ENGAA 2020
Section 2

Model Solutions

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1 Spring P has spring constant $1.0\,\text{N cm}^{-1}$ and spring Q has spring constant $3.0\,\text{N cm}^{-1}$.

The two springs are connected in series.

The springs are stretched by 6.0 cm in total.

What is the extension of spring P?

(The springs have negligible mass and obey Hooke's law.)

A 1.5 cm

B 2.0 cm

C 3.0 cm

D 4.0 cm

E 4.5 cm

Splitting extension in ratio of spring constants:

$1:3 \rightarrow \frac{3}{2} : \frac{9}{2}$

$P:Q \rightarrow Q:P$

The ratios are reversed because greater spring constant means less extension.

$\Rightarrow$ extension of P is $\frac{9}{2} = 4.5$ cm

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2 A single strand of wire has a radius of 2.0×10^{-4} m and length 15 m. The resistivity of the material from which the wire is made is $4.8 \times 10^{-7}\,\Omega$ m.

Twelve strands of this wire are connected in parallel to make a cable.

What is the resistance of the cable?

A $\dfrac{\pi}{2160}\Omega$

B $\dfrac{\pi}{180}\Omega$

C $\dfrac{\pi}{15}\Omega$

D $\dfrac{15}{\pi}\Omega$

E $\dfrac{180}{\pi}\Omega$

F $\dfrac{2160}{\pi}\Omega$

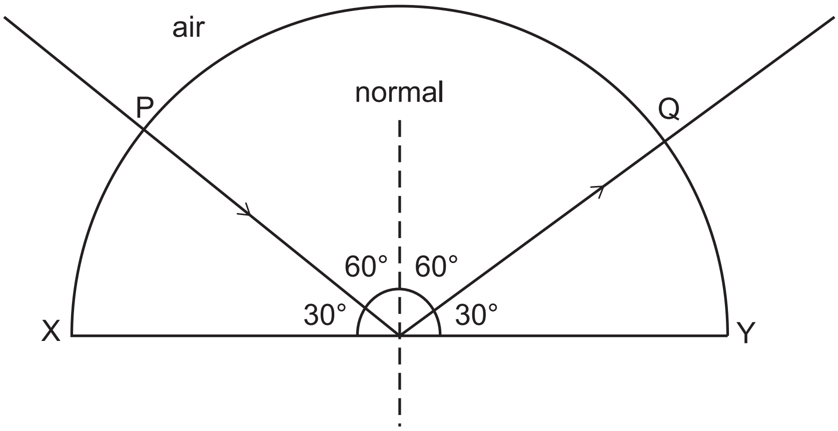
$\rho = 4.8 \times 10^{-7}\ \Omega m$

$A = \pi \times (2 \times 10^{-4})^2 \times 12 = \pi \times 4.8 \times 10^{-7}\ m^2$

$l = 15\ m$

$R = \dfrac{\rho l}{A} = \dfrac{4.8 \times 10^{-7} \times 15}{4.8 \times 10^{-7} \times \pi} = \dfrac{15}{\pi}\ \Omega$

3 A ray of light is directed into a semicircular transparent block, entering at P. The direction of the ray is adjusted until it strikes the centre of the flat face XY of the block at the critical angle and reflects to Q as shown.



The length of XY is L.

The speed of light in air is c.

What is the time taken by the light to travel from P to Q in the block?

A $\frac{L\sqrt{3}}{2c}$

B $\frac{L}{c}$

C $\frac{2L}{c\sqrt{3}}$

D $\frac{L\sqrt{3}}{c}$

E $\frac{2L}{c}$

F $\frac{4L}{c\sqrt{3}}$

Speed of light in block = c . sin θ

$v = c \sin 60 = c\frac{\sqrt{3}}{2}$

P → middle = radius = $\frac{L}{2}$

∴ distance = $2 \times \frac{L}{2} = L$

$t = \frac{d}{s} = \frac{L}{c\frac{\sqrt{3}}{2}} = \frac{2L}{c\sqrt{3}}$

4 A solid cube with sides of length 20 cm is made from material with density 2000 kg m^{-3}. The cube is suspended, in equilibrium, from an initially unstretched spring, and this results in the spring gaining strain energy of 3.2 J.

What is the spring constant of the spring?

(gravitational field strength = 10 N kg^{-1}; the spring obeys Hooke's law)

A 40 N m^{-1}

B 80 N m^{-1}

C 400 N m^{-1}

D 800 N m^{-1}

E 4000 N m^{-1}

F 8000 N m^{-1}

mass = density × volume

mass = $2000 \times (20 \times 10^{-2})^3$

$= 16$ kg

Strain energy $= \frac{1}{2} f x$

$f = kx$, $x = \frac{f}{k}$

$\Rightarrow E = \frac{1}{2} f \frac{f}{k} = \frac{f^2}{2k}$

$\therefore 3.2 = \frac{(16 \times 10)^2}{2k}$

$k = \frac{160^2}{6.4} = 4000$ N m^{-1}

5 A projectile is fired upwards from the ground at an angle of 60° to the vertical at a speed of $20\,m\,s^{-1}$.

It travels a horizontal distance d and lands with a downwards vertical component of velocity of $4.0\,m\,s^{-1}$ on ground that is height h above the starting point of the projectile.

