ENGAA 2020
Section 1

Model Solutions

PART A Mathematics and Physics

1 A soldering iron has a copper tip of mass 2.0 g.

The tip is heated with 30 W of thermal power. In 50 s, the temperature of the tip increases by 200 °C.

How much energy is transferred from the tip to the surroundings in this time?

(specific heat capacity of copper = 400 J kg^{-1} °C^{-1})

A 160 J

B 500 J

C 1340 J

D 1500 J

E 1660 J

F 1840 J

G 2500 J

$Q = mc\Delta\theta = 0.002 \times 400 \times 200$

$= 160$ J

(energy taken in by tip)

$\Rightarrow$ energy left = energy transferred to surroundings

= energy supplied − 160

$= (30 \times 50) - 160$

$= 1340$ J

2 The admission charge to a cinema is different for adults and children.

Admission for 2 adults and 3 children costs £20.

Admission for 4 adults and 4 children costs £34.

What does admission cost for 6 adults and 2 children?

let a = adult cost

let c = child cost

A £27

B £29

C £33

D £39

E £44

F £48

G £72

$2a + 3c = 20$

$4a + 4c = 34$

$\Rightarrow$ solve simultaneously

$4a + 6c = 40$

$-\ 4a + 4c = 34$

$0a + 2c = 6$

$c = 3$

$\Rightarrow 2a + 3 \times 3 = 20$

$a = 5.5$

$\therefore 6a + 2c = 18 + 11 = 39$

3 Uranium-238 $\left({}^{238}_{92}\mathrm{U}\right)$ decays by a series of alpha and beta (β^-) emissions to become the stable isotope lead-206 $\left({}^{206}_{82}\mathrm{Pb}\right)$.

How many beta (β^-) particles are emitted in the decay of one uranium-238 nucleus to lead-206?

A 6

B 8

C 10

D 12

E 14

F 16

1 beta : ${}^{0}_{-1}\beta$, 1 alpha : ${}^{4}_{2}\alpha$

$238 - 206 = 32$

$\frac{32}{4} = 8$ alpha

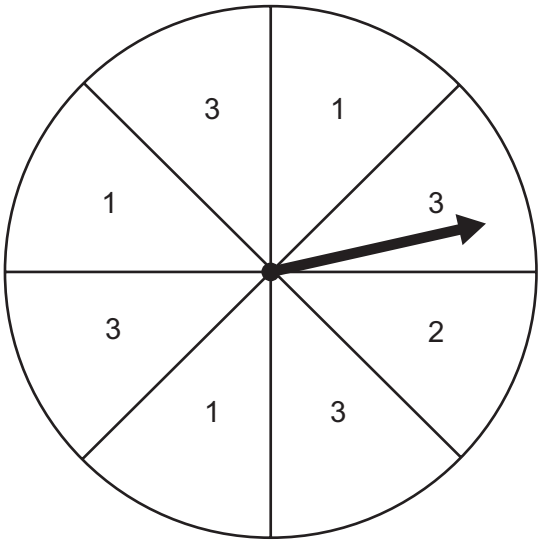
effect of 8 × α on atomic number $= 2 \times 8 = 16$

But $92 - 16 - 82 = -6$

one β is -1 $\rightarrow$ $-6 =$ 6 β decays

4 A fair spinner has eight equal sections.

Each section has one number written on it, as shown.



The spinner is spun twice, and the two numbers scored are added.

What is the probability that the sum of the two numbers is 5?

A $\frac{1}{8}$

B $\frac{5}{8}$

C $\frac{1}{16}$

D $\frac{3}{16}$

E $\frac{25}{64}$

F $\frac{55}{64}$

5 could be only be made by 3+2 or 2+3

$P(3+2) = \frac{4}{8} \times \frac{1}{8} = \frac{1}{16}$

$P(2+3) = \frac{1}{8} \times \frac{4}{8} = \frac{1}{16}$

$\therefore P(5) = \frac{1}{16} \times 2 = \frac{1}{8}$

5 A dc electricity transmission system uses an undersea cable to send electricity from one country to another. On a particular day, the first country supplies electricity at a voltage of 400 kV and 2000 A to the transmission system. The second country receives electricity from the transmission system at 160 kV and 4000 A.

What is the percentage efficiency of the system and how much energy is wasted every minute?

	efficiency %	*energy wasted every minute / J*
A	20	9.6×10^{9}
B	20	3.84×10^{10}
C	20	4.8×10^{10}
D	80	9.6×10^{9}
E	80	3.84×10^{10}
F	80	4.8×10^{10}

supplied :
$P = IV = 400000 \times 2000$
$= 8 \times 10^{8}$
received :
$= 160\,000 \times 4000$
$= 6.4 \times 10^{8}$

$\% = \frac{6.4}{8} \times 100 = 80\%$

energy lost: $(8 - 6.4) \times 10^{8} = 1.6 \times 10^{8}$ W

$\therefore$ per minute $= 1.6 \times 10^{8} \times 60 = 9.6 \times 10^{9}$ J

6 Consider the four lines with the following equations.

1 $2x + 6y = 3 \rightarrow y = \frac{3}{6} - \frac{2}{6}x$, $m = -\frac{1}{3}$

2 $9y = 3x - 4 \rightarrow y = \frac{1}{3}x - \frac{4}{9}$, $m = \frac{1}{3}$

3 $2y = 6x + 3 \rightarrow y = 3x + \frac{3}{2}$, $m = 3$

$-\frac{1}{3} \times 3 = -1$

4 $4x + 6y - 9 = 0$

Which two lines are perpendicular?

$\Rightarrow$ in form $y_1 = m_1 x_1 + c_1$
$y_2 = m_2 x_2 + c_2$

for lines to be $\perp$: $m_1 \times m_2 = -1$

A 1 and 2

B 1 and 3

C 1 and 4

D 2 and 3

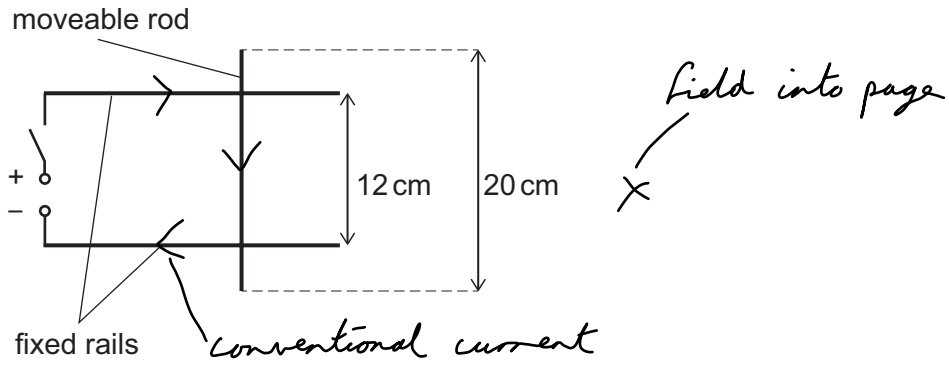
E 2 and 4

F 3 and 4

7 Two fixed horizontal metal rails are side by side and 12 cm apart. The rails are connected to a dc power supply by a switch that is initially open.

