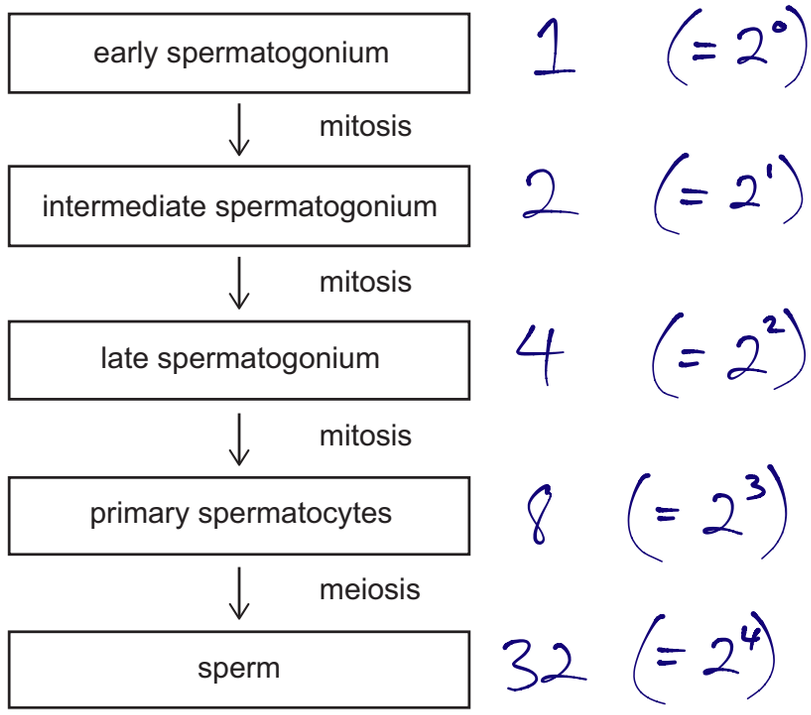
4 generations ⇒ 3 'levels' of litters

Parents
|
Litter
|
Litters' Litter
|
Litters' Litters' Litter

$8^3 = 512$

≈ ½ of them are male (XY) ⇒ $\frac{512}{2} = 256$

60 The diagram shows four steps in the process of human sperm production. For each step only one complete division takes place.



Assuming no mutations and that all of the cells survive, what will be the maximum number of haploid cells originating from a single early spermatogonium?

A 1

B 2

C 8

D 16

E 32

F 64

61 Which of the following conditions is/are required by the cells near the tip of a plant shoot in order for the tip to grow towards light from one direction?

1 sufficient glucose

2 uneven distribution of plant hormone

3 sufficient oxygen

True: required for aerobic respiration, to release energy for cell division

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

2. True, this allows the plant to change its direction of growth towards the Sun.

62 A cell from the epithelium of an animal was removed. The cytoplasm of this cell can be considered as a 2% sugar solution. The living cell was placed in a 4% sugar solution.

Which of the following statements is/are correct?

1 At equilibrium, the sugar concentration in the cell was 6%.

2 Water continued to move across the cell membrane after equilibrium was reached.

3 Osmosis was most rapid when the cell was first placed in the solution.

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

1) False, $2\% < \text{Final}_{\text{conc.}}\ \text{sugar} < 4\%$

2) True, osmosis will occur until there is 0 water potential gradient

3) True, Rate of osmosis is largest when water potential gradient is steepest.

63 A piece of DNA is made up of two complementary strands, each 25 bases long.

14% of the bases are adenine.

Which two statements are correct?

1 Adenine and cytosine together make up 25 bases.

2 Adenine and guanine together make up 50% of the bases.

3 There are 14 thymine bases present.

4 36 of the bases are guanine.

A 1 and 2 only

B 1 and 3 only

C 1 and 4 only

D 2 and 3 only

E 2 and 4 only

F 3 and 4 only

↳ Total = 25 × 2 = 50 bases

→ 14% × 50 ⇒ 7 are adenine

⇒ 7 must be thymine [Complementary to adenine]

1) Out of the 50, # C and G = 50 − # A&T
= 50 − 14
= 36

#C = #G = $\frac{36}{2}$ = 18

#A & C = 7 + 18 = 25

∴ TRUE

2) #A & G = #A & C = 25 = 50% of 50

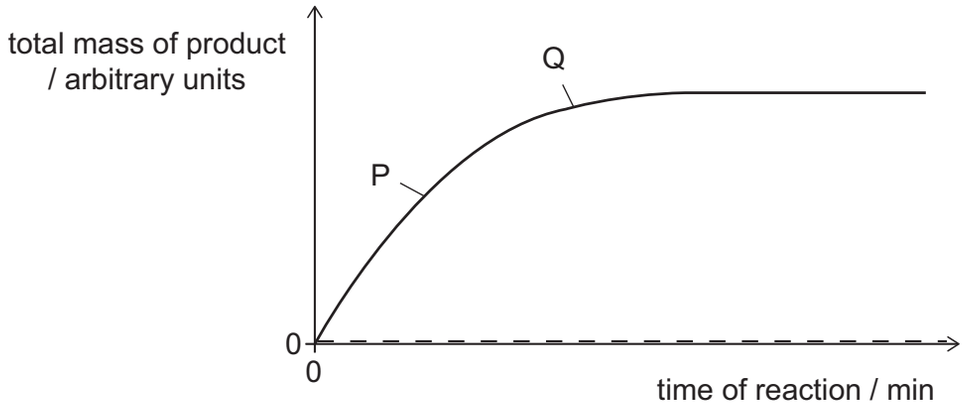
∴ TRUE

3) #T = #A = 7 ∴ FALSE

4) #G = 18 ∴ FALSE

64 In a laboratory, the activity of two lipase enzymes on the same type of lipid was studied. One lipase enzyme was produced from a mutation in the gene that coded for the original enzyme. The mutation occurred in the sequence for three adjacent amino acids called serine, aspartic acid and histidine. The graph shows the results of this study.

Key
——— lipase
- - - mutated lipase



Which of the following statements is/are correct?

1 The serine, aspartic acid and histidine amino acids could be in the active site of the enzyme.

2 All mutations affecting the region coding for serine, aspartic acid and histidine amino acids will be expected to have the same effect.

