## ENGAA 2017 Section 1

## Model Solutions

1

$$
\begin{aligned}
& \frac{(\sqrt{12}+\sqrt{3})^{2}}{(\sqrt{12}-\sqrt{3})^{2}} \\
& =\frac{(2 \sqrt{3}+\sqrt{3})^{2}}{(2 \sqrt{3}-\sqrt{3})^{2}} \\
& =\frac{(3 \sqrt{3})^{2}}{(\sqrt{3})^{2}} \\
& =\frac{27}{3} \\
& =9
\end{aligned}
$$

## 2

The area underneath the graph represents displacement, and the gradient represents acceleration. The car is therefore decelerating where the gradient is negative; for $110 \leq t \leq 130$. Therefore:

$$
\begin{aligned}
\text { Distance } & =\frac{1}{2} \cdot(30+20) \cdot(20) \\
& =500 \mathrm{~m}
\end{aligned}
$$

3

$$
\begin{aligned}
2 x^{2} & \geq 15-x \\
2 x^{2}+x-15 & \geq 0 \\
(2 x-5)(x+3) & \geq 0 \\
\text { Critical values: } x=\frac{5}{2}, x & =-3
\end{aligned}
$$



$$
\therefore x \leq-3, x \geq \frac{5}{2}
$$

4
$\rho=\frac{m}{v}$ where $\rho$ is density, $m$ is mass, $v$ is volume.
Heating the liquid increases the thermal vibration of water molecules, thereby increasing the spacing between adjacent molecules. This leads to an increase in volume, and therefore a decrease in density, for a fixed mass of water. Therefore statements 2 and 3 are true. The less dense, hot water rises because it is lighter, allowing cooler water to replace it. This process creates a convection current.

For fixed $v$, as $\rho$ decreases, $m$ must also decrease, so statement 1 is false.

5

Making $x$ the subject:

$$
\begin{aligned}
y & =3\left(\frac{x}{2}-1\right)^{2}-5 \\
\frac{y+5}{3} & =\left(\frac{x}{2}-1\right)^{2} \\
1 \pm \sqrt{\frac{y+5}{3}} & =\frac{x}{2} \\
x & =2 \pm 2 \sqrt{\frac{y+5}{3}}
\end{aligned}
$$

6

$$
\begin{aligned}
\text { GPE gained } & =m g h \\
& =1200 \cdot 10 \cdot 1 \\
& =12000
\end{aligned}
$$

Work-energy principle:

$$
12000=28000-\text { Energy lost }
$$

$\therefore$ Energy lost $=16000 J$

7

$$
\begin{align*}
& 2 x+5 y=P  \tag{1}\\
& 3 x+2 y=Q \tag{2}
\end{align*}
$$

To find $y$, the cost of one pear, eliminate $x$ from both equations:

$$
\begin{align*}
& 3 \times(1): 3 P=6 x+15 y  \tag{3}\\
& 2 \times(2): 2 Q=6 x+4 y \tag{4}
\end{align*}
$$

subtract (4) from (3)

$$
\begin{aligned}
11 y & =3 P-2 Q \\
y & =\frac{3 P-2 Q}{11}
\end{aligned}
$$

8


Figure 1: $x$ is the distance between Q and the source
$t_{1}=$ the time taken for a gamma ray to travel $D_{1}$
$t_{2}=$ the time taken for a gamma ray to travel $D_{2}$
Time difference $=t_{2}-t_{1}=4.0 \times 10^{-10} s$

$$
\begin{aligned}
\text { distance } & =\text { speed } \times \text { time } \\
\therefore D_{2}-D_{1} & =c \times\left(t_{2}-t_{1}\right) \\
& =\left(3 \times 10^{8}\right) \times\left(4 \times 10^{-10}\right) \\
& =12 \times 10^{-2} m \\
& =12 c m \\
D_{1} & =1.5-x \\
D_{2} & =1.5+x \\
D_{2}-D_{1} & =(1.5+x)-(1.5-x) \\
12 c m & =2 x \\
\therefore x & =6 c m
\end{aligned}
$$