A freely moveable metal rod of length 20 cm is placed on the rails as shown in the diagram. The diagram shows the arrangement seen from above.

The angle between the rod and the rails is 90°.



The whole arrangement is placed in a uniform magnetic field of magnitude 0.50 T that is directed perpendicularly into the page.

The moveable rod has a weight of 0.40 N. $m = 0.04$ kg

The switch is now closed. As a result, there is a current of 2.4 A in the circuit and the rod moves.

What is the initial magnitude of the acceleration of the rod and what is its direction?

(gravitational field strength = 10 N kg^{-1})

length of current carrying wire in field

$F = BIL$

$= 0.5 \times 2.4 \times 12 \times 10^{-2}$

$= 0.1444$

$F = ma$, $a = \frac{F}{m} = \frac{0.1444}{0.04}$

$= 3.6$ m s^{-2}

	acceleration / m s^{-2}	*direction*
A	0.36	to the left
B	0.36	to the right
C	0.60	to the left
D	0.60	to the right
E	3.6	to the left
F	3.6	to the right
G	6.0	to the left
H	6.0	to the right

from fleming's left hand rule, force is to the right

Sum of roots $= -\frac{b}{a}$

8 Find the sum of the solutions of

$$2\left(\frac{x}{4}+3\right)^2-\left(\frac{x}{4}+3\right)-36=0$$

A 2

B $\frac{3}{2}$

C $\frac{1}{2}$

D -4

E -13

F -22

G -26

H -34

$2\left(\frac{x^2}{16}+9+\frac{3x}{2}\right)-\frac{x}{4}-3-36=0$

$\frac{x^2}{8}+3x-\frac{x}{4}+18-3-36=0$

$\frac{x^2}{8}+\frac{11}{4}x-21=0$

$-\frac{b}{a}=-\frac{11}{4}\div\frac{1}{8}=-22$

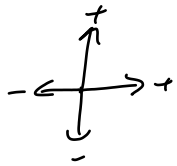
9 Two trolleys are moving towards each other along a straight horizontal track.

One trolley has mass 8.0 kg and is travelling to the right at 4.0 m s^{-1}.

The other trolley has mass 2.0 kg and is travelling to the left at 1.0 m s^{-1}.

When the trolleys collide they stick together.

How much kinetic energy is transferred to other forms of energy in the collision?



A 2.0 J

B 18 J

C 20 J

D 28 J

E 35 J

F 40 J

G 45 J

H 65 J

Σmv before $=\Sigma mv$ after

$8\times4+2\times-1=(8+2)\times v$

$v=\frac{32-2}{10}=3\ ms^{-1}$

ΣKE before $=\frac{1}{2}\times8\times4^2+\frac{1}{2}\times2\times1^2$

$=65\ J$

KE after $=\frac{1}{2}\times10\times3^2=45\ J$

$\Delta KE=65-45=20\ J$

10 When the expression

$$(2x + 3)^2 - (x - 3)^2$$

is written in the form $p(x + q)^2 + r$, where p, q and r are constants, what is the value of r?

A -27

B -9

C 0

D 3

E 15

$4x^2 + 9 + 12x - (x^2 + 9 - 6x)$

$3x^2 + 18x$

$3(x^2 + 6x)$

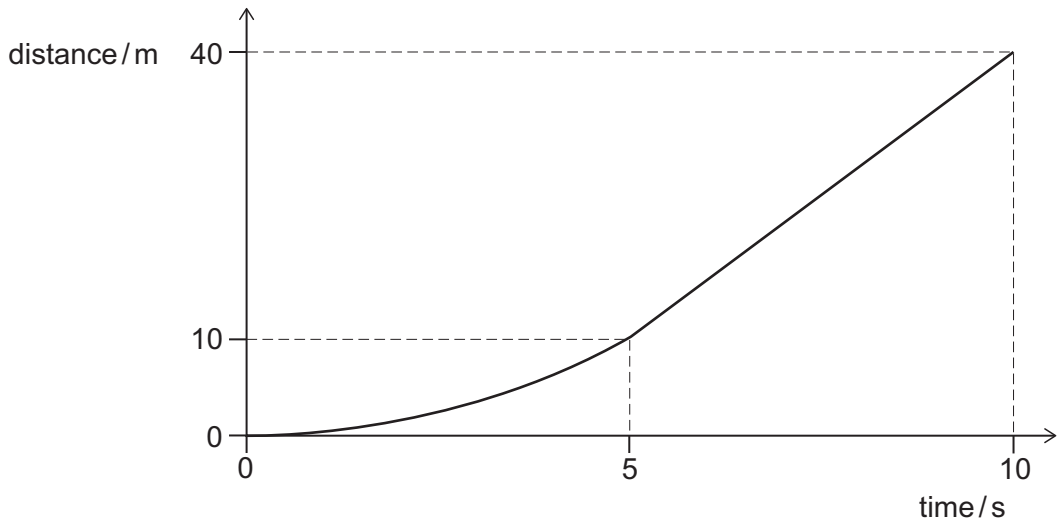
$3((x+3)^2 - 3^2)$

$3(x+3)^2 - 3^3$

$r = -27$

11 A car of mass 800 kg travels in a straight line along a horizontal road.

The car accelerates **non-uniformly** from rest for 5.0 seconds and then moves at constant speed, as shown in the distance–time graph:



What is the average resultant force acting on the car over the time for which it is accelerating?

↳ only curved section

A 320 N

B 480 N

C 640 N

D 960 N

E 1600 N

F 3200 N

G 4800 N

$$a = \frac{\text{final speed} - \text{initial speed}}{\text{time}}$$

$$\text{final speed} = \frac{d}{t} = \frac{40-10}{10-5} = 6$$

$$a = \frac{6-0}{5} = 1.2$$

$$f = ma = 800 \times 1.2 = 960\,N$$

12 The number of pairs of winter boots sold on a day is inversely proportional to the cube of the outside temperature on that day, measured in °C.

On a day when the outside temperature is 8 °C, 250 pairs of boots are sold.

The next day, when the outside temperature is x °C, the number of pairs of boots sold is 700% more than on the previous day.

What is the value of x?

A 2

B 4

C $\frac{8}{\sqrt[3]{7}}$

D $8\sqrt[3]{7}$

E 16

13 P and Q are two fixed points on the surface of the ocean which are 6.0 m apart.

An ocean wave travels in the direction P to Q.

The wave has a frequency of 0.50 Hz and travels at a constant speed.