3 At point Q on the graph, the pH of the reaction mixture will be higher than at P.

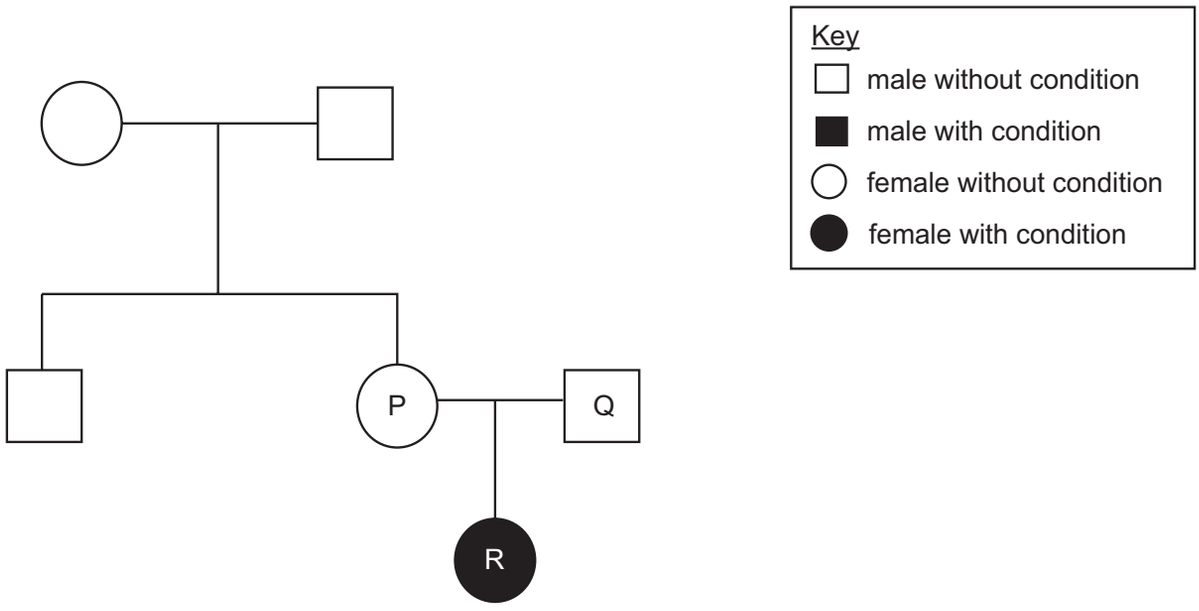
A none of them
B 1 only
C 2 only
D 3 only
E 1 and 2 only
F 1 and 3 only
G 2 and 3 only
H 1, 2 and 3

1) TRUE, as the shape of the active site determines whether or not the enzyme can bind with the substrate and catalyse the reaction

2) FALSE, it depends on what the mutation changes and the sequence / order of amino acids.

3) Lipids $\xrightarrow{\text{Lipase}}$ Fatty Acids + Glycerol
lowers pH at Q
FALSE

65 The family tree shows a family affected by a dominant genetic condition. All people who carry the mutation show symptoms of the condition.



Which of the following statements could explain the presence of the dominant condition in female R?

1 The mutation occurs in P.

2 The mutation occurs in Q's father.

3 The mutation occurs in R.

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

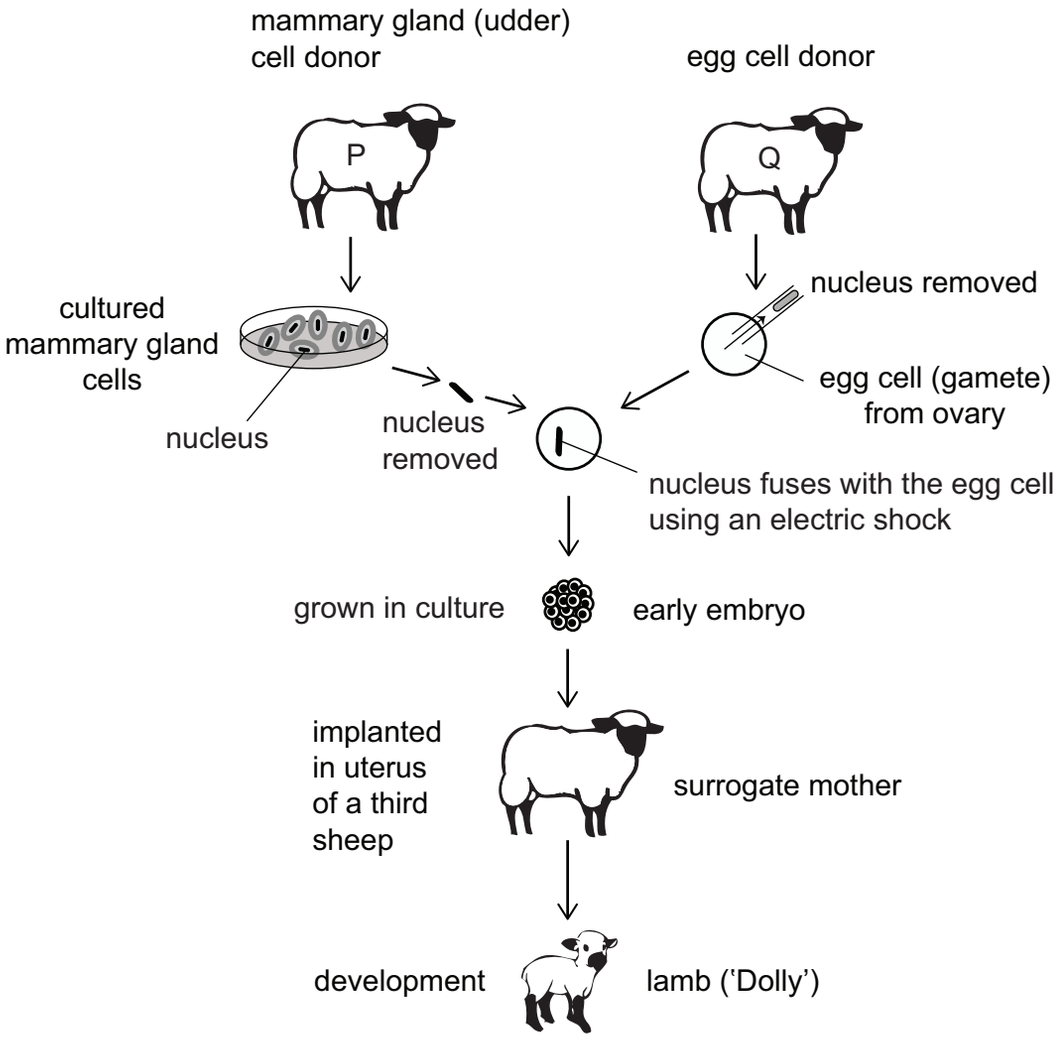
1) TRUE. A mutation occuring in P will be passed to her offspring

2) FALSE. Q would have shown symptoms of the condition in this case

3) TRUE. R may acquire the condition through mutation

66 Dolly the sheep was born in 1996. She was unusual because she had no biological father.

The diagram shows how she was produced.



Which of the following processes had to occur to produce Dolly?