What are d and h?

(gravitational field strength = $10\,N\,kg^{-1}$; assume that air resistance is negligible)

	d / m	h / m
A	$6.0\sqrt{3}$	4.2
B	$6.0\sqrt{3}$	5.8
C	$10\sqrt{3} - 4.0$	4.2
D	$10\sqrt{3} - 4.0$	14.2
E	$10\sqrt{3} + 4.0$	5.8
F	$10\sqrt{3} + 4.0$	14.2
G	$14\sqrt{3}$	4.2
H	$14\sqrt{3}$	5.8

Vertical:

$x = h$

$u = 20\cos 60$

$v = -4$

$a = -10$

$t =$

$v^2 = u^2 + 2ax$

$(-4)^2 = (20\cos 60)^2 + 2 \times -10h$

$16 = 100 - 20h$

$h = 4.2\,m$

To find d, we need time of flight:

$v = u + at$

$-4 = 20\cos 60 + -10t$

$-14 = -10t$

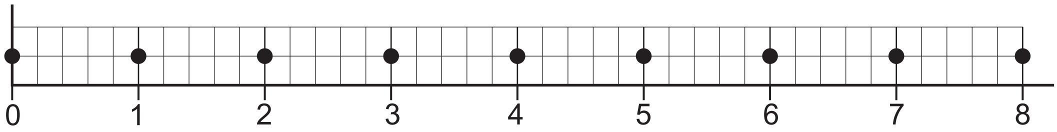
$t = 1.4\,s$

Horizontally: $s = \frac{d}{t}$, $d = st = 20\sin 60 \times 1.4$

$= 28 \times \frac{\sqrt{3}}{2}$

$= 14\sqrt{3}\,m$

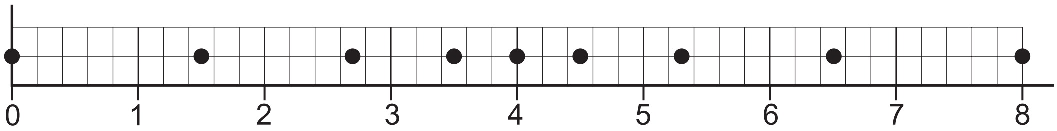
6 Diagram 1 shows the positions of nine equally spaced particles in a medium.



distance along medium / m

Diagram 1

Diagram 2 shows the positions of the same nine particles, at a particular time, while a longitudinal wave is travelling through the medium.



distance along medium / m

Diagram 2

What is the amplitude of the wave?

A 0.4 m

B 0.5 m

C 0.6 m

D 0.7 m

E 2.0 m

F 4.0 m

G 6.0 m

H 8.0 m

Amplitude = maximum displacement from rest

Particle at 2 m in diagram has the greatest displacement, = 0.7 m

7 A spaceship with mass 8.0×10^4 kg travels at constant velocity and has 1.0×10^{12} J of kinetic energy.

An external impulse of 8.0×10^7 kg m s^{-1}, lasting for 2.0 s, is applied to the spaceship acting in the opposite direction to the motion of the spaceship.

What is the average rate of loss of kinetic energy of the spaceship during the application of the impulse?

A 9.5×10^{10} W

B 1.8×10^{11} W

C 2.2×10^{11} W

D 3.2×10^{11} W

E 3.6×10^{11} W

F 7.2×10^{11} W

$KE = \frac{1}{2} m v^2$

$v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2 \times 1 \times 10^{12}}{8 \times 10^4}} = 5000 \, ms^{-1}$

Impulse = change in momentum = $mv - mu$

$-8 \times 10^7 = v_2 \times 8 \times 10^4 - 5000 \times 8 \times 10^4$

$\Rightarrow v_2 = \frac{-8 \times 10^7 + 4 \times 10^8}{8 \times 10^4} = 4000 \, ms^{-1}$

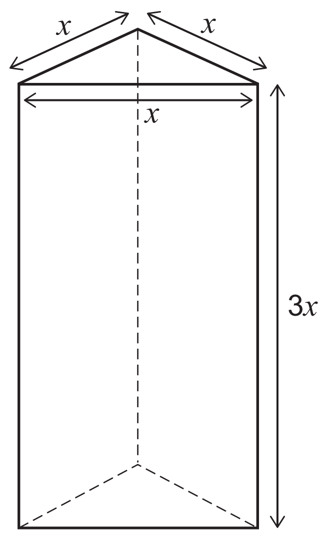
$\therefore \Delta KE = \frac{1}{2} m (v_2^2 - v_1^2)$