A wave peak passes Q at time $t = 0$ s.

The next wave peak travelling towards Q passes P at time $t = 0.80$ s.

What is the speed of the wave?

A 2.1 ms^{-1}

B 3.4 ms^{-1}

C 5.0 ms^{-1}

D 7.5 ms^{-1}

E 20 ms^{-1}

14 In a sale, all prices are reduced by 25%.

A customer calculates the pre-sale price of a bicycle incorrectly by increasing the marked sale price by 25%.

The customer's calculated pre-sale price is incorrect by £15.

What is the correct pre-sale price of the bicycle?

let x = original price
y = sale price
z = customer's calculated price

A £180

B £195

C £210

D £225

E £240

$x \times 0.75 = y$

$y \times 1.25 = z \Rightarrow y = 0.8z$

sub $y = 0.8z$ into $0.75x = y$

$\Rightarrow 0.75x = 0.8z \Rightarrow z = 0.9375x$

z is 93.75% of x, so £15 = 6.25% of x

$\therefore x = 15 \div 0.0625 = 240$

15 A parachutist of mass 80.0 kg drops from a plane travelling at 40.0 m s^{-1}, 2000 m above the Earth's surface.

The parachutist hits the ground at a speed of 5.00 m s^{-1}.

How much work is done by the parachutist against drag forces during the fall?

(Take the Earth's gravitational field strength to be 10.0 N kg^{-1}.)

A 1 535 000 J

B 1 624 000 J

C 1 649 000 J

D 1 663 000 J

E 1 726 000 J

work done = ΔE

E before = GPE + KE

$= 80 \times 10 \times 2000 + \frac{1}{2} \times 80 \times 40^2$

$= 1664000$ J

E after = GPE + KE

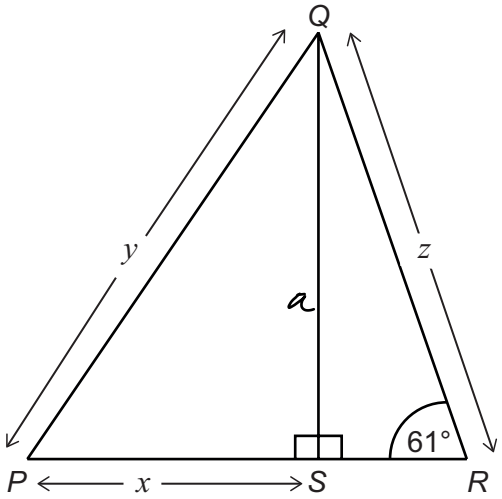
$= 0 + \frac{1}{2} \times 80 \times 5^2$

$= 1000$ J

$\Rightarrow \Delta E$ = work done $= 1664000 - 1000$

$= 1,663,000$ J

16



[diagram not to scale]

In the diagram, QS is perpendicular to PR.

$PS = x$ cm
$PQ = y$ cm
$QR = z$ cm

angle $QRS = 61°$

PSR is a straight line.

Which one of the following is an expression for the length z, in cm?

let $QS = a$

$a = \sqrt{y^2 - x^2}$

$\sin 61 = \frac{\text{opp}}{\text{hyp}} = \frac{a}{z} = \frac{\sqrt{y^2 - x^2}}{z}$

$\Rightarrow z = \frac{\sqrt{y^2 - x^2}}{\sin 61}$

A $\sqrt{y^2 + x^2}\sin 61°$

B $\sqrt{y^2 - x^2}\sin 61°$

C $\sqrt{y^2 + x^2}\cos 61°$

D $\sqrt{y^2 - x^2}\cos 61°$

E $\dfrac{\sqrt{y^2 + x^2}}{\sin 61°}$

F $\dfrac{\sqrt{y^2 - x^2}}{\sin 61°}$

G $\dfrac{\sqrt{y^2 + x^2}}{\cos 61°}$

H $\dfrac{\sqrt{y^2 - x^2}}{\cos 61°}$

17 A light spring of unstretched length 0.10 m has a spring constant of 20 N m^{-1}. The spring is suspended so that it is vertical and a load of mass 0.050 kg is attached to the end of the spring.

The load is pulled vertically downwards until the length of the spring is 0.30 m. The load is then released.

What is the speed of the load at the instant that the spring returns to its unstretched length?

(gravitational field strength = 10 N kg^{-1}; assume that resistive forces are negligible)

A 0 m s^{-1}

B 4.0 m s^{-1}

C 6.0 m s^{-1}

D 12 m s^{-1}

E 16 m s^{-1}

F $\sqrt{6}$ m s^{-1}

G $\sqrt{12}$ m s^{-1}

H $\sqrt{30}$ m s^{-1}

At start : energy = elastic $= \frac{1}{2}kx^2$

$= \frac{1}{2} \times 20 \times (0.3-0.1)^2$

$= 0.4$ J

At end : energy = kinetic + GPE

$0.4 = \frac{1}{2} \times 0.05 \times v^2 + 0.05 \times 10 \times 0.2$

$0.3 = 0.025 v^2$

$v = \sqrt{\frac{0.3}{0.025}} = \sqrt{12}$

18 Two vertices of a square are at (1, 1) and (3, 5).

What is the difference between the perimeters of the largest and smallest possible squares that can be drawn with these points as two of their vertices?

A 0

B $4\sqrt{3}(2-\sqrt{2})$

C $4\sqrt{3}(\sqrt{2}-1)$

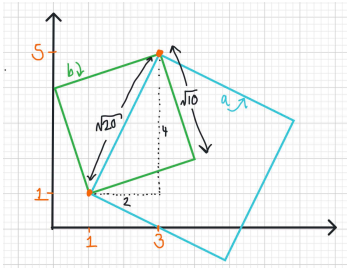
D $4\sqrt{5}(2-\sqrt{2})$

E $4\sqrt{5}(\sqrt{2}-1)$

F $4\sqrt{13}(2-\sqrt{2})$

G $4\sqrt{13}(\sqrt{2}-1)$

H $4\sqrt{3}\sqrt{5}(2-\sqrt{}$



a : side length $= \sqrt{(3-1)^2+(5-1)^2}$

$= \sqrt{20}$

perimeter $= 4 \times \sqrt{20} = \sqrt{320}$

$= 8\sqrt{5}$

b : side length $= \sqrt{20} \times \cos 45 = \frac{\sqrt{2} \times \sqrt{20}}{2}$

$= \sqrt{10}$

$\therefore$ perimeter $= 4\sqrt{10}$

difference $= 8\sqrt{5} - 4\sqrt{10} = 4\sqrt{5}(2-\sqrt{2})$

no resistance

19 A rocket travelling in space is burning its fuel at a constant rate. By expelling the burnt fuel through a nozzle, the engine is applying a constant force to the rocket.

What is happening to the magnitude of the acceleration of the rocket?