1 genetic engineering

2 mitosis

3 meiosis

4 differentiation

A 1 and 2 only

B 2 and 3 only

C 1, 2 and 4 only

D 1, 3 and 4 only

E 2, 3 and 4 only

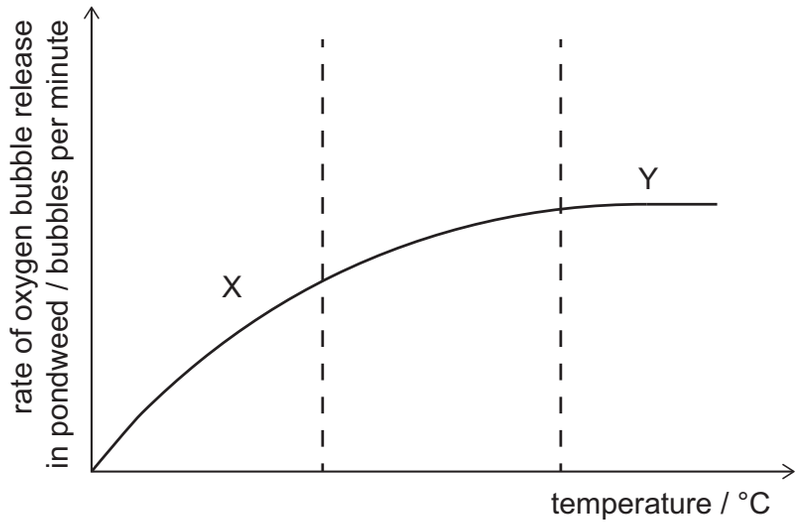
1) False. No genes were modified or manipulated by humans.

2) TRUE. For growth from embryo

3) TRUE. For formation of egg cell in donor

4) TRUE. For specialisation of cells as embryo developed into foetus.

67 A student investigated the rate of oxygen bubble release from a pondweed plant at different temperatures. The rates are shown below, with two sections of the graph marked X and Y. In each investigation all other factors were kept constant.



Which of the following statements about sections X and Y is/are correct?

1 In section X, the kinetic energy of the reaction molecules is increasing with increasing temperature.

2 In section Y, temperature is the factor which limits that rate of oxygen production.

3 In section Y, the plant's enzymes may have denatured.

4 Section Y represents the plant's maximum possible rate of oxygen production under any conditions.

A 1 only

B 2 only

C 3 only

D 4 only

E 1 and 2 only

F 1 and 4 only

G 2 and 3 only

H 3 and 4 only

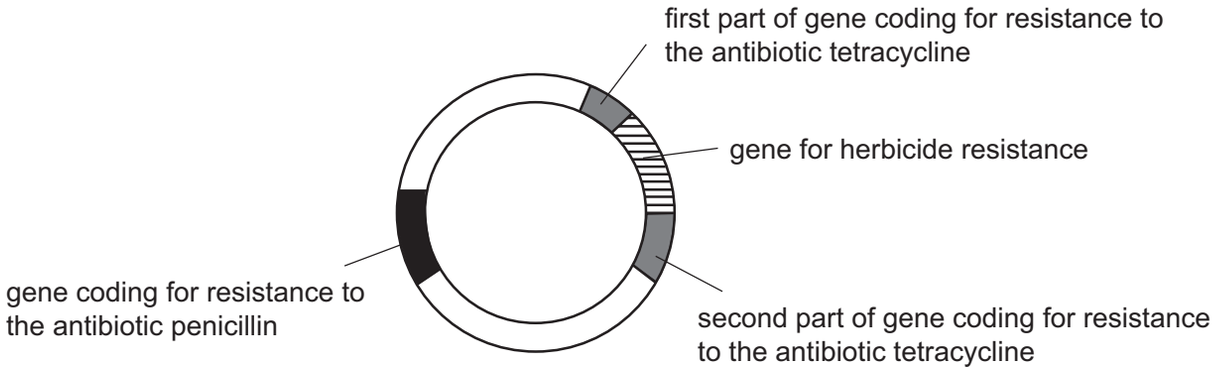
1) TRUE: Rising Temperature increases Kinetic Energy of reactant Molecules. This increases the Rate of reaction

2) FALSE: Temperature is still allowed to rise as you move along Section Y. It is NOT a limiting factor in this case

3) FALSE: If plant enzymes denatured, rate of reaction would have decreased.

4) FALSE: Temperature is NOT the only factor influencing oxygen production [pH, CO_2 conc. etc. also play a role]

68 The diagram below shows a circular piece of bacterial DNA called a plasmid that has been made recombinant by the insertion of a plant gene for herbicide resistance.



Bacteria containing only the recombinant plasmid had to be identified from bacteria that contained only the original plasmid. The original plasmid contained a gene coding for resistance to the antibiotic penicillin and a gene coding for resistance to the antibiotic tetracycline.

These bacteria were grown on two sets of agar plates. One set used agar that contained the antibiotic penicillin and the other set used agar that contained the antibiotic tetracycline.

Which row in the table correctly shows the growth of these bacteria on the two sets of agar?

	bacteria containing only	*growth on agar containing penicillin*	*growth on agar containing tetracycline*
A	recombinant plasmid	yes	yes
	original plasmid	no	no
B	recombinant plasmid	yes	yes
	original plasmid	yes	no
C	recombinant plasmid	yes	no
	original plasmid	yes	yes
D	recombinant plasmid	no	no
	original plasmid	yes	yes
E	recombinant plasmid	yes	yes
	original plasmid	no	yes
F	recombinant plasmid	no	no
	original plasmid	no	no
G	recombinant plasmid	yes	no
	original plasmid	no	yes

Original would be resistant to BOTH antibiotics

Recombinant would NOT be resistant to tetracycline as its resistance gene was broken and had herbicide resi

69 A student analysed a gene sequence that had been identified in four different types of organism. The gene codes for a functional protein. A section of the gene's DNA is shown below. The rest of the DNA from this gene (not shown) is identical in all four different types of organism.

organism	*DNA sequence*					
human	ACG	CCT	CGT	CAC	GCT	AAA
oak tree	ACG	GAA	TAT	GTA	GCT	AAA
mushroom	ACG	GAA	CTC	TTA	GCT	AAA
E.coli bacterium	ACG	TAC	GAT	GGG	GCT	AAA

The student then made the following conclusions:

1 This gene does not code for chlorophyll.

2 This gene may be found in the nucleus or cytoplasm.

3 The protein that this gene codes for is likely to be more similar in plants and fungi than in the other organisms.

Which of these conclusions is/are correct?