$= \frac{1}{2} \times 8 \times 10^4 (4000^2 - 5000^2)$

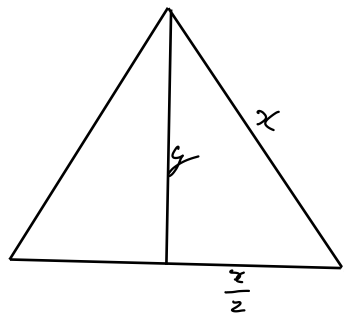
$= -3.6 \times 10^{11}$ J

This means 3.6×10^{11} J are lost in 2 seconds, giving a rate of $1.8 \times 10^{11} \, J s^{-1}$

8 The diagram shows a solid triangular prism.



Cross section :



$y = \sqrt{x^2 - \left(\frac{x}{2}\right)^2}$

$= \sqrt{\frac{3x^2}{4}}$

$= \frac{\sqrt{3}}{2}x$

The sides of the triangular cross section of the prism are of length x.

The height of the prism is $3x$.

The uniform density of the prism is ρ.

The gravitational field strength is g.

What is the minimum pressure the prism can exert when it rests on level ground?

Pressure $= \frac{\text{force}}{\text{area}}$, so minimum occurs on the largest face

Largest face area $= 3x \times x = 3x^2$

Force = mass × gravity

= density × volume × gravity (as $\rho = \frac{m}{V}$)

V = cross section × length

$= \frac{1}{2}\left(\frac{x}{2} \times \frac{\sqrt{3}}{2}x\right) \times 2 \times 3x$

$= \frac{3\sqrt{3}}{4}x^3$

∴ Pressure $= \frac{\rho \times \frac{3\sqrt{3}}{4}x^3 \times g}{3x^2} = \frac{\sqrt{3}\rho g x}{4}$

A $3\rho g$

B $3\rho g x$

C $\frac{\rho g}{4}$

D $\frac{\rho g x}{4}$

E $\frac{\sqrt{3}\rho g}{4}$

F $\frac{\sqrt{3}\rho g x}{4}$

9 An apple of mass m_a is placed on a uniform metre rule with the centre of gravity of the apple at the 10 cm mark. The rule is balanced on a pivot placed at the 35 cm mark.

The apple is replaced with an orange of mass m_o. The rule now balances with the pivot at the 40 cm mark.

What is the ratio $\frac{m_a}{m_o}$?

A $\frac{5}{9}$

B $\frac{4}{5}$

C $\frac{5}{6}$

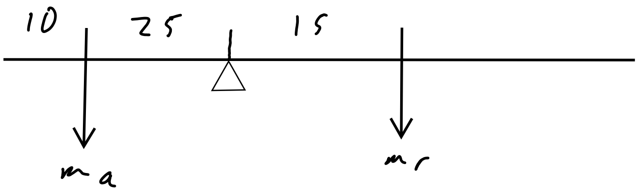
D $\frac{6}{5}$

E $\frac{5}{4}$

F $\frac{9}{5}$

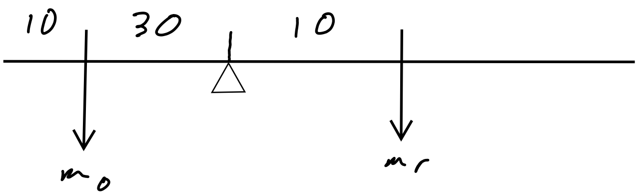
Apple distance to pivot = 35 − 10 = 25

Orange distance to pivot = 40 − 10 = 30



$\Rightarrow m_a \times 25 = m_r \times 15$

$m_a = \frac{15}{25} m_r = \frac{3}{5} m_r$



$\Rightarrow m_o \times 30 = m_r \times 10$

$m_o = \frac{10}{30} m_r = \frac{1}{3} m_r$

$\therefore$ ratio $= \dfrac{\frac{3}{5} m_r}{\frac{1}{3} m_r} = \frac{9}{5}$

10 A cyclist travels at a constant speed of $12\,\text{m}\,\text{s}^{-1}$ on level ground. During this time the power needed to maintain a constant speed is 900 W. The total weight of the cyclist and bicycle is 850 N.

The cyclist now cycles up a slope at the same constant speed. The slope is at an angle of 30° to the horizontal.

What is the driving force on the bicycle as it travels up the slope?

(Assume that the magnitude of the resistive forces is constant.)