A It is increasing at an increasing rate.

B It is increasing at a constant rate.

C It is increasing at a decreasing rate.

D It is not changing.

E It is decreasing at an increasing rate.

F It is decreasing at a constant rate.

G It is decreasing at a decreasing rate.

$f = ma$

f constant; a decreasing as fuel is burned

This causes the same force to have an increased effect.

20 The quadratic equation $2x^2 - px - 4 = 0$, where p is a positive constant, has two solutions that differ by 6.

What is the value of p?

A 2

B $4\sqrt{7}$

C 12

D $4\sqrt{11}$

E $4\sqrt{34}$

F $6\sqrt{30}$

Quadratic formula: $x = \frac{(-b) \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore$ difference of solutions $= \frac{-b + \sqrt{b^2 - 4ac}}{2a} - \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow 6 = \frac{p + \sqrt{p^2 - 4 \times 2 \times -4}}{2 \times 2} - \frac{p - \sqrt{p^2 - 4 \times 2 \times -4}}{2 \times 2}$

$6 = \frac{p + \sqrt{p^2 + 32} - p + \sqrt{p^2 + 32}}{4}$

$4 = 2\sqrt{p^2 + 32}$

$12 = \sqrt{p^2 + 32}$

$p^2 = 12^2 - 32$

$p^2 = 112$

$p = \sqrt{112}$

$p = \sqrt{16}\sqrt{7}$

$= 4\sqrt{7}$

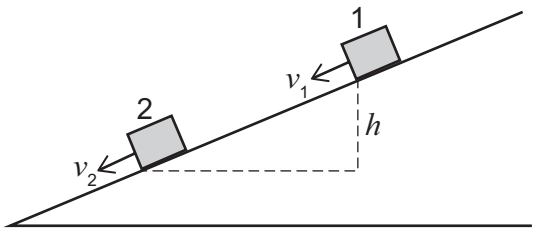
$\frac{112}{16} = \frac{28}{4} = \frac{14}{2} = 7$

PART B Advanced Mathematics and Advanced Physics

21 A block of mass m slides down a rough slope.

At position 1 the velocity of the block is v_1.

At position 2, which is a vertical distance h below position 1, the velocity of the block is v_2.



Which expression gives the work done against friction by the block as it slides from position 1 to position 2?

(gravitational field strength = g; assume that air resistance is negligible)

A $mgh + \frac{1}{2}m(v_2^2 - v_1^2)$

B $mgh - \frac{1}{2}m(v_2^2 - v_1^2)$

C $mgh + \frac{1}{2}m(v_2 - v_1)^2$

D $mgh - \frac{1}{2}m(v_2 - v_1)^2$

E $\frac{1}{2}m(v_2^2 - v_1^2) - mgh$

F $\frac{1}{2}m(v_1^2 - v_2^2) - mgh$

G $\frac{1}{2}m(v_2 - v_1)^2 - mgh$

H $\frac{1}{2}m(v_1 - v_2)^2 - mgh$

Work done = change in energy of block

Energy at 1 $= mgh + \frac{1}{2}mv_1^2$

Energy at 2 $= \frac{1}{2}mv_2^2$

change $= mgh + \frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2$

$= mgh - \frac{1}{2}m(v_2^2 - v_1^2)$

22 $(x - 1)$ and $(x - 2)$ are both factors of $x^4 + ax^3 + bx^2 - 12x + 4$

What are the values of a and b?

A $a = -6$ and $b = -23$

B $a = -6$ and $b = 13$

C $a = 6$ and $b = -11$

D $a = 6$ and $b = 1$

Sub in $x = 1$ and $x = 2$:

$x = 1$: $1^4 + a \times 1^3 + b \times 1^2 - 12 \times 1 + 4 = 0$

$a + b - 7 = 0$

$\Rightarrow a = 7 - b$

$x = 2$: $2^4 + a \times 2^3 + b \times 2^2 - 12 \times 2 + 4 = 0$

$8a + 4b - 4 = 0$

(sub in $a = 7 - b$)

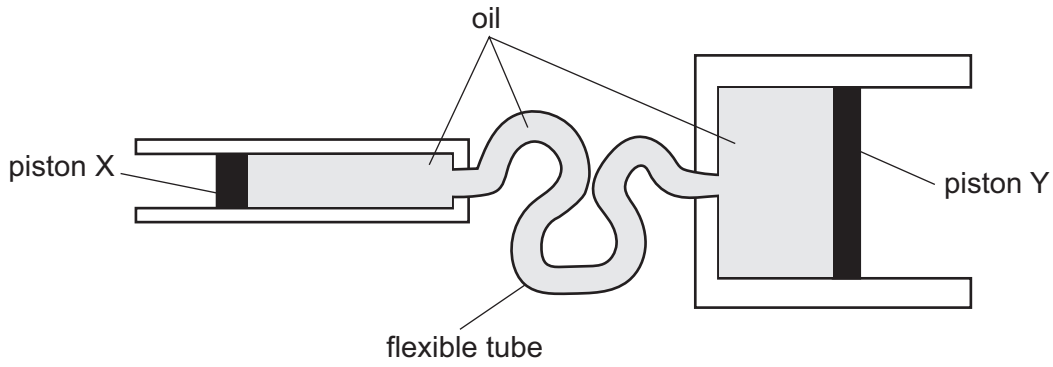
$56 - 8b + 46 - 4 = 0$

$52 = 4b$

$b = 13$

$\Rightarrow a = 7 - 13 = -6$

23 The braking system of a car includes two cylinders containing an incompressible oil, linked by a flexible tube that also contains oil. There is a freely moving piston in each cylinder. One piston is labelled X, and the other is labelled Y in the diagram.



When the driver presses on the brake pedal, a force is exerted on piston X. The pressure produced in the oil by this force is transmitted through the oil so that it causes a force to act on piston Y. This presses the brakes against the moving parts.

The diameter of piston X is 4.0 cm. The diameter of piston Y is 12.0 cm.

The driver exerts a force of 36.0 N on piston X and it moves a distance of 5.4 cm to the right.

What is the resultant force on piston Y and how far does it move along the cylinder?

	force on piston Y / N	*distance moved by piston Y* / cm
A	4.0	0.60
B	4.0	48.6
C	12.0	1.80
D	12.0	16.2
E	108	1.80
F	108	16.2
G	324	0.60
H	324	48.6

Length factor $x \rightarrow y$:

$\frac{12}{4} = 3$

$\therefore$ area factor $x \rightarrow y$:

$3^2 = 9$

This means the force will be 9x greater, as it has to produce the same pressure over an area 9x greater.

The work done is equal on both pistons, so as the force is 9x larger, the distance must be 9x smaller

$36 \times 9 = 324$ N

$\frac{5.4}{9} = 0.6$ cm

24 What is the area of the region enclosed between the curve $y = \frac{1}{2}x^2$, the line $y = -x$, and the lines $x = 1$ and $x = 3$?