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

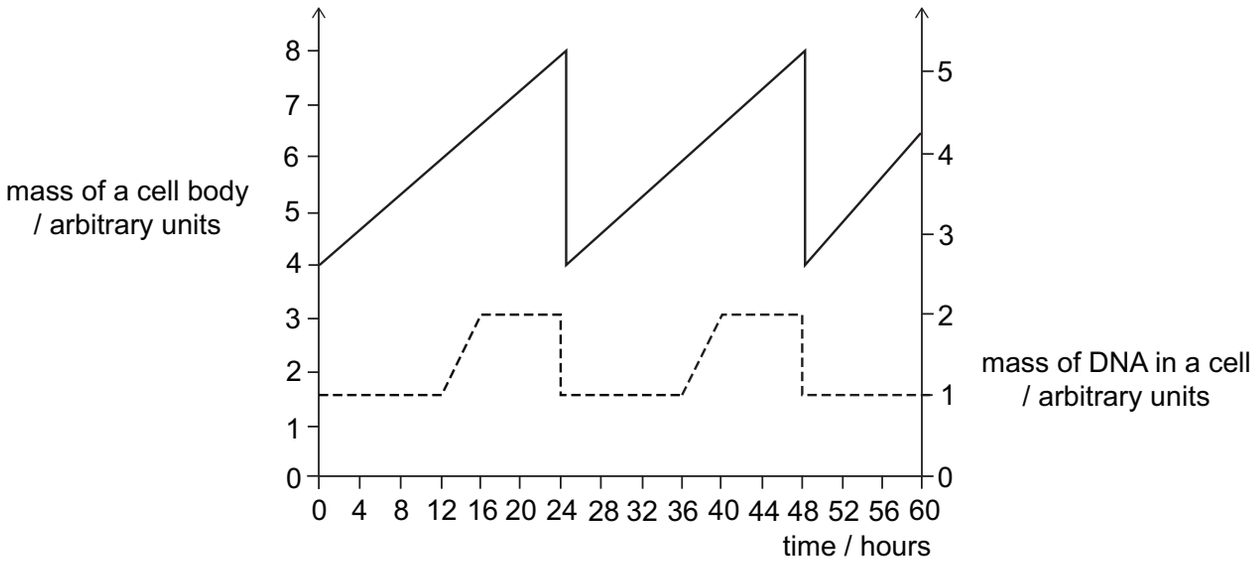
1) TRUE. The chlorophyll gene would have only been present in the oak tree

2) True, as those are the only components of a Fungus (which also Happen to be present in all the other organisms)

3) TRUE. 4 of the 6 triplets above occur identically for the fungus and oak tree. It is 3/6 for any other 2 organisms

70 The graphs represent the changes in the mass of a healthy human body cell and in the mass of the DNA of that cell over time.

Key
——— mass of cell
----- mass of DNA



Using the graph, which of the following statements is/are correct?

1 Mitosis takes place at 12 and 36 hours.

2 The graph shows two cell divisions.

3 The next cell division should take place at 72 hours.

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

1) FALSE: DNA starts to double up at 12 and 36 h,

2) TRUE: 2 big dips in mass

3) TRUE: Big dip occurs every 24h

71 Bt pesticide is used by farmers to kill insect pests. However, widespread use has resulted in the evolution of resistance to this pesticide. A recessive allele causes resistance.

Scientists have suggested that in areas where the Bt pesticide is used, a small number of fields are left untreated. These untreated fields are known as *refugia*. This method has been shown to slow down evolution of resistance to the pesticide.

Which of the following statements explain why refugia could slow down the evolution of resistance to Bt pesticide?

1 When resistant insects breed with pesticide-sensitive insects that do not have the allele for resistance, the offspring produced will be sensitive to the pesticide.

2 When fewer insects are exposed to pesticide, fewer mutations occur that produce alleles for resistance.

3 The refugia help to maintain genetic variation in the population of insect pests.

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

1) TRUE: Since resistance is recessive, The offspring will be pesticide sensitive

2) FALSE: The rate at which insects mutate does not depend on whether or not whether or not pesticides were used

3) TRUE: It provides a region where pesticide-sensitive insects can still live.

72 A diploid plant cell divides by mitosis.

After mitosis of this cell, a mutation occurs that changes the genotype of **one** of the daughter cells. This mutant daughter cell produces a non-functional enzyme instead of the functional enzyme produced by the other daughter cell. This mutation has no effect on the phenotype of the plant or the number or length of chromosomes in the plant cell.

Which of the following statements describe the daughter cells after the mutation has occurred?

1 The chromosomes in the nucleus of each daughter cell will contain the same genes.

2 Both daughter cells will contain the same alleles for every gene.

3 The sequence of bases along each allele will be the same in each daughter cell.

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

1) TRUE: Yes, as the mutant still has the gene to produce the enzyme (albeit non-functional)

2) FALSE: There will be changes to the alleles of the gene corresponding to proteins of the enzyme in the mutant

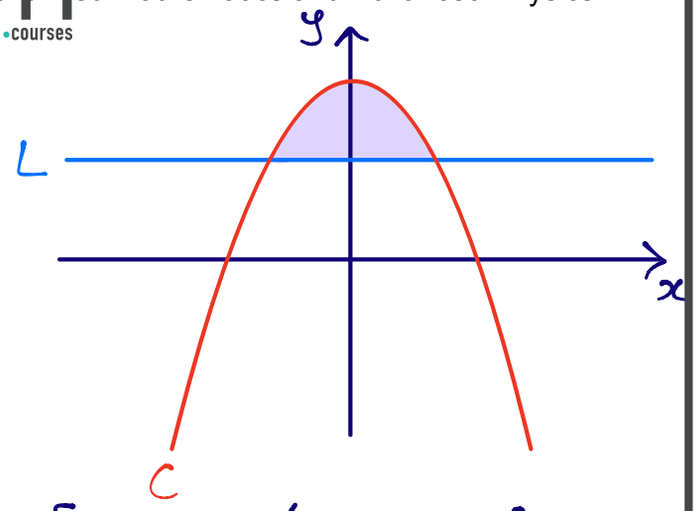
3) FALSE: The mutation changes the sequence of bases for the mutant

PART E Advanced Mathematics and Advanced Physics

73 Curve C has equation $y = 9 - x^2$

Line L has equation $y = 5$

What is the area enclosed between C and L?



A $\frac{32}{3}$

B $\frac{62}{3}$

C $\frac{92}{3}$

D $\frac{122}{3}$

E $\frac{152}{3}$

At intersection points:
$5 = 9 - x^2 \Rightarrow x = \pm 2$

If we shift C down 5 units, C': $y = 4 - x^2$

Area we're looking for $= \int_{-2}^{2} 4 - x^2 \, dx$

$= 2\int_{0}^{2} 4 - x^2 \, dx = 2\left[4x - x^3/3\right]_0^2 = 2\left[8 - 8/3\right]$

$= \frac{32}{3}$

74 An aircraft moves from rest with uniform acceleration along a horizontal runway. After travelling 1600 m it reaches a speed of 80 m s^{-1}.

What is the acceleration of the aircraft?

A 0.025 m s^{-2}

B 0.050 m s^{-2}

C 0.10 m s^{-2}

D 0.50 m s^{-2}

E 2.0 m s^{-2}

F 4.0 m s^{-2}

G 10 m s^{-2}

H 20 m s^{-2}

$s = 1600$
$u = 0$
$v = 80$
$a = ?$

$v^2 = u^2 + 2as$

$80^2 = 0^2 + 2 \times 1600 \times a$

$a = \frac{6400}{3200} = 2.0 \text{ m s}^{-2}$

75 How many solutions of the equation $2\sin^3\theta = \sin\theta$ lie in the interval $-\frac{\pi}{2} \leq \theta \leq \pi$?