9

$$
\begin{aligned}
P & \propto Q^{2} \\
P & =k Q^{2} \\
2 & =16 k \\
k & =\frac{1}{8} \\
P & =\frac{1}{8} Q^{2} \\
\therefore P & =\frac{1}{8} \cdot\left(\frac{10}{R}\right)^{2} \\
& =\frac{100}{8 R^{2}} \\
& =\frac{25}{2 R^{2}}
\end{aligned}
$$

$$
Q \propto \frac{1}{R}
$$

$$
Q=\frac{k}{R}
$$

$$
2=\frac{k}{5}
$$

$$
k=10
$$

$$
Q=\frac{10}{R}
$$

10
a) False as $w+y+z=240$ due to the conservation of nucleon number
b) True as it is a rearrangement of the above equation
c) False as neutrons do not contribute to proton number, therefore $x=40$ due to the conservation of proton number
d) False as $94=54+x+0$
e) False as it combines both mass and proton number
f) False as it combines both mass and proton number

11

$$
\begin{aligned}
2 & -\frac{x^{2}\left(9 x^{2}-4\right)}{x^{3}(2-3 x)} \\
& =2-\frac{x^{2}(3 x+2)(3 x-2)}{-x^{3}(3 x-2)} \\
& =2+\frac{(3 x+2)}{x} \\
& =2+\frac{2}{x}+3 \\
& =5+\frac{2}{x}
\end{aligned}
$$

## 12

$$
\begin{aligned}
\text { Gain in GPE } & =20 \cdot g \cdot 6=1200 J \\
0.8 & =\frac{1200 J}{\text { total input from motor }}
\end{aligned}
$$

$$
\text { Total energy input by motor }=1500 J
$$

$$
\text { Power of motor }=I V=\frac{E}{t}
$$

$$
I \cdot 12 \cdot 5=1500 J
$$

$$
I=25 A
$$

## 13

$$
\begin{aligned}
2^{3+2 x} \cdot 4^{x} \cdot 8^{-x} & =4 \sqrt{2} \\
2^{3+2 x} \cdot 2^{2 x} \cdot 2^{-3 x} & =2^{2} \cdot 2^{0.5} \\
3+2 x+2 x-3 x & =2+0.5 \\
x & =-0.5
\end{aligned}
$$

## 14

A is not possible because alpha decay results in P-4, Q-2. To get an atomic number of Q-1 you would need one beta decay. But as the atomic mass number does not change in beta decay, its not possible for atomic mass to be P .

## 15

|  | Boys | Girls | Total |
| :---: | :---: | :---: | :---: |
| German | 2 Y | $2 X+Y-35$ | $?$ |
| French |  | X |  |
| Spanish | Y | $35-Y$ | 35 |
| Total |  | 3 X | 100 |

$$
\begin{aligned}
\text { Number of girls studying Spanish } & =35-Y \\
\text { Number of girls studying German } & =3 X-X-(35-Y) \\
& =2 X+Y-35 \\
\text { Total number of German students } & =(2 X+Y-35)+2 Y \\
& =2 X+3 Y-35
\end{aligned}
$$

## 16

Atomic Radius $(\mathrm{Ra})=3 \times 10^{4}$ Nuclear Radius (Rn)

$$
\begin{aligned}
\frac{R a}{R n} & =3 \times 10^{4} \\
\rho & =\frac{m}{v} \\
\rho & =\frac{m}{\frac{4}{3} \pi r^{3}}
\end{aligned}
$$

$\therefore$ For a constant mass, $\rho \propto \frac{1}{r^{3}}$

$$
\begin{aligned}
\frac{\rho_{a}}{\rho_{n}} & =\left(\frac{R n}{R a}\right)^{3} \\
& =\left(3 \times 10^{4}\right)^{-3}
\end{aligned}
$$

## 17

$$
\begin{aligned}
\frac{360}{n}-4 & =\frac{360}{n+3} \\
(360-4 n)(n+3) & =360 n \\
-4 n^{2}-12 n+1080+360 n & =360 n \\
4 n^{2}+12 n-1080 & =0 \\
n^{2}+3 n-270 & =0 \\
(n+18)(n-15) & =0 \\
n & =15
\end{aligned}
$$

18

$$
\begin{aligned}
v & =f \lambda \\
\text { Time period } & =\frac{2\left(t_{2}-t_{1}\right)}{3} \\
\text { Frequency } & =\frac{3}{2\left(t_{2}-t_{1}\right)} \\
\lambda & =2\left(x_{2}-x_{1}\right) \\
v & =2\left(x_{2}-x_{1}\right) \cdot \frac{3}{2\left(t_{2}-t_{1}\right)} \\
& =\frac{3\left(x_{2}-x_{1}\right)}{t_{2}-t_{1}}
\end{aligned}
$$