A 75 N

B 350 N

C 500 N

D $(425\sqrt{3} - 75)$ N

E 775 N

F $(425\sqrt{3} + 75)$ N

G 925 N

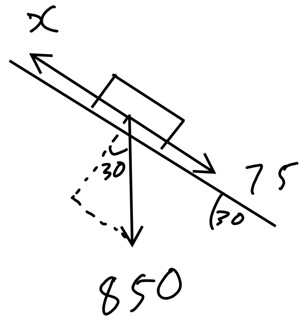
$d = st = 12 \times 1 = 12\text{ m}$

900 J work done against friction in 12 m

Work = force × distance

$900 = F \times 12$

$F = 75\text{ N}$ (force of friction)



N2L along slope: $x = 75 + 850\sin 30$

$= 75 + 425$

$= 500\text{ N}$

11 Three identical resistors can be combined in four different arrangements.

One of the arrangements has a resistance of 18 Ω.

A different arrangement has a resistance of 8.0 Ω.

What are the resistances of the other two arrangements?

(All three resistors contribute to the total resistance in all arrangements.)

A 2.0 Ω and 4.0 Ω

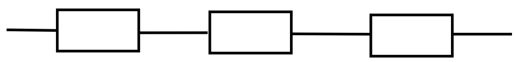
B 2.0 Ω and 9.0 Ω

C 4.0 Ω and 12 Ω

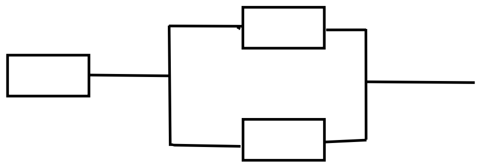
D 4.0 Ω and 36 Ω

E 36 Ω and 162 Ω

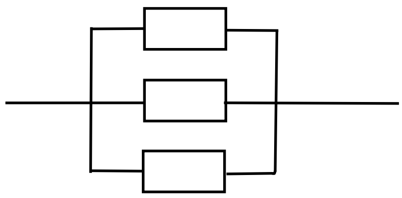
F 81 Ω and 162 Ω



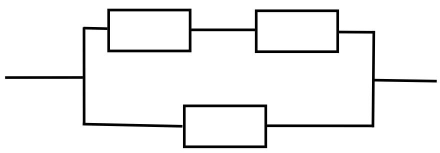
$R_1 = 3R$



$R_2 = \frac{3}{2}R$



$R_3 = \frac{1}{3}R$



$R_4 = \frac{2}{3}R$

If $R = 12$: $3R = 36$

$\frac{3}{2}R = 18$

$\frac{2}{3}R = 8$

$\frac{1}{3}R = 4$

12 A 4.0 kΩ fixed resistor is connected in series with a light dependent resistor (LDR) across a 100 V dc power supply.

The current in the LDR is 5.0 mA.

The intensity of light falling on the LDR now decreases and the voltage across the fixed resistor changes by 50%.

What is the change in the resistance of the LDR as a result of the change in intensity?

A 8.0 kΩ

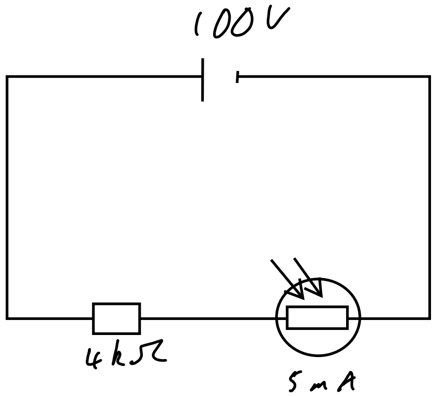
B 12 kΩ

C 16 kΩ

D 20 kΩ

E 32 kΩ

F 36 kΩ



Over 4kΩ resister:

$V = IR = 5\times10^{-3} \times 4\times10^{3} = 20V$

$\therefore$ Ratio of voltages $= 20 : 80 = 1:4$

This gives an initial LDR resistance of $4k\Omega \times 4 = 16k\Omega$

Intensity decrease ⇒ LDR resistance increase, so voltage change across 4kΩ is a decrease.

This gives a new voltage ratio: $10 : 90 = 1:9$

In a potential divider: $V_{out} = V_{in} \times \frac{R_2}{R_1 + R_2}$

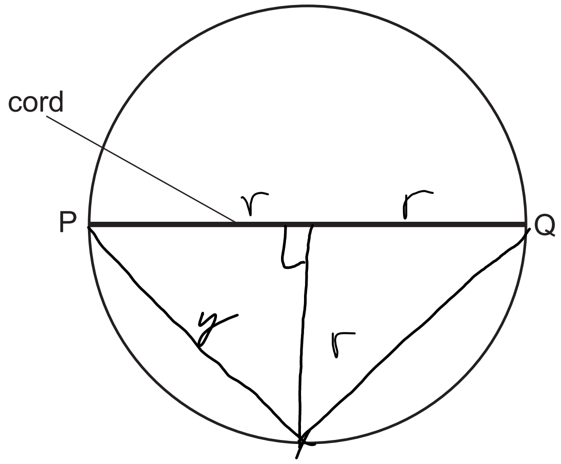
Here $V_{out} = V$ over 4kΩ and $V_{in} = 100V$ and $R_1 = LDR$, $R_2 = 4k\Omega$:

$10 = 100 \times \frac{4000}{R_1 + 4000}$

$R_1 = \frac{100 \times 4000}{10} - 4000 = 36000\Omega$

$\therefore \Delta R = 36000 - 16000 = 20000\Omega$

13 An elastic cord with spring constant k is fixed to two points P and Q on the diameter of a ring so that the cord is taut but unstretched. The radius of the ring is r.