A $\frac{1}{3}$

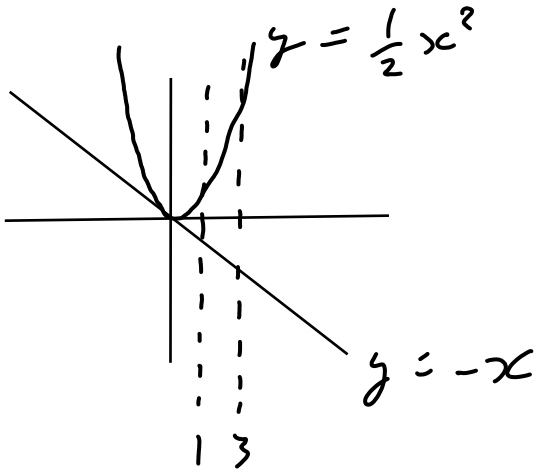
B 2

C 4

D 6

E $\frac{25}{3}$

F $\frac{28}{3}$



$y = -x$ is below $y = \frac{1}{2}x^2$ in the region (1 to 3), so area enclosed can be found by finding the difference between integrals:

$$\text{area} = \int_1^3 \left(\frac{1}{2}x^2\right)dx - \int_1^3 (-x)\,dx$$

$$= \int_1^3 \left(\frac{1}{2}x^2 + x\right)dx$$

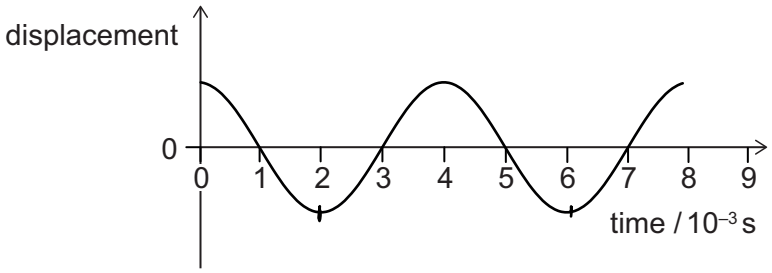
$$= \left[\frac{x^3}{6} + \frac{x^2}{2}\right]_1^3$$

$$= \left(\frac{3^3}{6} + \frac{3^2}{2}\right) - \left(\frac{1^3}{6} + \frac{1^2}{2}\right)$$

$$= \frac{9}{2} + \frac{9}{2} - \frac{2}{3}$$

$$= \frac{25}{3}$$

25 The graph shows how the displacement due to a wave in air varies with time.



The speed of the wave in air is 300 m s^{-1}.

The wave now travels into water.

$$\frac{\text{wave speed in air}}{\text{wave speed in water}} = 0.2$$

What is the wavelength of the wave in water?

A $\frac{1}{6}$ m

B $\frac{2}{9}$ m

C $\frac{5}{6}$ m

D $\frac{9}{10}$ m

E $\frac{10}{9}$ m

F $\frac{6}{5}$ m

G $\frac{9}{2}$ m

H 6 m

from the graph, 4 sec = 1 wave

1 wave = $300 \times 4 \times 10^{-3} = 1.2$ m in air

wave speed $\propto \frac{1}{\text{wavelength}}$

This means the change in wave speed of $\times 0.2$ from air to water causes a $\times \frac{1}{0.2}$ change in wavelength:

$1.2 \times \frac{1}{0.2} = 6$ m

26 A line with non-zero gradient m is reflected in the line $y = x$

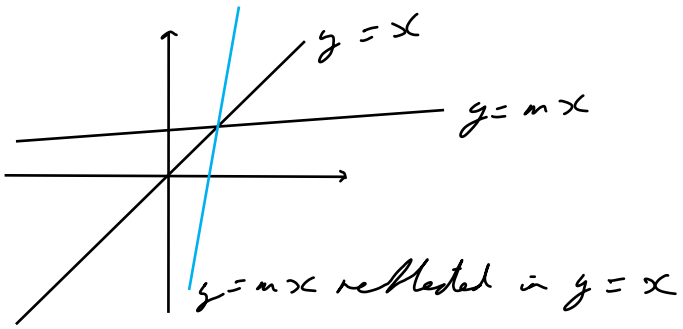
What is the gradient of the reflected line?

A m

B $-m$

C $\frac{1}{m}$

D $-\frac{1}{m}$

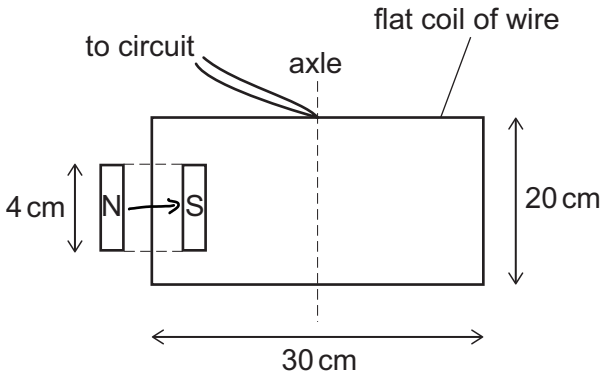


This gradient is not negative, so the answer is not B or D. It is different to m, so it must be $\frac{1}{m}$, C.

27 A flat rectangular coil of wire with sides of length 30 cm and 20 cm is freely pivoted about an axle. The axle passes through the middle of the sides of length 30 cm.

Part of the coil is between the poles of a U-shaped magnet as shown in the diagram. The poles are 4.0 cm long. The magnetic field can be considered uniform between the poles, and zero elsewhere.

The coil is connected to a power supply so that there is a current in it.



[diagram not to scale]

The current is 0.60 A and the magnetic flux density is 0.050 T. There are 50 turns of wire in the coil.

What is the moment about the axle, in N cm, produced by the magnetic force acting on the coil?

A 0.018 N cm

B 0.036 N cm

C 0.045 N cm

D 0.90 N cm

E 1.8 N cm

F 2.25 N cm

G 4.5 N cm

moment = force × perpendicular distance

$f = BIL = 0.05 \times 0.6 \times (6 \times 10^{-2} \times 50)$

$f = 0.06\ N$

(length of wire in field, L, = length of poles × number of turns)

$\Rightarrow$ moment $= 0.06 \times 15$

$= 0.90\ N\ cm$

28 The sum of the first 20 terms of an arithmetic progression is 50.

The sum of the next 20 terms of the arithmetic progression is –50.

What is the sum of the first 100 terms of the arithmetic progression?