19


From diagram, $\angle C R L=40^{\circ}$ due to alternate angles rules
Triangle CRL is isosceles, therefore $\angle C L R=40^{\circ}$
$\therefore x=080^{\circ}$

$$
\begin{aligned}
P & =I V \\
I & =\frac{150}{12} \\
Q & =I t \\
& =\frac{150}{12} \cdot 20 \cdot 60 \\
& =15000 C
\end{aligned}
$$

21


Take 4 as a reference,
Angle between 4 and $8=30 \times 4=120$
Angle between 4:00 and 4:40 $=30 \times \frac{2}{3}=20$
Angle between the hour and minute hand $=100^{\circ}$

## 22

Mass of freight train $\left(M_{f}\right)=7 \times 30+3 \times 130=600$
Mass of passenger train $\left(M_{p}\right)=2 \times 70+x \times 10=140+10 x$
By the Principle of Conservation of Linear Momentum, we know that the two trains have momentum equal in magnitude:

$$
\begin{aligned}
2 M_{f}-5 M_{p} & =0 \\
\frac{2}{5} \cdot 600 & =140+10 x \\
x & =10
\end{aligned}
$$

## 23

$$
\begin{aligned}
\frac{x}{4+x} \cdot \frac{x-1}{3+x} & =\frac{1}{3} \\
3 x(x-1) & =(4+x)(3+x) \\
3 x^{2}-3 x & =12+7 x+x^{2} \\
2 x^{2}-10 x-12 & =0 \\
x^{2}-5 x-6 & =0 \\
(x-6)(x+1) & =0 \\
x=6 &
\end{aligned}
$$

## 24

$$
\begin{aligned}
K E & =0.5 \times 72 \times 5^{2} \\
& =900 J \\
& \therefore 1 \text { is false }
\end{aligned}
$$

Each second, loss of height is 5 m :

$$
\begin{aligned}
\text { GPE lost per second } & =72 \times 10 \times 5 \\
& =3600 J s^{-} 1 \\
& \therefore 2 \text { is true }
\end{aligned}
$$

The two forces are not of the same type and act on the same body, therefore 3 is false

25


$$
\begin{aligned}
\text { radius } & =\sqrt{x^{2}+\left(\frac{1}{2} x\right)^{2}} \\
& =\sqrt{\frac{5}{4} x^{2}} \\
\text { Area } & =\frac{1}{2} \pi r^{2}-x^{2} \\
& =\frac{1}{2} \pi\left(\frac{5}{4} x^{2}\right)-x^{2} \\
& =x^{2}\left(\frac{5 \pi-8}{8}\right)
\end{aligned}
$$

## 26

If there are initially N molecules of x and N molecules of Y :
6 hours later X undergoes 2 half lives, so there are $\frac{N}{4}$ molecules 6 hours later Y undergoes 3 half lives, so there are $\frac{N}{8}$ molecules Number of Z $=2 \mathrm{~N}-\frac{3}{8} N=\left(\frac{13}{8}\right) N$

Fraction of mixture made of $\mathrm{Z}=\left(\frac{13}{8}\right) N \div 2 N$

$$
=\frac{13}{16}
$$

27

$$
\begin{aligned}
\text { Volume of metal } & =\left(\pi \cdot 5^{2}-\pi \cdot 4^{2}\right) \cdot 16 \\
& =144 \pi \mathrm{~cm}^{3} \\
\text { Mass } & =8 \mathrm{~g} / \mathrm{cm}^{3} \cdot 144 \pi \mathrm{~cm}^{3} \\
& =1152 \pi g
\end{aligned}
$$

28

$$
\begin{aligned}
K E_{y} & =\frac{1}{2} m v^{2} \\
K E_{x} & =\frac{1}{2} \cdot \frac{4 m}{5} \cdot\left(\frac{3 v}{2}\right)^{2} \\
& =K E_{y} \cdot\left(\frac{4}{5} \cdot \frac{9}{4}\right) \\
& =1.8 \cdot K E_{y}
\end{aligned}
$$