The midpoint of the cord is then pulled and fixed to a point on the ring halfway between P and Q.

What is the energy stored in the elastic cord?

A $\frac{1}{2}kr^2$

B $2kr^2$

C $\frac{1}{2}(\sqrt{2}-1)kr^2$

D $2(\sqrt{2}-1)kr^2$

E $\frac{1}{2}(3-2\sqrt{2})kr^2$

F $2(3-2\sqrt{2})kr^2$

$E = \frac{1}{2}kx^2$

$PQ = 2r$

$y = \sqrt{r^2+r^2} = r\sqrt{2}$

$\therefore$ new length of cord $= 2r\sqrt{2}$

$\therefore$ extension $= 2r - 2r\sqrt{2}$

$= 2r(1-\sqrt{2})$

$E = \frac{1}{2}k \times (2r(1-\sqrt{2}))^2$

$= \frac{1}{2}k(4r^2(1+2-2\sqrt{2}))$

$= 2(3-2\sqrt{2})kr^2$

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14 An object of mass M experiences a resultant force of magnitude F. The force acts in a single horizontal direction with a magnitude that varies with time t according to

$$F = X + Y\sqrt{t}$$

where X and Y are constants.

The object is at rest at $t = 0$.

What is the magnitude of the momentum of the object at time $t = T$?

A $T\left(X + \frac{2}{3}Y\sqrt{T}\right)$

B $T\left(X + Y\sqrt{T}\right)$

C $\frac{T}{M}\left(X + \frac{2}{3}Y\sqrt{T}\right)$

D $\frac{T}{M}\left(X + Y\sqrt{T}\right)$

E $\frac{Y}{2\sqrt{T}}$

F $\frac{Y}{2M\sqrt{T}}$

$f = ma$

$f = M\frac{dv}{dt}$

$\frac{dv}{dt} = \frac{f}{m} = \frac{X + Y\sqrt{t}}{m}$

$\Rightarrow v = \int\left(\frac{X + Y\sqrt{t}}{M}\right)dt$

As momentum $= mv$:

$Mv = \cancel{m}\int\left(\frac{X + Y\sqrt{t}}{\cancel{M}}\right)dt$

$Mv = \int(X + Y\sqrt{t})\,dt$

$Mv = Xt + \frac{2}{3}Yt^{\frac{3}{2}} + C$

at $t = 0$, $v = 0$ $\therefore C = 0$

$Mv = Xt + \frac{2}{3}Yt^{\frac{3}{2}}$

at $t = T$:

$Mv = XT + \frac{2}{3}YT^{\frac{3}{2}}$

$Mv = T\left(X + \frac{2}{3}YT^{\frac{1}{2}}\right)$

15 A trolley of mass 3.0 kg is moving horizontally along a smooth track. Its displacement x from a point at time t is given by the equation:

$$x = 8 + 4t + 2t^2$$

where x is in metres and t is in seconds.

How much work is done on the trolley between times $t = 0$ and $t = 5.0$ s?

A 12 J

B 24 J

C 78 J

D 270 J

E 840 J

F 864 J

G 936 J

Work = ΔKE (as track is smooth + horizontal)