A –750

B –350

C –50

D $-\frac{159}{8}$

E $\frac{159}{8}$

F 50

G 350

H 750

$S_{20} = 50 = \frac{1}{2} \times 20(2a + (20-1)d)$

$50 = 20a + 190d$

$S_{40} - S_{20} = -50 = \frac{1}{2} \times 40(2a + (40-1)d) - (20a + 190d)$

$-50 = 40a + 780d - 20a + 190d$

$-50 = 20a + 590d$

Solve simultaneously:

$$\begin{array}{r} 50 = 20a + 190d \\ -\quad -50 = 20a + 590d \\ \hline 100 = 0a - 400d \end{array}$$

$d = -\frac{1}{4}$

$\Rightarrow 50 = 20a + 190 \times -\frac{1}{4}$

$a = \frac{39}{8}$

$S_{100} = \frac{1}{2} \times 100\left(2 \times \frac{39}{8} + (100-1) \times -\frac{1}{4}\right)$

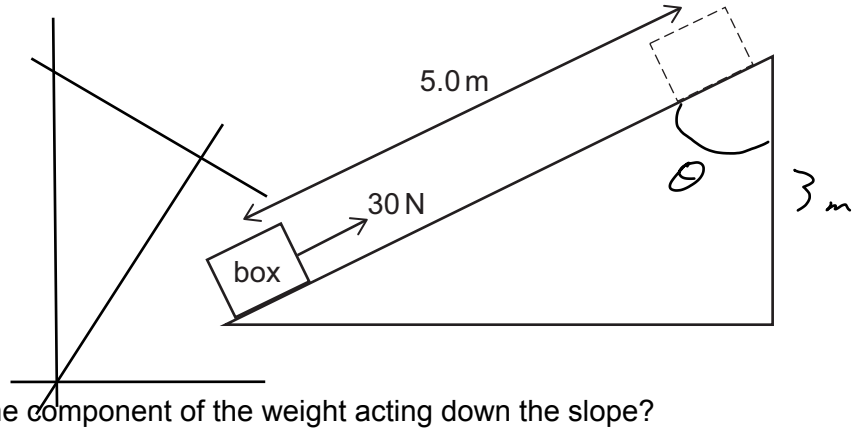
$= 50\left(\frac{39}{4} - \frac{99}{4}\right)$

$= 50\left(-\frac{60}{4}\right)$

$= -750$

29 A box of mass 3.0 kg is pulled a distance 5.0 m directly up a smooth slope by a constant applied force of 30 N acting parallel to the slope.

The initial speed of the box is 3.0 m s^{-1} and the final speed is 7.0 m s^{-1}. Its acceleration is constant.



What is the component of the weight acting down the slope?

(gravitational field strength = 10 N kg^{-1}; air resistance is negligible)

A 12 N

B 18 N

C 22 N

D 28 N

E 29 N

F 42 N

G 90 N

Work done = ΔGPE + ΔKE

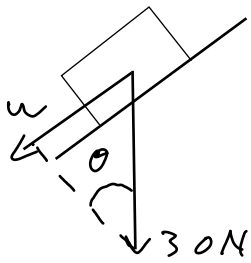
$fd = mg\Delta h + \frac{1}{2}m\Delta v^2$

$30 \times 5 = 3 \times 10 \times \Delta h + \frac{1}{2} \times 3 \times (7^2 - 3^2)$

$150 = 30\Delta h + 60$

$\Delta h = \frac{150 - 60}{30} = 3\text{ m}$

from diagram: $\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{3}{5}$



$\Rightarrow \sin\theta = \frac{w}{30}$

$\frac{3}{5} = \frac{w}{30}$

$w = \frac{3}{5} \times 30 = 18\text{ N}$

30 The line L with equation $y = mx + c$, where $m > 0$ and $c \geq 0$, passes through the point $(2, 4)$.

A line is drawn through the point $(2, 4)$ perpendicular to L.

The triangle enclosed between the two lines and the y-axis has area 5 square units.

What is the **larger** of the two possible values of m?

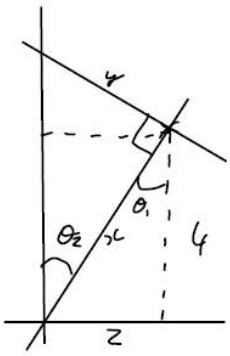
A −0.5

B 0.5

C 1.25

D 2

E 5



This is the greatest possible value of m. If this diagram is correct, this will be the larger value of m.

area $= \frac{bh}{2} = 5 \Rightarrow bh = 10$

$x = \sqrt{4^2 + 2^2} = 2\sqrt{5}$

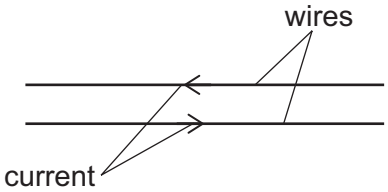
then xy must $= 10$, $y = \frac{10}{2\sqrt{5}} = \sqrt{5}$

If $\theta_1 = \theta_2$ then this is correct

$\theta_1 = \tan^{-1}\left(\frac{2}{4}\right) = \tan^{-1}\left(\frac{1}{2}\right)$. $\theta_2 = \tan^{-1}\left(\frac{\sqrt{5}}{2\sqrt{5}}\right) = \tan^{-1}\left(\frac{1}{2}\right)$

Consistent, ∴ this m $\left(\frac{4}{2} = 2\right)$ is the largest

31 Electrical power is supplied through a dc transmission line that consists of two metal wires. Each wire is 8.0 km long and has a cross-sectional area of 1.0 cm^2.



The resistivity of the metal from which the wires are made is $2.5 \times 10^{-7}\,\Omega\,m$.

Electrical power is transmitted to the transmission line at a potential difference of 24 000 V.

At what rate is energy being wasted as heat in the wires when the power supplied to the transmission line is 120 kW?

A 0.40 W

B 0.80 W

C 1.6 W

D 250 W

E 500 W

F 1000 W

G 1.44×10^7 W

H 5.76×10^7 W

$\rho = \frac{RA}{L} \Rightarrow R = \frac{\rho L}{A}$

$R = \frac{2.5 \times 10^{-7} \times (8 \times 10^3 \times 2)}{1 \times 10^{-4}} = 40\,\Omega$

$P = IV$, $I = \frac{P}{V}$

$I = \frac{120 \times 10^3}{24000} = 5$

Power dissipated as heat $= I^2 R = 5^2 \times 40$

$= 1000$ w

32 P and Q are two different geometric progressions.

The 3rd term of each geometric progression is 4.

The 5th term of each geometric progression is 2.

What is the modulus of the difference between the sums to infinity of P and Q?