$\Rightarrow \sin\theta(2\sin^2\theta - 1) = 0$

A 2

B 3

C 4

D 5

E 6

F 7

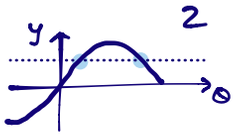
$\sin\theta = 0$

$\Rightarrow \theta = 0, \pi$

$2\sin^2\theta - 1 = 0$

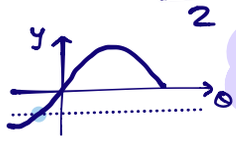
$\sin\theta = \pm\sqrt{\frac{1}{2}} = \frac{\pm\sqrt{2}}{2}$

$\sin\theta = \frac{\sqrt{2}}{2}$



$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

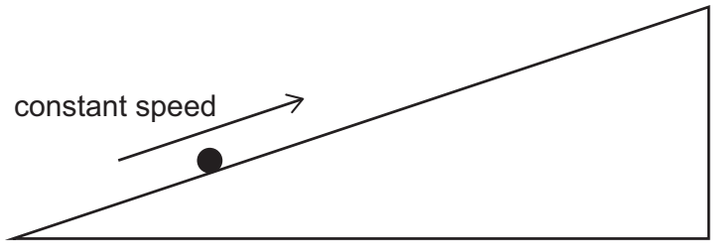
$\sin\theta = \frac{-\sqrt{2}}{2}$



$\theta = -\frac{\pi}{4}$

$\therefore$ 5 solutions

76 The diagram represents a mass that is moving in a straight line at constant speed up a slope of constant gradient.



Which statement about the forces acting on the mass **must** be correct?

A All the forces acting on the mass are equal in magnitude. ✗ Vectors could be different mag. but still cancel eachother out

B Only three forces act on the mass. ✗ 5 : Weight, Reaction, Driving, Friction, Air Resistance

C The force of friction on the mass is equal to the driving force. ✗ Driving Force = Resistance + Air res + Component of weight acting down the slope

D The weight of the mass acts in the opposite direction to the contact force. ✗ Same direction (↓)

E There is no air resistance acting on the mass. ✗ Not enough info. to make this assumption

F There is no resultant force acting on the mass. ✓ Constant Velocity $\Leftrightarrow$ In equilibrium

77 The line $y = x + k$, where k is a constant, is a tangent to the curve $y = 3x^2 - 2x + 1$

What is the value of k?

A -2

B -1

C $\frac{1}{4}$

D $\frac{1}{3}$

E $\frac{1}{2}$

F $\frac{3}{4}$

G 1

H 2

$\Rightarrow$ Only one solution to :

$3x^2 - 2x + 1 = x + k$

$3x^2 - 3x + 1 - k = 0$

$x = \frac{3 \pm \sqrt{9 - 12(1-k)}}{6}$

One solution $\Rightarrow$ Part under square root $= 0$

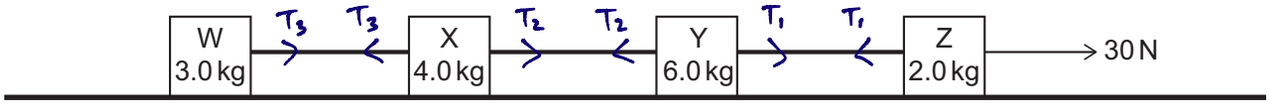
$9 - 12(1-k) = 0$

$1 - k = \frac{3}{4}$

$k = \frac{1}{4}$

78 The diagram shows four objects W, X, Y and Z, of masses 3.0 kg, 4.0 kg, 6.0 kg and 2.0 kg respectively, connected by light, inextensible rods.

The objects are pulled along a smooth, horizontal surface by a constant force of 30 N in the direction indicated.



What is the tension in the rod connecting X and Y?

A 8.0 N

B 10 N

C 12 N

D 14 N

E 16 N

Considering Z,

$F_{net} = ma \Rightarrow 30 - T_1 = 2.0a$

$T_1 = 30 - 2a$

Considering Y,

$T_1 - T_2 = 6a$

$T_2 = T_1 - 6a = 30 - 8a$

$a_x = a_y = a_z = a = \frac{30}{3+4+6+2} = 2.0\ ms^{-2}$ $\Rightarrow$ acceleration of each mass $=$ acc. of full body

In extensible rods

$\Rightarrow T_2 = 30 - 8 \times 2 = 14\ N$

79 In a particular arithmetic progression:

- the 13th term is six times the 1st term $u_{13} = 6u_1$
- the 11th term is 1 less than twice the 5th term $u_{11} = 2u_5 - 1$

What is the 3rd term of the progression?

A −14.5

B −11

C $\frac{29}{19}$

D 3.5

E 11

F 14.5

In an arithmetic progression (aka arithmetic series)

$u_n = u_1 + (n-1)d$ — d = common difference between consecutive terms

$u_{13} = u_1 + 12d = 6u_1$

$\Rightarrow u_1 = \frac{12}{5}d$

$u_{11} = u_1 + 10d = 2u_5 - 1$

$\Rightarrow \frac{12}{5}d + 10d = 2u_1 + 8d - 1$

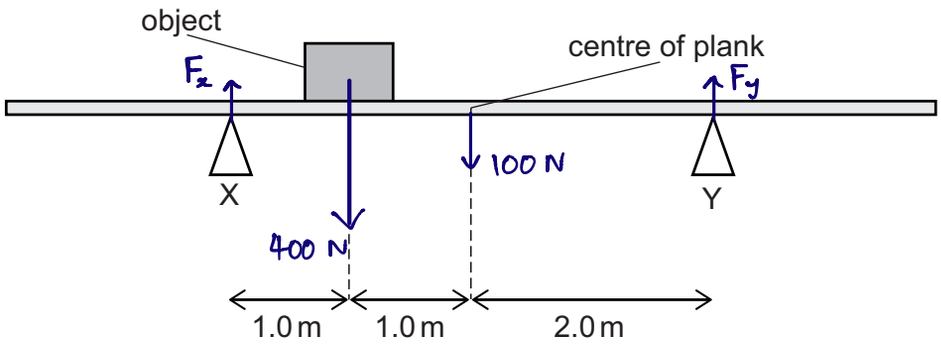
$\frac{62}{5}d = \frac{24}{5}d + \frac{40d}{5} - 1 \Rightarrow d = 5/2$

$\therefore u_3 = u_1 + 2d$

$= \frac{12}{5} \times \frac{5}{2} + 2 \times \frac{5}{2}$

$= 6 + 5 = 11$

80 An object of mass 40 kg is placed on a uniform, horizontal plank of mass 10 kg between two supports X and Y as shown in the diagram.



What is the contact force at X?