29

$$
\begin{aligned}
1-\left(\frac{3+\sqrt{3}}{6-2 \sqrt{3}}\right)^{2} & =1-\frac{6(2+\sqrt{3})}{24(2-\sqrt{3})} \\
& =\frac{4(2-\sqrt{3})}{4(2-\sqrt{3})}-\frac{(2+\sqrt{3})}{4(2-\sqrt{3})} \\
& =\frac{6-5 \sqrt{3}}{4(2-\sqrt{3})} \\
& =\frac{(6-5 \sqrt{3})(2+\sqrt{3})}{4(4-3)} \\
& =-\frac{3}{4}-\sqrt{3}
\end{aligned}
$$

30

$$
\begin{aligned}
\text { Additional anti-clockwise moment } & =400 \cdot 10 \cdot 5 \\
& =20000 \mathrm{Nm}
\end{aligned}
$$

$\therefore$ Additional clockwise moment needed to balance $=20000 \mathrm{Nm}$

$$
\begin{aligned}
20000 & =2000 \cdot 10 \cdot x \\
x & =1 m
\end{aligned}
$$

Direction is to the right to provide a greater clockwise moment

$$
\begin{aligned}
2 \sin x+1 & =0 \\
\sin x & =-\frac{1}{2} \\
x & =210,330 \\
2 \cos 2 x & =1 \\
\cos 2 x & =\frac{1}{2} \\
2 x & =60,300,420 \\
x & =30,150,210
\end{aligned}
$$

$$
\therefore k=210
$$

30,150 , and 210 are solutions for at least one of the equations in the range $0 \leq x \leq 210 \therefore 3$ values of $x$

## 32

- Acceleration is constant (g) so the graph should comprise of straight lines (not B, C, H)
- The ball is thrown up into the air so line must be above time axis initially (not F,G)
- When the ball falls back down the velocity is negative (not E, D)
- A is the correct graph


## 33

$$
\begin{aligned}
3^{2 x+1} & =2 \cdot 3^{x+1} \\
\frac{3^{2 x+1}}{3^{x+1}} & =2 \\
3^{x} & =2 \\
x & =\log _{3} 2 x
\end{aligned}
$$

## 34

Resultant force is zero because the aircraft is climbing at a "constant speed" so acceleration is zero.

## 35

$$
\begin{aligned}
\text { Length of arc } & =r \theta \\
r \cdot \frac{11 \pi}{6} & =22 \pi \\
r & =12 \\
\text { Area QOR (Major Sector) } & =\frac{1}{2} r^{2} \theta \\
& =\frac{1}{2} \cdot(12)^{2} \cdot \frac{11 \pi}{6} \\
& =132 \pi
\end{aligned}
$$

$$
\begin{aligned}
\text { Area POS (Triangle) } & =\frac{1}{2} \cdot P Q \cdot S Q \cdot \sin (\angle P Q S) \\
& =\frac{1}{2} \cdot(12+18) \cdot(12+18) \cdot \sin \frac{\pi}{6} \\
& =225
\end{aligned}
$$

$$
\therefore \text { Total Area }=132 \pi+225
$$

## 36

Taking moments about the pivot:

$$
\begin{aligned}
60 \cdot 10 \cdot 2 & =F \sin 60 \cdot 4 \\
F & =\frac{300}{\sin 60}
\end{aligned}
$$

37

$$
\begin{aligned}
\frac{\left(8^{p}\right)^{3}}{\left(\left(\frac{1}{2}\right)^{2 q}\right)^{2}} & =\frac{8^{3 p}}{\left(\frac{1}{2}\right)^{4 q}} \\
& =\frac{\left(2^{3}\right)^{3 p}}{\left(2^{-1}\right)^{4 q}} \\
& =\frac{2^{9 p}}{2^{-4 q}} \\
& =2^{9 p+4 q}
\end{aligned}
$$

$$
\begin{aligned}
\log _{2}\left(2^{9 p+4 q}\right) & =(9 p+4 q)\left(\log _{2} 2\right) \\
& =9 p+4 q
\end{aligned}
$$