P: $a, ar, 4, ar^3, 2$

Q: $a, ar, 4, ar^3, 2$

A 0

B 8

C $8\sqrt{2}$

D 16

E $16\sqrt{2}$

F 32

G $32\sqrt{2}$

$2 = ar^4$

$\frac{4 = ar^2}{0.5 = r^2}$

$r = \pm\sqrt{\frac{1}{2}}$

$\Rightarrow a = \frac{4}{\left(\sqrt{\frac{1}{2}}\right)^2} = 8$

$\Delta S_\infty = \frac{8}{1-\sqrt{\frac{1}{2}}} - \frac{8}{1-\left(-\sqrt{\frac{1}{2}}\right)} = 16\sqrt{2}$

33 A tennis ball travelling at $24.0\,\text{m s}^{-1}$ is hit by a racket. As a result of the impact, the ball returns back along its original path having undergone a change in velocity of $48.0\,\text{m s}^{-1}$. The acceleration of the ball whilst in contact with the racket is constant with magnitude $6000\,\text{m s}^{-2}$.

What is the total distance travelled by the ball whilst in contact with the racket?

A 0.00 cm

B 4.80 cm

C 9.60 cm

D 14.4 cm

E 19.2 cm

$u = 24\ \text{ms}^{-1}$, $v = -24\ \text{m s}^{-1}$, $a = -6000\ \text{ms}^{-2}$

Using $v^2 = u^2 + 2ax$ we can find x:

$$x = \frac{v^2 - u^2}{2a}$$

However as the modulus of u and v are equal, this gives $x = 0$.

Instead we have to split the journey in two, one journey is the deceleration to $0\,\text{ms}^{-1}$ and the acceleration:

$$x = \frac{0 - u^2}{2a} + \frac{0 - v^2}{2a}$$

$$x = \frac{0 - 24^2}{2\times -6000} + \frac{0 - (-24)^2}{2\times -6000} = 0.096\,\text{m}$$

$= 9.6$ cm

34 The curve

$$y = x^3 + 3\sqrt{5}px^2 + 3px + 13$$

has two distinct turning points.

What are all the possible values of p?

A $p < 0$, $p > 0.2$

B $p \leq 0$, $p \geq 0.2$

C $0 < p < 0.2$

D $0 \leq p \leq 0.2$

E $p < 0$, $p > 1.2$

F $p \leq 0$, $p \geq 1.2$

G $0 < p < 1.2$

H $0 \leq p \leq 1.2$

Two distinct turning points $\Rightarrow \frac{dy}{dx}$ has two real roots.

$\frac{dy}{dx} = 3x^2 + 6\sqrt{5}px + 3p$

$3x^2 + 6\sqrt{5}px + 3p = 0$

Two real roots $\Rightarrow b^2 - 4ac > 0$.

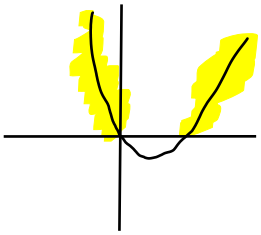
$(6\sqrt{5}p)^2 - 4 \times 3 \times 3p > 0$

$180p^2 - 36p > 0$

$36p(5p - 1) > 0$

$\Rightarrow p = 0$ or $p = \frac{1}{5} = 0.2$

We need the regions where $f(x) > 0$:

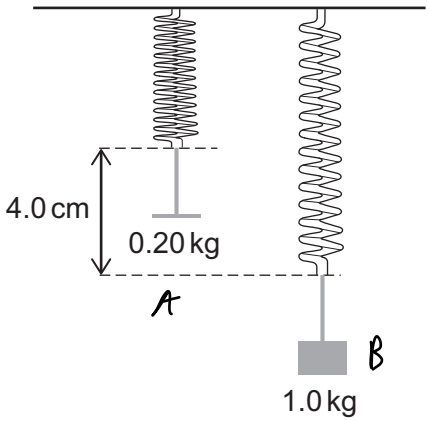


$\Rightarrow p > 0.2$

$p < 0$

35 The diagram shows the relative positions of two identical light springs, both in equilibrium.

The springs are supporting loads of mass 0.20 kg and 1.0 kg as shown.



[diagram not to scale]

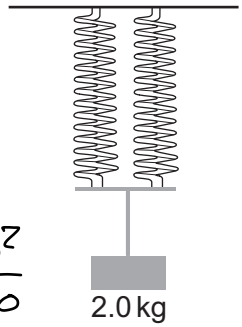
$f = kx$

$x = \frac{f}{k}$

$x_A + 0.04 = x_B$

$\frac{0.2g}{k} + 0.04 = \frac{1g}{k}$

The same two springs are now connected in parallel, supporting a 2.0 kg mass as shown.



[diagram not to scale]

$\frac{0.8g}{k} = 0.04$

$k = \frac{8}{0.04} = 200$

$\frac{1}{2} fx = \frac{1}{2} f \frac{f}{k} = \frac{1}{2} \frac{f^2}{k}$

$= \frac{1}{2} \times \frac{20^2}{400}$

In this parallel arrangement, what is the total strain energy in the springs?

(gravitational field strength = 10 N kg^{-1}; assume that the springs obey Hooke's law)

A 0.25 J

B 0.40 J

C 0.50 J

D 1.0 J

E 25 J

F 40 J

G 50 J

H 100 J

Initially, extension of A + 0.04 m = extension of B:

$x_A + 0.04 = x_B$

As $f = kx$, $x = \frac{f}{k}$ so:

$\frac{0.2 \times 10}{k} + 0.04 = \frac{1 \times 10}{k}$ (just 'k' as springs are identical)

$\Rightarrow k = \frac{0.8 \times 10}{0.04} = 200\ N m^{-1}$

In parallel, k adds up so in scenario 2, effectively $k = 400\ N m^{-1}$

$E = \frac{1}{2} fx = \frac{1}{2} f \frac{f}{k} = \frac{f^2}{2k} = \frac{(2 \times 10)^2}{2 \times 200} = \frac{1}{2} = 0.5\ J$

36 Find the number of solutions of the equation

$$14\cos^3 x + 10\sin^2 x \cos x = 13\cos x$$

in the range $-2\pi \leq x \leq 2\pi$

A 4

B 6

C 8

D 10

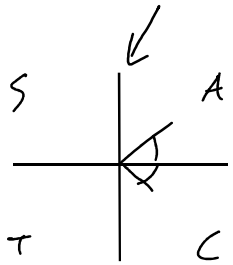
E 12

F 14

$14\cos^3 x + 10\sin^2 x\cos x - 13\cos x = 0$

$\cos x\left(14\cos^2 x + 10\sin^2 x - 13\right) = 0$

$\Rightarrow \cos x = 0$



$\cos^{-1}(0) = \frac{\pi}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}, -\frac{\pi}{2}, \frac{3\pi}{2}$

