Biomedical Admissions Test (BMAT)

Section 2: Mathematics
Questions by Topic

M5: Geometry

M5: Geometry - Questions by Topic

Mark scheme and explanations at the end

1 This diagram shows a sector of a circle, with centre *O* and radius 5. The arc length of the sector is 10.

Calculate the area of this sector.

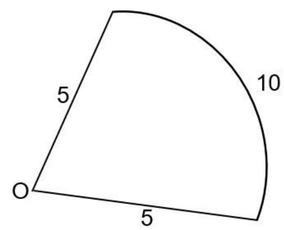


Diagram not drawn to scale

A 25π

B $\frac{25}{2}$

C 25

D 50

E $25\pi^2$

2 The curve $6y - 12 = 6x^2 - 18x$ is intersected at two points, A and B, by the straight line $y + x = 5$.

Find the coordinates of points A and B

A (2,3) and (-1,8)

B (-4,5) and (-5,4)

C (2,8) and (-1,3)

D (-1,6) and (3,2)

E (4,5) and (-4,-5)

3 In the figure below, YZ is the diameter of the circle, and XY=XZ. What is the area of the grey region?

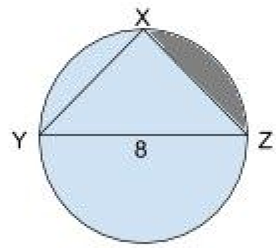


Diagram not drawn to scale

A $4\pi - 8$
B $8\pi - 8$
C π
D 4π
E 8π

4 The diagram below shows a cone and a frustum. The frustum is formed by removing the top 15 cm from the cone of height 60 cm. The frustum has an upper radius of 4 cm and a lower base radius of 8 cm.

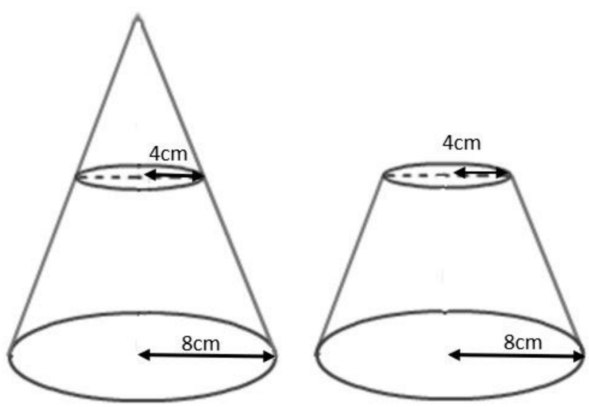


Diagram not to scale

Calculate the volume of the frustum that is formed from the cone.

A 1200π
B 1800π
C 3400π
D 3600π
E $10,8000\pi$

5 Margo wants to clean the window at the very top of her house which is 19 m high. She borrows a ladder from her neighbour Mark which is $4\sqrt{3}$ m long. She leans this ladder against the wall of her house, so that the foot of the ladder is 1.25 m from the base of the wall. The ladder doesn't reach the window.

How far away, in metres, is the top of the ladder from the window at the top of Margo's house?

A $19 - \frac{\sqrt{743}}{4}$

B $\frac{19-\sqrt{743}}{4}$

C $19 + \frac{\sqrt{743}}{4}$

D $\frac{19+\sqrt{743}}{4}$

E *More information needed*

6 The diagram shows an equilateral triangle ABC sitting within square $ABED$. The square has sides of length $4\ cm$.

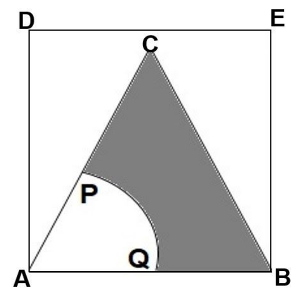
P is the midpoint of AC

Q is the midpoint of AB

APQ is a sector of a circle, centre A

Calculate the area of the shaded region

Diagram not drawn to scale



A $\frac{2}{3}\pi - 4\sqrt{3}$

B $\frac{2}{3}\pi - 8$

C $8 - \frac{2}{3}\pi$

D $4\sqrt{3} - \frac{2}{3}\pi$

E $2 - \frac{8}{3}\pi$

7 What is the circumference of a circle with an area of 18π?

A More information needed

B $\sqrt{6\pi}$

C 6π

D $6\pi\sqrt{2}$

E $\pi\sqrt{2}$

8 Julie has gone shopping. She sees a can of Baked Beans on the shelf in the shop and finds herself wondering what the surface area of the cylindrical can is. The area of the circle forming the base and lid of the can is 10π and the height h of the can is $8\ cm$.