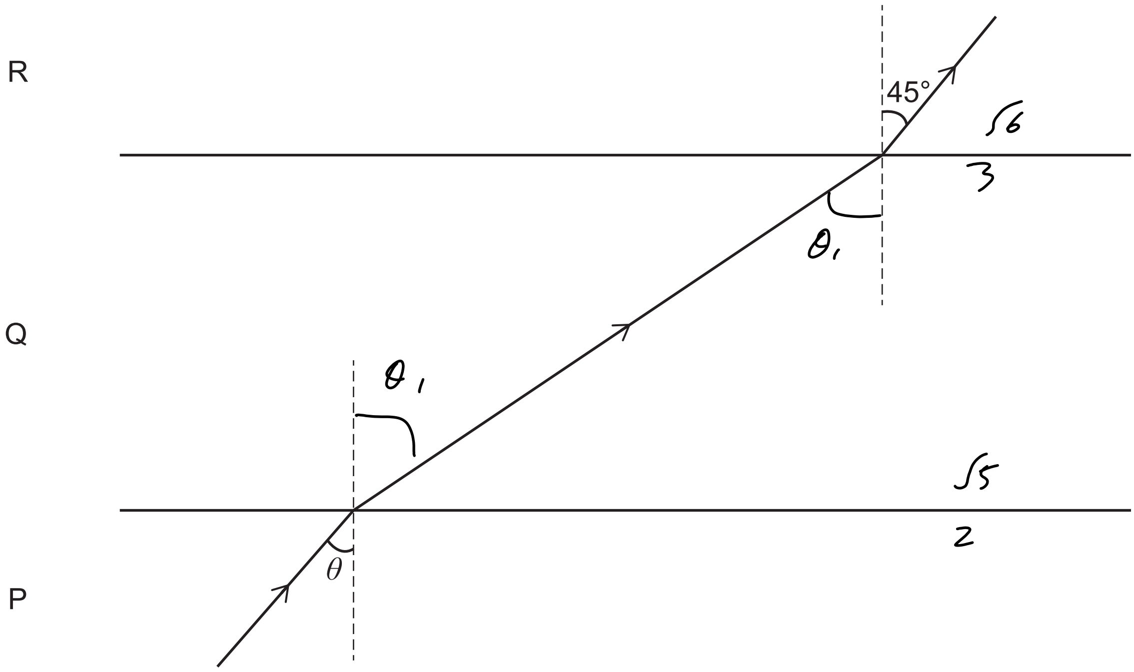
$v = \frac{dx}{dt} = 4 + 4t$

$\Delta KE = \frac{1}{2} m \Delta v^2 = \frac{1}{2} \times 3 \left((4 + 4 \times 5)^2 - (4 + 4 \times 0)^2 \right)$

$= \frac{3}{2} (24^2 - 16)$

$= 840$ J

16 The diagram shows a ray of light passing through three mediums, P, Q and R. The boundaries between the three mediums are parallel.



[diagram not to scale]

The ratio of the speed of light in medium P to the speed of light in medium Q is $2:\sqrt{5}$

The ratio of the speed of light in medium Q to the speed of light in medium R is $3:\sqrt{6}$

What is the value of $\sin\theta$?

A $\frac{\sqrt{2}}{2}$

B $\frac{\sqrt{3}}{2}$

C $\frac{\sqrt{3}}{6}$

D $\frac{\sqrt{5}}{5}$

E $\frac{\sqrt{15}}{5}$

F $\frac{\sqrt{15}}{6}$

$\sin\theta_1 = \sin 45 \times \frac{3}{\sqrt{6}}$

$= \frac{\sqrt{2}}{2} \times \frac{3}{\sqrt{6}}$

$= \frac{\sqrt{3}}{2}$

$\sin\theta = \sin\theta_1 \times \frac{2}{\sqrt{5}}$

$= \frac{\sqrt{3}}{2} \times \frac{2}{\sqrt{5}}$

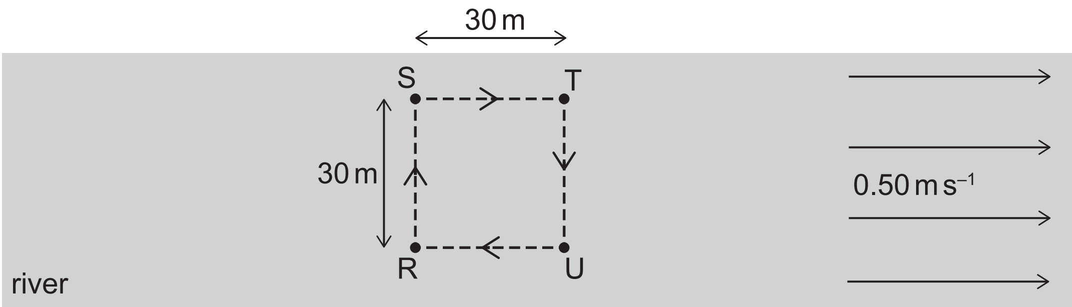
$= \sqrt{\frac{3}{5}}$

$= \frac{\sqrt{15}}{5}$

cc BY NC ND

17 Water in a wide river flows at a constant speed of $0.50\,\mathrm{m\,s^{-1}}$. A swimmer swims around a square path of side 30 m marked out by 4 posts R, S, T and U which are fixed to the river bed, as shown.

The swimmer has a constant speed of $1.0\,\mathrm{m\,s^{-1}}$ relative to the water.



How long does it take for the swimmer to swim around the square path once?