38

$$
\begin{array}{ll}
u=40 m s^{-1} & \\
s=20 m & \\
s=?
\end{array}
$$

$$
\begin{aligned}
v^{2} & =u^{2}+2 a s \\
& =40^{2}+2 \cdot(-14.4) \cdot 20 \\
& =1024
\end{aligned}
$$

$$
\therefore v=\sqrt{1024}=32
$$

39

$$
\text { Let } \begin{aligned}
& f(x)=x^{3}+p x^{2}+q x+6 \\
& \qquad \begin{aligned}
f^{\prime}(x) & =3 x^{2}+2 p x+q \\
f^{\prime}(2) & =12+4 p+q=0(1) \\
f^{\prime}(4) & =48+8 p+q=0(2)
\end{aligned}
\end{aligned}
$$

$$
(2)-(1): 36+4 p=0
$$

$$
4 p=-36
$$

$$
p=-9
$$

$$
q=24
$$

## 40

Resolving forces on the suspended load, we see that the tension in the string is 10 N because the load is in equilibrium and therefore experiences no resultant force. The pulley is frictionless and the string inextensible, therefore the tension throughout the string is 10 N . The force meter reads 10 N .

41

$$
\begin{aligned}
\text { Area } & =\frac{1}{2} \cdot(8-3 x) \cdot(4 x) \cdot \sin 60 \\
& =\frac{\sqrt{3}}{4} \cdot 4 x \cdot(8-3 x) \\
& =\sqrt{3}\left(8 x-3 x^{2}\right)
\end{aligned}
$$

Completing the Square:

$$
\begin{aligned}
& -3 \sqrt{3}\left(x-\frac{4}{3}\right)^{2}+\frac{16 \sqrt{3}}{3} \\
& \therefore \text { maximum area }=\frac{16 \sqrt{3}}{3}
\end{aligned}
$$

## 42

Using Work-Energy Principle:

$$
\begin{aligned}
M E_{i} & =4 J \\
M E_{f} & =3.2 J \\
\text { Work done against resistive forces } & =4-3.2 \\
& =0.8 J
\end{aligned}
$$

## 43

Using binomial expansions:

$$
\begin{aligned}
\text { Coefficient of } x^{4} & =\binom{6}{4} \cdot 2^{2} \cdot(3 x)^{4} \\
& =4860
\end{aligned}
$$

Once differentiated:

$$
\text { Coefficient of } \begin{aligned}
x^{3} & =4860 \times 4 \\
& =19440
\end{aligned}
$$

44
$u=13 \mathrm{~ms}^{-1}$ due to the conservation of energy

$$
\begin{gathered}
a=10 m s^{-2} \\
s=6 m \\
t=?
\end{gathered}
$$

$$
\begin{aligned}
s & =u t+\frac{1}{2} a t^{2} \\
6 & =13 t+\frac{1}{2}(10) t^{2} \\
0 & =5 t^{2}+13 t-6 \\
& =(5 t-2)(t+3) \\
\therefore t & =0.4 s
\end{aligned}
$$

45

$$
\begin{aligned}
\frac{4}{3} & =\frac{1}{1-r} \\
4(1-r) & =3 \\
4-2 \sin 2 x & =3 \\
1 & =2 \sin 2 x \\
\sin 2 x & =\frac{1}{2} \\
2 x & =\frac{\pi}{6}, \frac{5 \pi}{6}, \frac{13 \pi}{6}, \frac{17 \pi}{6} \\
x & =\frac{\pi}{12}, \frac{5 \pi}{12}, \frac{13 \pi}{12}, \frac{17 \pi}{12}
\end{aligned}
$$

$\therefore$ In the given range $x=\frac{13 \pi}{12}, \frac{17 \pi}{12}$

> Work Done = area under F-d graph

$$
\begin{aligned}
& =\frac{1}{2} \cdot 0.4 \cdot 192 \\
& =38.4 \mathrm{~J}
\end{aligned}
$$

