Useful Formulae for ENGAA

Physics

Electricity

$$R_T = R_1 + R_2 + \cdots + R_n$$

Effective resistance within a series circuit

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

Effective resistance within a parallel circuit

$$V_{out} = \frac{R_1}{R_1 + R_2} \cdot EMF$$

where V_{out} is the PD across the resistor, R_1

$$P = IV = \frac{V^2}{R} = I^2R$$

where P is electrical power dissipated across a component, V is the PD across it and I is the current through it

$$V = IR$$

$$Q = It$$

$$W = QV$$

$$R = \frac{\rho L}{A}$$

where ρ is resistivity, L is the length of wire and A is the cross-sectional area

Waves

$$Frequency = \frac{1}{Time\ Period}$$

$$v = f\lambda$$

$$\lambda = \frac{ax}{D}$$

λ is the wavelength, a is the slit to slit separation, x is the fringe separation and D is the slit to screen distance

$$n\,\lambda = \ d \ sin\theta$$

n is the order of the maxima, d is the slit separation, θ is the angle between the light and the horizontal

Kinematics

When acceleration is not constant:

$$a = \frac{dv}{dt}$$

$$v = \frac{ds}{dt} = \int a \ dt$$

$$s \ = \int v \ dt$$

For a constant acceleration use suvat equations:

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$v \ = \ u \ + \ at$$

Forces and Equilibrium

$$\text{Moment} \ = \ \text{Force} \ \times \text{Perpendicular Distance}$$

$$F \ \leq \mu R$$

For an object in equilibrium, where R is the normal contract force and F is friction

$$F \ = \ \mu R$$

At the point of sliding

$$\text{Magnitude of Resultant Force, F} = \sqrt{x^2 + y^2}$$

$$x \ = \ Fcos\,\theta$$

$$y \ = \ Fsin\,\theta$$

where x is the force in horizontal plane and y is force in vertical

$$Centripetal\ Force = \frac{mV^2}{R}$$

Newton's Laws

$$F = ma$$

where F is the resultant force acting on the body

$$\text{Momentum} = \text{mass} \times \text{velocity}$$

where total momentum in system remains constant unless an external force acts

$$\Delta P = Ft$$

Change in Momentum = Area under F-t graph

$$F = \frac{dp}{dt}$$

$$Power = \frac{Energy}{Time}$$

Energy

$$KE = \frac{1}{2}mv^2$$

$$GPE = mgh$$

$$W = fd$$

where d is the distance travelled in direction of the force in m

$$ME_f - ME_i = \text{Work done by Driving Force} - \text{Work Done by Resistive Forces}$$

where ME_f is the Final mechanical energy, ME_i is the Initial mechanical energy

$$\text{Mechanical Energy} = KE + GPE$$

$$\text{Efficiency} = \frac{\text{useful output}}{\text{total input}} \times 100\%$$

Materials

$$Density = \frac{mass}{volume}$$

$$Stress = \frac{Extension}{Original\ Length}$$

$$Strain = \frac{Force}{Cross - Sectional\ Area}$$

$$Young's\ Modulus = \frac{Stress}{Strain}$$

$$F = kx$$

$$EPE = \frac{1}{2}Fx = \frac{1}{2}kx^2$$

Area underneath a F-x graph

Radioactivity

Half-life = time taken for number of undecayed nuclei or the activity of a sample to halve

$$\text{Number of half} - \text{lives occurred, n} = \frac{\text{time elapsed}}{\text{half} - \text{life of a sample}}$$

$$A = \left(\frac{1}{2}\right)^n \cdot A_0$$

The final activity, A, after n half-lives, of a sample with initial activity A_0

$$N = \left(\frac{1}{2}\right)^n \cdot N_0$$

Amount of undecayed nuclei left, N, after n half-lives, of a sample with initial number of undecayed nuclei N_0

Mathematics

Ratios and Proportionality

$$\text{If } x: y = a: b, \text{ then } \frac{x}{y} = \frac{a}{b}$$

where a and b are numbers

$$x \propto y \Rightarrow \frac{x_1}{y_1} = \frac{x_2}{y_2} = k$$

x is directly proportional to y so their quotient is a constant ratio, k

$$x \propto \frac{1}{y} \Rightarrow x_1 y_1 = x_2 y_2 = k$$

x is inversely proportional to y so their product is a constant ratio, k

$$\text{Given } x = yz, \text{for a constant } z, x \propto y$$

Algebra and Functions

For a quadratic of form $ax^2 + bx + c = 0$, roots are:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant, d = $b^2 - 4ac$

d = 0 for repeated roots, d < 0 for no real roots, d > 0 for 2 distinct real roots

$$\text{If } f(a) = 0, \text{then} (x - a) \text{ is a factor of the equation } f(x) = 0$$

$$\text{If } f(a) < 0, f(b) > 0, \text{the root is in the interval } [a, b]$$

Sequences and Series

For Arithmetic Progressions with first term, a, and common difference, d:

$$U_n = a + (n-1)d$$

$$S_n = \frac{1}{2}n[2a + (n-1)d]$$

For Geometric Progressions with first term, a, and common ratio, r:

$$U_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \quad \text{for } -1 < r < 1$$

Binomial Expansions

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

Geometry

$$\text{Circumference of circle} = 2\pi r$$

$$\text{Area of a Circle} = \pi r^2$$

$$\text{Volume of a circle} = \frac{4}{3}\pi r^3$$

$$\text{Exterior Angle} = \frac{360}{n}$$

For a regular polygon

Coordinate Geometry

$$\text{Equation of a circle: } (x-a)^2 + (y-b)^2 = r^2$$

For a circle with centre (a,b) and radius, r

If two lines are perpendicular, the product of their gradients = -1

Trigonometry

Angle	**0°**	**30°**	**45°**	**60°**	**90°**
sinϑ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cosϑ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tanϑ	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

From degrees to radians: multiply by π and divide by 180
From radians to degrees: divide by π and multiply by 180

For θ in radians;

$$\text{Length of Arc} = \text{r}\theta$$

$$\text{Area of a sector} = \frac{1}{2}\text{r}^2$$

For a triangle with angles A,B and C in radians or degrees and sides a, b, c;

$$\text{Area of a Triangle} = \frac{1}{2}\text{absinC}$$

$$\frac{\text{sinA}}{\text{a}} = \frac{\text{sinB}}{\text{b}} = \frac{\text{sinC}}{\text{c}}$$

Trigonometric Identities;

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Exponentials and Logarithms

$$\log a + \log b = \log ab$$

$$log a - log b = log\frac{a}{b}$$

$$log a^b = b log a$$

Calculus

Increasing function: $f'(x) > 0$

Decreasing function: $f'(x) < 0$

Stationary Point: $f'(x) = 0$

Maximum point: $f''(x) < 0$

Minimum point: $f''(x) > 0$