A $(60 + 24\sqrt{5})$ s

B $(60 + 40\sqrt{3})$ s

C $(80 + 24\sqrt{5})$ s

D $(80 + 40\sqrt{3})$ s

E 120 s

F 140 s

Velocities:

$S \to T = 1.5i$

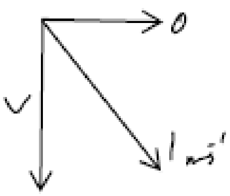
$T \to U = 0.5i - j$

$U \to R = -0.5i$

$R \to S = 0.5i + j$

S → T

For the swim a



The resulting speed, $v = \sqrt{1^2 - 0.5^2}$

$= \frac{\sqrt{3}}{2}\ \mathrm{ms^{-1}}$

$\therefore t = \frac{30}{\sqrt{3}/2} = \frac{60}{\sqrt{3}}$ per side

$\Rightarrow \frac{120}{\sqrt{3}}$ for both $= \frac{120}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{120}{3}\sqrt{3} = 40\sqrt{3}$

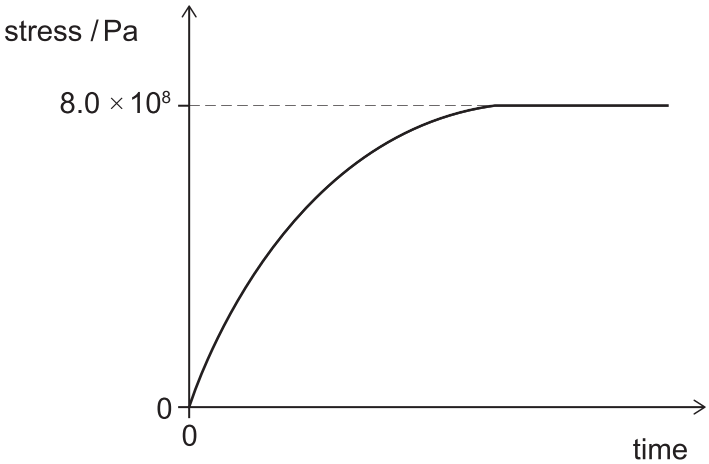
Total $= (80 + 40\sqrt{3})$ s

$\therefore t = \frac{30}{\sqrt{3}/2} = \frac{60}{\sqrt{3}}$ per side

$\Rightarrow \frac{120}{\sqrt{3}}$ for both $= \frac{120}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{120}{3}\sqrt{3} = 40\sqrt{3}$

Total $= (80 + 40\sqrt{3})$ s

18 The stress in a steel cable increases with time and is then maintained at a constant value, as shown. The wire does not reach its limit of proportionality.



The table shows properties of the steel used in the cable and the dimensions of the cable.

length / m	*cross-sectional area* / m^2	*Young modulus* / Pa
4.0	2.0×10^{-4}	2.0×10^{11}

How much work was done to stretch the cable?

A 320 J

B 1.28 kJ

C 2.56 kJ

D 320 kJ

E 640 kJ

F 1.60 MJ

G 6.40 MJ

$Y = \frac{fL}{Ax}$, $f = \frac{YAx}{L}$

$\frac{dW}{dx} = f$, $dW = f \cdot dx$

$W = \int f \, dx$

$W = \int \frac{YA}{L} x \, dx$

$W = \frac{1}{2} \frac{YA}{L} x^2$

$Y = \frac{fL}{Ax}$, $x = \frac{f}{A} \cdot \frac{L}{Y} = \text{stress} \times \frac{L}{Y}$

$x = 8 \times 10^8 \times \frac{4}{2 \times 10^{11}} = 0.016 \text{ m}$

$\therefore W = \frac{1}{2} \times \frac{2 \times 10^{11} \times 2 \times 10^{-4}}{4} \times 0.016^2 = 1280 \text{ J}$

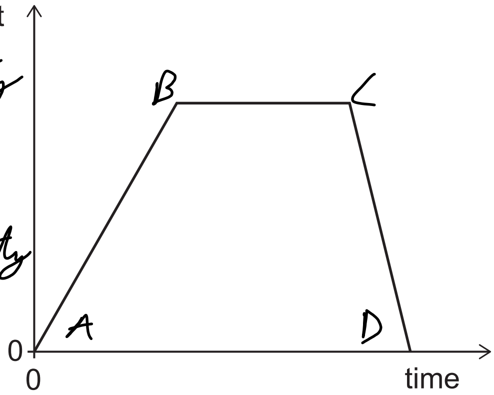
19 The following graph shows how the displacement of an object travelling along a straight, horizontal track varies with time.