or $14\cos^2 x + 10\sin^2 x - 13 = 0$

$14\cos^2 x + 10(1 - \cos^2 x) - 13 = 0$ $\qquad \sin^2 x = 1 - \cos^2 x$

$14\cos^2 x + 10 - 10\cos^2 x - 13 = 0$

$4\cos^2 x - 3 = 0$

$\cos^2 x = \frac{3}{4}$

$\cos x = \pm\sqrt{\frac{3}{4}}$

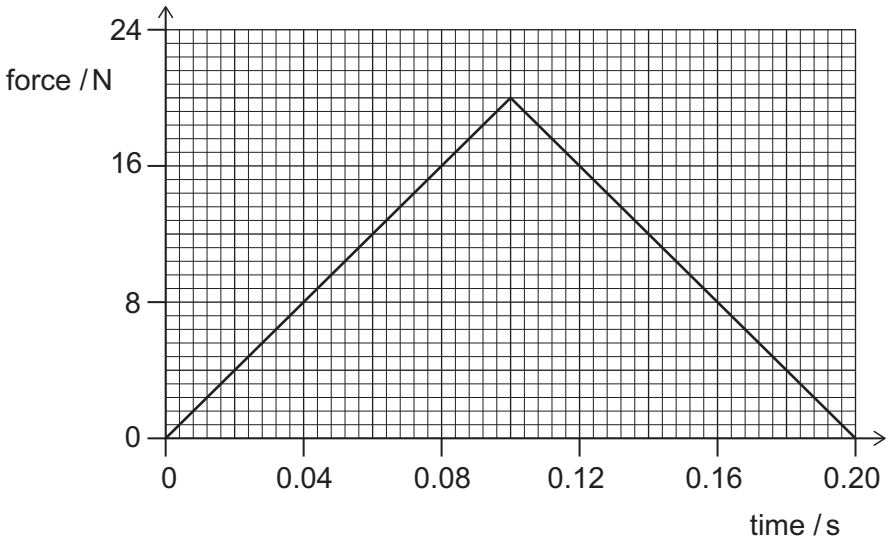
$\cos x = \pm\frac{\sqrt{3}}{2}$

Each of these will also give 4 solutions in the range

$\Rightarrow 4 \times 3 = 12$

37 An object of mass 2.5 kg is at rest at time = 0 s. A resultant force acts on the object in a constant direction.

The magnitude of the resultant force acting on the object varies with time as shown by the graph.



What is the kinetic energy of the object at time = 0.20 s?

A 0 J

B 0.80 J

C 1.0 J

D 1.6 J

E 2.0 J

F 3.2 J

G 6.4 J

Average force $= \frac{\text{max} - \text{min}}{2} = \frac{20 - 0}{2} = 10\,N$

Impulse $= ft = 10 \times 0.2 = 2\,Ns$

Impulse also $= \Delta mv$

$\Rightarrow \Delta mv = ft$

$v_2 m - v_1 m = 2$

$v_2 \times 2.5 - 0 \times 2.5 = 2$

$2.5 v_2 = 2$

$v_2 = \frac{2}{2.5} = 0.80\,J$

38 Find the product of the real roots of the equation

$$\left(\log_{10} x^2\right)^2 + \log_{10} x = 3$$

A $10^{-\frac{3}{2}}$

B 10^{-1}

C $10^{-\frac{1}{2}}$

D $10^{-\frac{1}{4}}$

E $10^{\frac{3}{5}}$

F 10^{1}

$(2\log_{10} x)^2 + \log_{10} x - 3 = 0$

$4(\log_{10} x)^2 + \log_{10} x - 3 = 0$

$\Rightarrow \log_{10} x = \frac{-1 \pm \sqrt{1^2 - 4\times 4\times -3}}{2\times 4}$

(we know these roots will be real as the discriminant > 0)

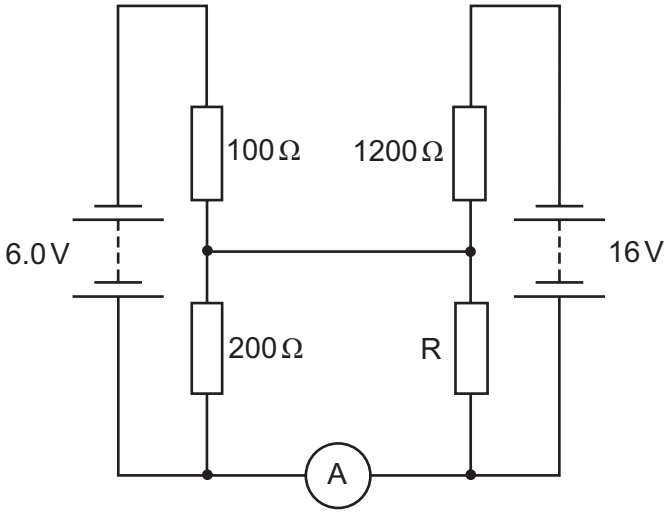
$\log_{10} x = \frac{-1 \pm 7}{8}$

$\Rightarrow \log_{10} x = \frac{3}{4}$ or $\log_{10} x = -1$

$x = 10^{\frac{3}{4}}$ $\quad x = 10^{-1}$

Product $= 10^{\frac{3}{4}} \times 10^{-1} = 10^{-\frac{1}{4}}$

39 In the following circuit, the ammeter records a current of zero.



What is the resistance of resistor R?

A 0 Ω

B 200 Ω

C 300 Ω

D 400 Ω

E 600 Ω

F 1200 Ω

G 1800 Ω

H 2400 Ω

Zero current ⇒ no net potential difference

⇒ Voltage over R = voltage over 200Ω

Left side:

Split 6 V in the ratio of resistances:

$\frac{6}{2+1} = 2$, across 200Ω, $V = 2 \times 2 = 4V$

Right side:

V across R must = 4V, leaving 12V for the 1200Ω

Ratio of voltage: 12 : 4

∴ Ratio of resistance: 1200 : 600

40 Find the maximum value of the gradient of the curve with equation

$$y = 2 - 4x + 4x^{\frac{3}{2}} - x^2$$

where $x > 0$

A -4

B $-\frac{8}{9}$

C $\frac{1}{2}$

D 2

E 4

Maximum value of $\frac{dy}{dx}$ occurs at the same x-value as $\frac{d^2y}{dx^2} = 0$:

$$\frac{dy}{dx} = -4 + 6x^{\frac{1}{2}} - 2x$$

$$\frac{d^2y}{dx^2} = 3x^{-\frac{1}{2}} - 2$$

END OF TEST

$$3x^{-\frac{1}{2}} - 2 = 0$$

$$x^{-\frac{1}{2}} = \frac{2}{3}$$

$$\sqrt{x} = \frac{3}{2}$$

$$x = \frac{9}{4}$$

at $x = \frac{9}{4}$: $\frac{dy}{dx} = -4 + 6\left(\frac{9}{4}\right)^{\frac{1}{2}} - 2 \times \frac{9}{4}$

$$= \frac{6x^3}{2} - \frac{18}{4} - 4$$

$$= 5 - \frac{9}{2}$$

$$= \frac{1}{2}$$