(gravitational field strength = 10 N kg^{-1})

A 15 N

B 35 N

C 150 N

D 250 N

E 300 N

F 350 N

G 375 N

Taking moments about Y,

$\sum$ ↻ moments = $\sum$ ↺ moments

$4F_x = 400 \times 3 + 100 \times 2$

$F_x = \frac{1200 + 200}{4} = 350$ N

81 Evaluate

$$\log_2\left(\frac{5}{4}\right) + \log_2\left(\frac{6}{5}\right) + \log_2\left(\frac{7}{6}\right) + \cdots + \log_2\left(\frac{64}{63}\right)$$

A -2

B 3

C 4

D 6

E $\log_2(3!)$

F $\log_2 60$

$= \log_2(5) - \log_2(4) + \log_2(6) - \log_2(5)$
$+ \log_2(7) - \log_2(6) + \ldots + \log_2(64) - \log_2(63)$

$= \log_2(64) - \log_2(4) = \log_2\left(\frac{64}{4}\right)$

$= \log_2(16)$

$= \log_2(2^4) = 4$

82 An object X of mass 2.0 kg is initially moving at a speed of 4.5 m s^{-1} on a smooth, horizontal surface.

A 5.0 N force is applied to X in the direction of its motion for 3.0 seconds.

A short time later it collides head on with, and sticks to, a stationary object Y of mass 3.0 kg.

What is the speed of X and Y as they move off together after the collision?

A 1.8 m s^{-1}

B 3.0 m s^{-1}

C 3.6 m s^{-1}

D 4.8 m s^{-1}

E 5.4 m s^{-1}

Initial momentum of X $= 2 \times 4.5 = 9.0$ kg m s^{-1}

Momentum just before collision = Initial momentum + Impulse Experienced

$= 9 + 5 \times 3 = 24$ kg m s^{-1}

Speed of X+Y $= \frac{\text{Momentum of X+Y}}{(2+3)\text{ kg}} = \frac{16}{5} = 4.8$ m s^{-1}

83 Circle C has equation $(x+3)^2 + (y-2)^2 = 5$

The length of the tangent from the circle C to the point P is $5\sqrt{3}$

What is the shortest distance from P to C?

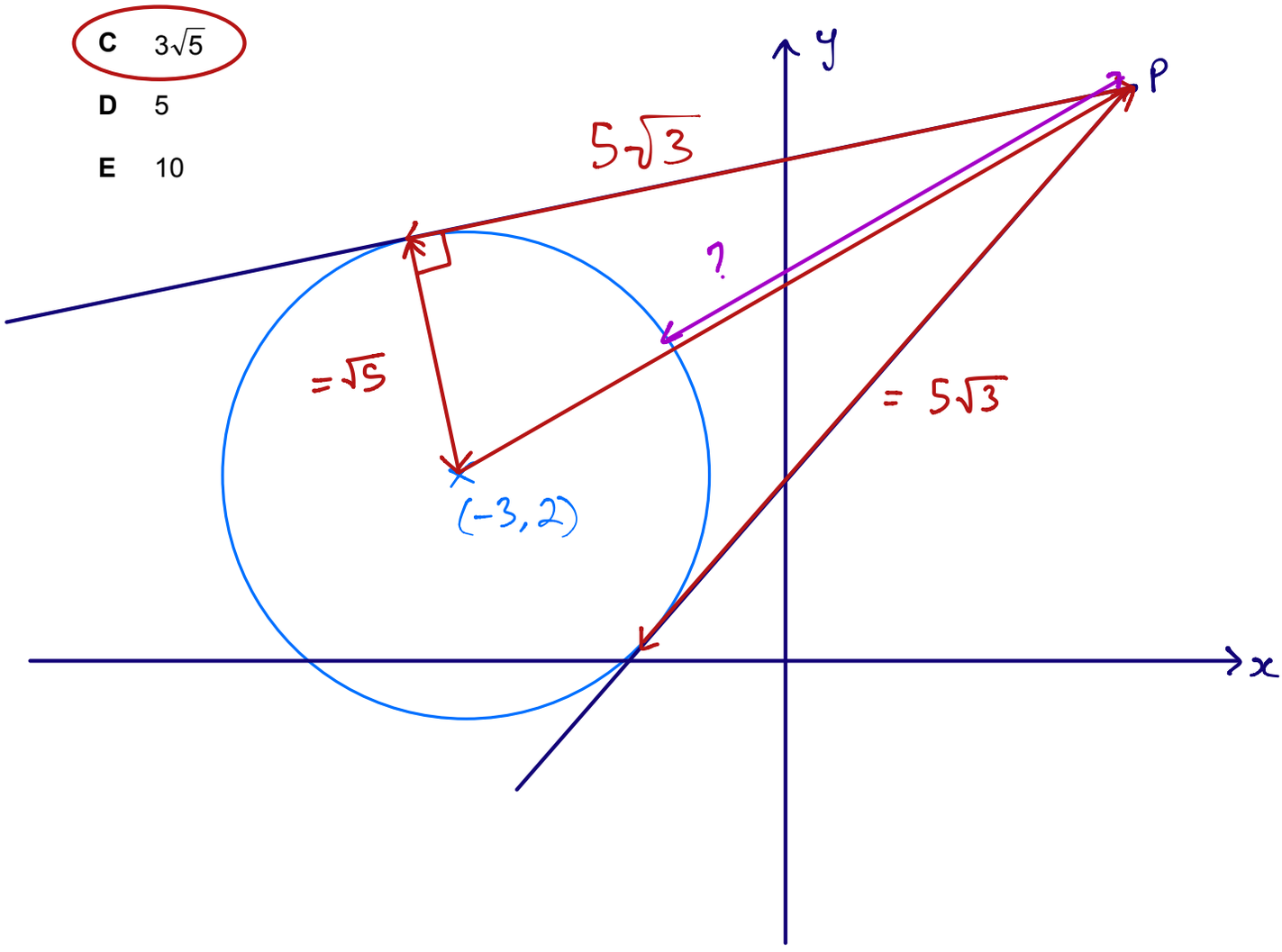
A $5\sqrt{3}$

B $5\sqrt{3} + \sqrt{5}$

C $3\sqrt{5}$

D 5

E 10



$$? = \sqrt{(5\sqrt{3})^2 + (\sqrt{5})^2} - \sqrt{5}$$

$$= \sqrt{75+5} - \sqrt{5}$$

$$= \sqrt{16\times 5} - \sqrt{5}$$

$$= 4\sqrt{5} - \sqrt{5}$$

$$= 3\sqrt{5}$$

Alternatively,

We know ? must be smaller than $3\sqrt{5}$

Option C is the only one $< 3\sqrt{5}$

→ +ve
−ve

84 Two solid spheres X and Y have masses m and $2m$ respectively. They travel in opposite directions towards each other along the same line with speeds v and $2v$ respectively and collide head on.