A → B : constant positive velocity

B → C : zero velocity

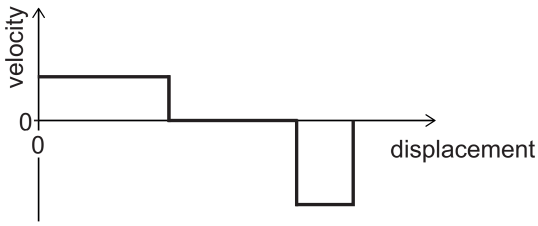
C → D : constant negative velocity

(negative velocity is greater than positive velocity)

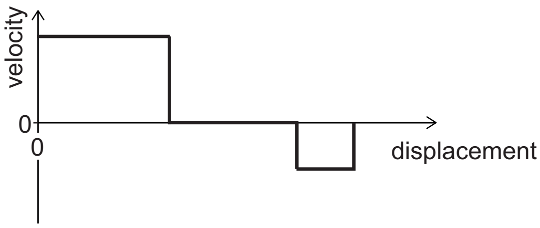


Which graph shows the velocity of this object against displacement?

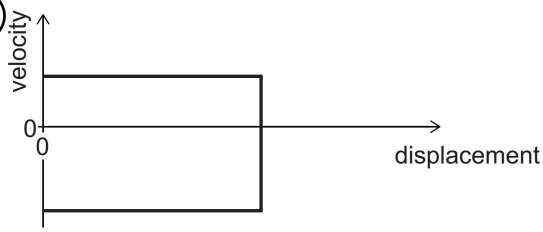
A



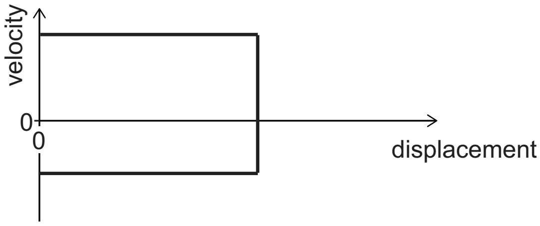
B



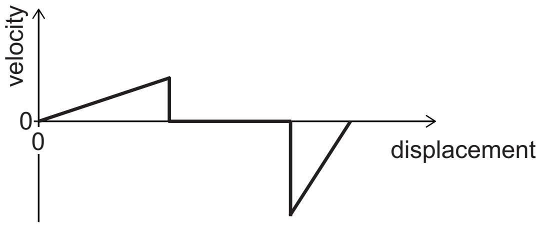
C



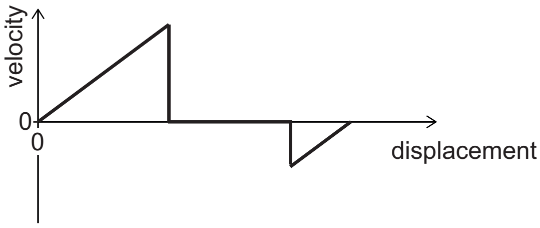
D



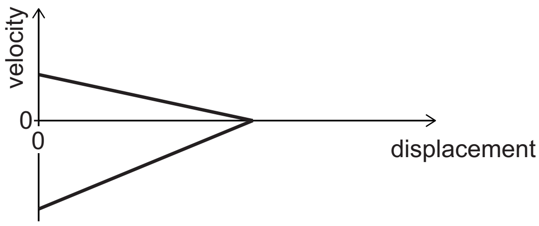
E



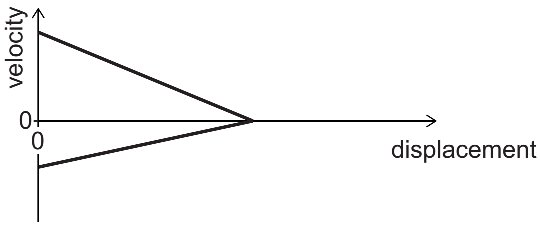
F



G



H



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20 A cell has emf E and internal resistance r that varies with current I according to:

$$r = kI^2$$

where k is a constant.

A variable resistor is connected to the terminals of the cell. The resistance of the variable resistor is adjusted.

Which expression gives the resistance of the variable resistor, in terms of k and E, that causes maximum power dissipation in it?

A $3\left(\frac{kE^2}{2}\right)^{\frac{1}{3}}$

B $3\left(\frac{kE^2}{4}\right)^{\frac{1}{3}}$

C $3\left(\frac{kE^2}{9}\right)^{\frac{1}{3}}$

D $3\left(\frac{kE^2}{16}\right)^{\frac{1}{3}}$

E $\left(2kE^2\right)^{\frac{1}{3}}$

F $\left(4kE^2\right)^{\frac{1}{3}}$

G $\left(9kE^2\right)^{\frac{1}{3}}$

H $\left(16kE^2\right)^{\frac{1}{3}}$

Maximum power dissipated occurs when external resistance = internal resistance, as per maximum power transfer theorem.

$E = V + Ir$

$R = \frac{V}{I}$, $V = RI$

$E = RI + Ir$

$I = \frac{E}{R+r}$

$P = I^2 R$

$P = \frac{E^2 R}{(R+r)^2}$

END OF TEST

at max, $R = r$.

$P = \frac{E^2 R}{(2R)^2}$

$P = \frac{E^2}{4R}$

$R = \frac{E^2}{4P}$

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