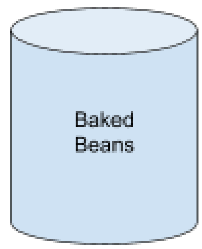


Diagram not drawn to scale

Calculate the surface area of the can of Baked Beans.

A More information needed
B $8\sqrt{10} + 10$
C $4\pi(4\sqrt{10} + 5)$
D $2\pi(8\sqrt{10} + 5)$
E $8\sqrt{10} + 5$

9 Calculate the surface area of the cardboard box in the diagram below.

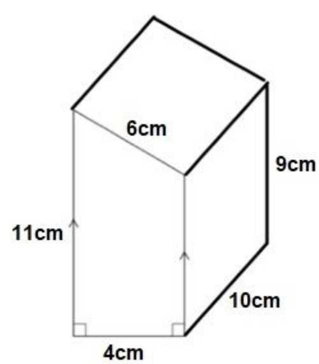


Diagram not drawn to scale

A $400\ cm^2$
B $360\ cm^2$
C $420\ cm^2$
D $380\ cm^2$
E $340\ cm^2$

10 Rectangle $ABCD$ is the base of the pyramid. Length $AB = 4\ cm$ and length $BC = 6\ cm$. The vertex of the pyramid is marked as point V. Length $VC = VB = VA = VD = 8\ cm$.

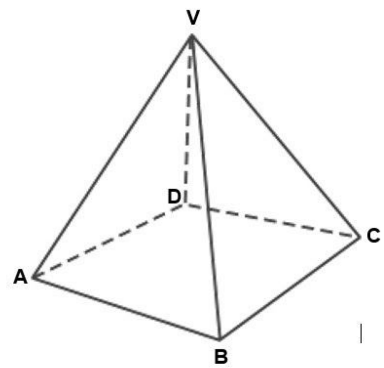


Diagram not drawn to scale

Calculate the vertical height, h, of this pyramid in cm.

A $\sqrt{51}$
B 28
C 23
D 13
E 8

11 From the top of a lighthouse, Ben can see his sister on the beach at angle of depression of 45°. If the lighthouse is 40m in height, how far is Ben's sister from the base of the lighthouse?

$$(\ sin45^\circ = \frac{\sqrt{2}}{2},\ tan45^\circ = 1,\ cos45^\circ = \frac{\sqrt{2}}{2})$$

A 40
B $40\sqrt{2}$
C $20\sqrt{2}$
D 20
E 15

12 Given that $cos(60) = 0.5$, find the length of side AB.

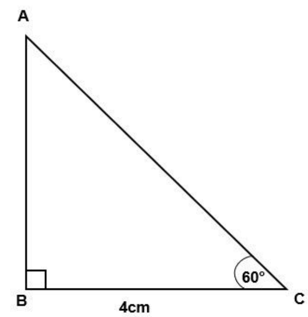


Diagram not drawn to scale

A 3
B $3\sqrt{4}$
C 4
D $\sqrt{3}$
E $4\sqrt{3}$

13 $ABCDE$ is a square-based pyramid with M as the midpoint of its base. The square base has sides of length 4 cm. The perpendicular height of the pyramid, AM, is 8 cm.

Find the angle edge AE makes with the base.

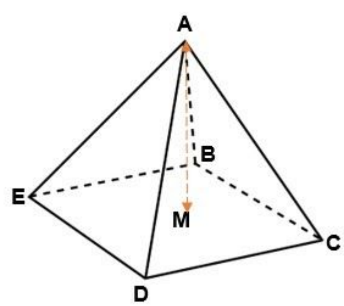


Diagram not drawn to scale

A $cos^{-1}(\frac{8}{\sqrt{8}})$
B $tan^{-1}(2\sqrt{2})$
C $sin^{-1}(\frac{8}{\sqrt{8}})$
D $cos(2\sqrt{2})$
E $tan(\frac{8}{\sqrt{8}})$

14 Mr Sliver has just won the lottery! With his prize money of £11,000 he decides to have his garden redecorated. Below are the blueprints for his new garden. The rectangle $WXYZ$ represents his new garden.