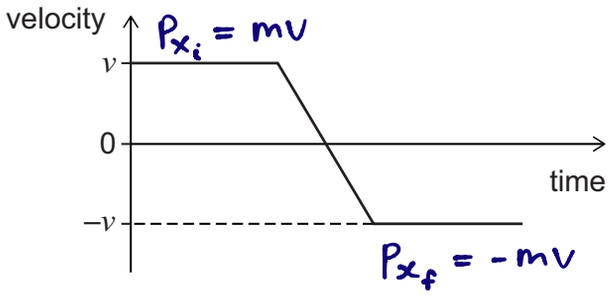
Initial momentum of system = $mv - 4mv = -3mv$

The graph shows the variation of velocity with time for sphere X before, during, and after the collision.

$P_{x_i} = mv$

$P_{y_i} = -4mv$

So NOT options A, C or E

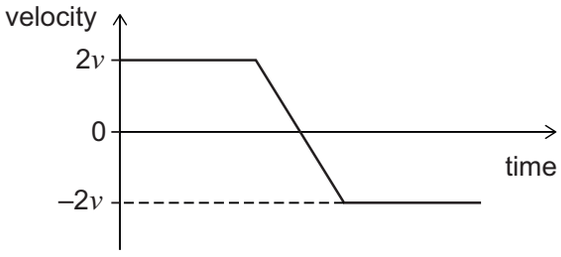


$P_{x_i} = mv$

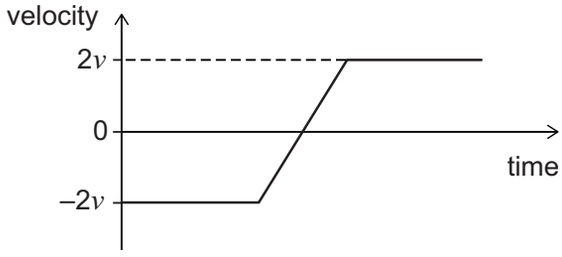
$P_{x_f} = -mv \Rightarrow P_{y_f} = -2mv$

Which sketch shows the variation of velocity with time for sphere Y?

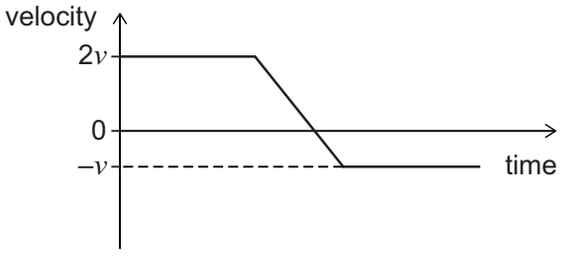
A



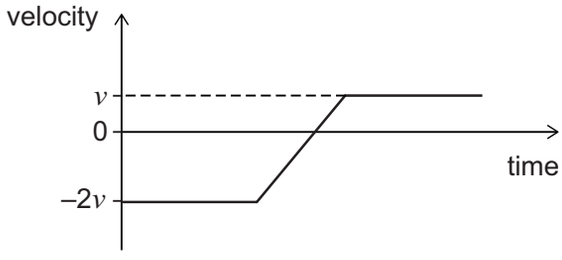
B



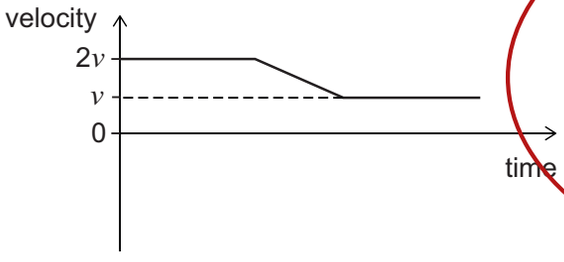
C



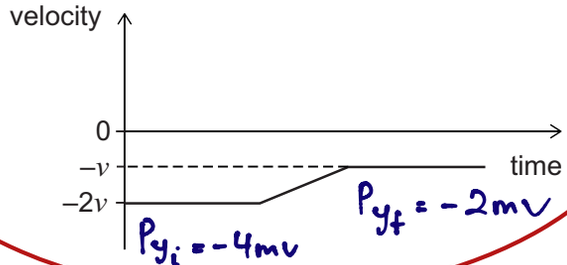
D



E



F



$P_{y_f} = -2mv$

$P_{y_i} = -4mv$

85 What is the coefficient of x^3 in the expansion of $(1-2x)^5(1+2x)^5$?

$(a-b)(a+b) \equiv a^2 - b^2$

$(1-2x)^5(1+2x)^5 = (1-2x)(1+2x)^5$

$= (1-4x^2)^5$

Expanding this only gives even powers of x:

$(4x^2)^0, (4x^2)^1, (4x^2)^2, \ldots, (4x^2)^5$

A −6400

B −640

C −80

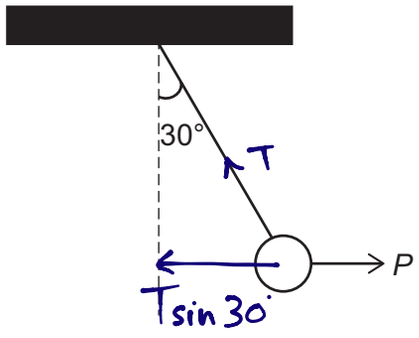
D 0

E 80

F 800

G 960

86 A metal ball suspended from a steel cable is held at rest by a horizontal force P. The cable makes an angle of 30° to the vertical as shown in the diagram. The cable exerts a force T on the ball.



In equilibrium

$\Rightarrow P = T_{\text{Horizontal}}$

$T\sin 30° = T/2$

What is the magnitude of P?

A $\frac{T}{2}$

B T

C $2T$

D $\frac{T}{\sqrt{2}}$

E $\frac{T}{\sqrt{3}}$

F $\frac{2T}{\sqrt{3}}$

G $\frac{\sqrt{3}T}{2}$

87 Given that

$$\int_0^2 x^m \, dx = \frac{16\sqrt{2}}{7} = \left[\frac{x^{m+1}}{m+1}\right]_0^2 = \frac{2^{m+1}}{m+1} \longrightarrow 1$$

and

$$\int_0^2 x^{m+1} \, dx = \frac{32\sqrt{2}}{9} = \left[\frac{x^{m+2}}{m+2}\right]_0^2 = \frac{2^{m+2}}{m+2} \longrightarrow 2$$

what is the value of m?