$\therefore$ Kinetic Energy $=38.4 \mathrm{~J}$
GPE gained $=$ KE loss

$$
\begin{aligned}
38.4 & =0.024 \cdot 10 \cdot h \\
\therefore h & =160 m
\end{aligned}
$$

$$
\begin{aligned}
& U_{1}=2 \\
& U_{2}=2 P+3 \\
& U_{3}=2 P^{2}+3 P+3 \\
& U_{4}=2 P^{3}+3 P^{2}+3 P+3=-7
\end{aligned}
$$

$$
\text { let } \mathrm{f}(\mathrm{p})=2 P^{3}+3 P^{2}+3 P+10
$$

$$
f(-2)=0 \therefore(\mathrm{P}+2) \text { is a factor }
$$

$$
\therefore P=-2
$$

$$
U_{2}=-1
$$

$$
U_{3}=5
$$

$$
\therefore \text { Total }=-1
$$

48


When angle of tilt is $20^{\circ}$ :
Resolving perpendicular to the slope: $R=m g \cos 20$
Resolving parallel to the slope: $m g \sin 20-\mu(m g \cos 20)=0$

$$
\begin{aligned}
\sin 20 & =\mu \cos 20 \\
\mu & =\tan 20
\end{aligned}
$$

When angle of tilt is $25^{\circ}$ :
Resolving perpendicular to the slope: $R=m g \cos 25$
Resolving parallel to the slope: $m g \sin 25-\mu(m g \cos 25)=m a$

$$
\begin{aligned}
g(\sin 25-\mu \cos 25) & =a \\
a & =g(\sin 25-\tan 20 \cdot \cos 25)
\end{aligned}
$$

## 49

From the answers we can surmise that the root to the equation on the numerator is $\mathrm{x}=1$ and $\mathrm{x}=4$. Then sketching $f(x)=(x-1)^{2}(x-4)$ and $g(x)=\frac{1}{x}$


- for $\mathrm{x}>4, \mathrm{f}(\mathrm{x})$ and $\mathrm{g}(\mathrm{x})$ both greater than 0 , so the function $>0$
- for $0 \leq \mathrm{x} \leq 4, \mathrm{f}(\mathrm{x})<0$ and $\mathrm{g}(\mathrm{x})>0$, so the function is $<0$
- for $\mathrm{x}<0, \mathrm{f}(\mathrm{x})$ and $\mathrm{g}(\mathrm{x})$ both less than 0 , so the function $>0$

50

- Resolving the component of weight (mg) parallel to the slope gives mgsin $\theta$
- To travel at a constant speed the resultant force on the suitcase must be zero
- $\therefore$ Frictional force acting up the slope (opposite to the parallel component of weight) is also $m g \sin \theta$

51

$$
\begin{aligned}
f(x) & =\sin x \\
\text { Stretch: } f(2 x) & =\sin (2 x) \\
\text { Translation: } f\left(2\left(x+\frac{\pi}{4}\right)\right) & =\sin \left(2 x+\frac{\pi}{2}\right)
\end{aligned}
$$

## 52

$$
\begin{aligned}
\text { Change in Momentum } & =\text { Area underneath } F-t \text { Graph } \\
& =F\left(t_{2}-t_{1}\right) \\
\text { Momentum before } & =m(-u) \\
\text { Momentum after } & =m v \\
F\left(t_{2}-t_{1}\right) & =m v-(-m u) \\
\therefore m v & =F\left(t_{2}-t_{1}\right)-m u
\end{aligned}
$$

## 53

Perpendicular implies the product of the gradients is -1

$$
\begin{aligned}
\left(2 p^{2}-p\right)(p-2) & =-1 \\
2 p^{3}-p^{2}-4 p^{2}+2 p+1 & =0 \\
\therefore f(p) & =2 p^{3}-5 p^{2}+2 p+1 \\
f(1) & =0 \therefore(p-1) \text { is a factor by the Factor Theorem } \\
\therefore f(p) & =(p-1)\left(2 p^{2}-3 p-1\right)
\end{aligned}
$$

Using the quadratic formula, the remaining two roots $=\frac{3 \pm \sqrt{9+8}}{4}$

$$
=\frac{3}{4} \pm \frac{\sqrt{17}}{4}
$$

We know that $\sqrt{16}=4$ so $\frac{\sqrt{17}}{4} \approx 1$
The greatest root is therefore roughly 1.75

## 54

For the situation in which the ball bounces off the ground at $\mathrm{t}=0.5$, and reaches maximum height.

$$
\begin{aligned}
v= & a=-10 & & t=\frac{1.3-0.5}{2}=0.4 \\
s & =v t-\frac{1}{2} a t^{2} & & v=u+a t \\
& =0-\left(\frac{1}{2}\right)(-10)(0.4)^{2} & & 0=u+(-10)(0.4) \\
& =0.8 & & u=4
\end{aligned}
$$