A $-\frac{11}{2}$

B $-\frac{9}{2}$

C $-\frac{22}{29}$

D $\frac{7}{22}$

E $\frac{5}{2}$

F $\frac{7}{2}$

2 ÷ 1 :

$$\frac{32\sqrt{2}}{9} \times \frac{7}{16\sqrt{2}} = \frac{2^{m+2}}{m+2} \times \frac{m+1}{2^{m+1}}$$

$$\frac{32\sqrt{2}}{9} \times \frac{7}{16\sqrt{2}} = \frac{2(m+1)}{m+2}$$

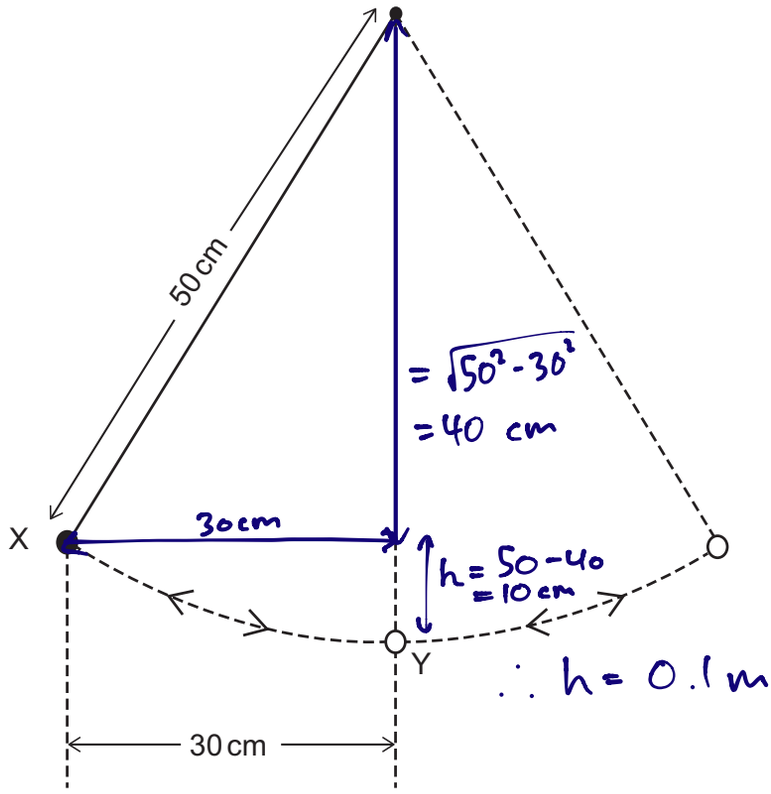
$$\frac{m+1}{m+2} = \frac{7}{9}$$

$$9m + 9 = 7m + 14$$

$$m = \frac{5}{2}$$

88 A pendulum bob of mass 10 g is suspended by a light, inextensible string of length 50 cm.

The bob is released from rest at position X.



$= \sqrt{50^2 - 30^2}$
$= 40$ cm
$h = 50 - 40$
$= 10$ cm
$\therefore h = 0.1$ m

What is the speed of the bob as it passes through position Y?

(gravitational field strength $g = 10\,\text{N kg}^{-1}$; assume that resistive forces are negligible)

A $\sqrt{2}\,\text{m s}^{-1}$

B $\sqrt{4}\,\text{m s}^{-1}$

C $\sqrt{6}\,\text{m s}^{-1}$

D $\sqrt{8}\,\text{m s}^{-1}$

E $\sqrt{10}\,\text{m s}^{-1}$

gpe lost = ke gained

$mgh = \frac{1}{2}mv^2$

$v = \sqrt{2gh}$

$= \sqrt{2 \times 10 \times 0.1}$

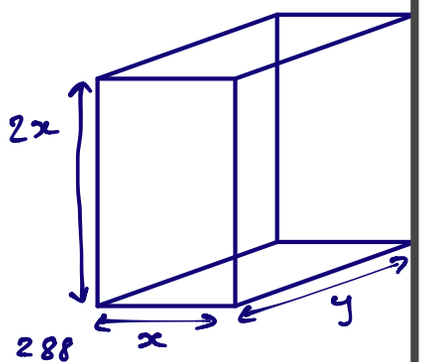
$= \sqrt{2}\ \text{m s}^{-1}$

89 The dimensions of a solid cuboid, in cm, are x, $2x$ and y

The volume of the cuboid is 576 cm^3.

At this volume, the surface area of the cuboid has its maximum value.

What is the area, in cm^2, of the face that has the largest area?



A $2(288)^{\frac{2}{3}}$

B 72

C 96

D 432

E $4(144)^{\frac{2}{3}}$

Volume $= 2x^2y = 576 \Rightarrow y = \frac{288}{x^2}$

Total Surface Area $= S = 2(2x \times x) + 2(x \times y) + 2(2x \times y)$

$= 4x^2 + 6xy = 4x^2 + \frac{1728}{x}$

$\frac{dS}{dx} = 8x - \frac{1728}{x^2} = 0$

$x^3 = \frac{1728}{8} = 216 \Rightarrow x = 6$

$y = \frac{288}{36} = 8$

∴ Biggest side
$= 2xy$
$= 2 \times 6 \times 8$
$= 96$

90 An object is thrown vertically upwards from ground level with an initial velocity of 40 $m\,s^{-1}$.

2.0 seconds later another object is released from a height above the ground and falls vertically from rest.

Both of the objects hit the ground at the same time.

From what height above the ground was the second object released?

(gravitational field strength g = 10 $N\,kg^{-1}$; air resistance can be ignored)

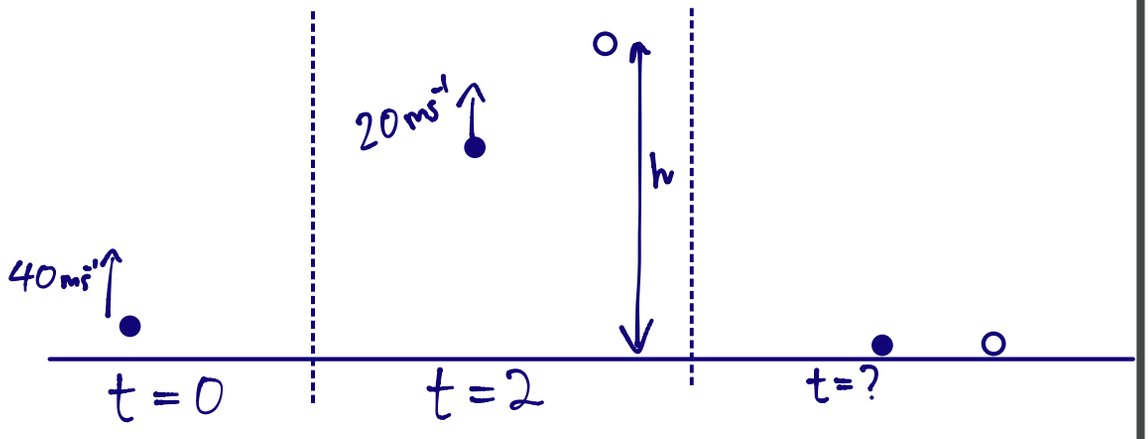
A 80 m

B 180 m

C 320 m

D 500 m

E 900 m



Time taken for 1st object to hit ground from launch $= \frac{\Delta v}{a} = \frac{40 - (-40)}{10} = 8.0$s

⇒ 2nd object has $8 - 2 = 6.0$s to reach the ground

$s = ?$
$u = 0$
$v =$
$a = 10$
$t = 6$

$s = ut + \frac{1}{2}at^2$

$= 0 \times 6 + \frac{1}{2} \times 10 \times 6^2 = 5 \times 36 = 180$ m