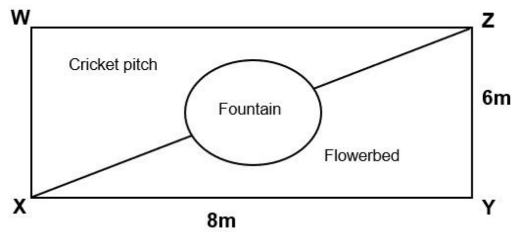


Diagram not to scale

The black lines show all the paved paths in his garden. Mr. Sliver will not permit anyone to walk in his garden unless they stick to walking on the paths. The circular path surrounds his new fountain at the centre of his garden. The fountain has a diameter of 4 m.

Calculate the shortest distance from X to Z, using only the paths shown.

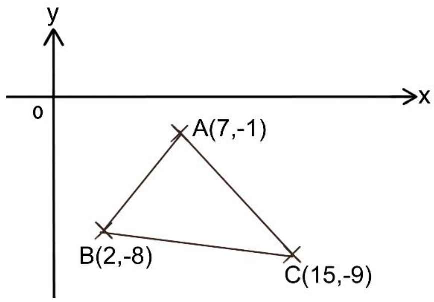
A $14 - 2\pi$
B $6 + 4\pi$
C $6 + 2\pi$
D $14 - 4\pi$
E $14 + 4\pi$

15 A triangle has vertices $(-3,-1)$, $(-6,-5)$, $(-1,-4)$. The triangle is reflected in the x-axis and rotated 180° clockwise about the origin.

What transformation can return the triangle to its original orientation?

A No transformation is needed
B Translate 6 units in the negative x direction
C Reflect in the y-axis
D Reflect in the line $y = x$
E Reflect in the x-axis

16 What are the coordinates of each vertex if the figure below is rotated 180° about the origin?



A A $(7,-1)$, B $(2,-8)$, C $(15,-9)$
B A $(-7,-1)$, B $(-2,-8)$, C $(-15,-9)$
C A $(7,1)$, B $(2,8)$, C $(15,9)$
D A $(-7,1)$, B $(2,8)$, C $(15,-9)$
E A $(-7,1)$, B $(-2,8)$, C $(-15,9)$

17 Silver has a density of 10.5 g/cm^3. A silver ball has a radius of 5 mm. The volume of a sphere is given by the formula $\frac{4}{3}\pi r^3$ where r is the radius of the sphere.

What is the mass of the silver ball?

A $\frac{7}{4}\pi\ grams$
B $1750\pi\ grams$
C $\frac{7}{2}\pi\ grams$
D $\frac{7}{32}\pi\ grams$
E $\frac{875}{4}\pi\ grams$

18 One side of a rectangle is $(16-\sqrt{7})\ cm$. The rectangle has an area of $996\ cm^2$.

What is the perimeter, in cm, of the rectangle?

A $160+6\sqrt{7}$
B $64+4\sqrt{7}$
C $80+3\sqrt{7}$
D $305-32\sqrt{7}$
E $96+2\sqrt{7}$

19 The length of a field measures 35 m. The diagonal of the field measures 65 m. 25 % of the field space will be used to grow corn.

What area of the field will be used to grow corn?

A $\frac{175}{2}\sqrt{30}$

B $350\sqrt{30}$

C $\frac{2275}{4}$

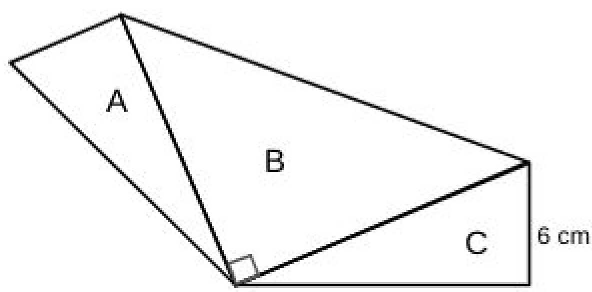
D $\frac{5\sqrt{218}}{4}$

E $70\sqrt{30}$

20 The diagram below is made up of right angled triangles.

Triangle C has an area of 24 cm^2.
B is an isosceles triangle.
Triangle A and C are similar.

[Diagram not to scale]



Calculate the perimeter of triangle A.

A 37.5 cm

B 24 cm

C 32 cm

D 26 cm

E 30 cm

Solutions

1 **C is the answer**

To be able to answer this question, you must know these four equations:

$$Area\ of\ circle = \pi r^2$$
$$Circumference\ of\ circle = \pi D = 2\pi r$$
$$Area\ of\ Sector = \frac{x^\circ}{360} \times Area\ of\ full\ circle$$
$$Length\ of\ arc = \frac{x^\circ}{360} x\ Circumference\ of\ full\ circle$$

We've been told arc length $= 10$ and radius = 5. Use this information to find out the angle x needed to work out the area of the sector.

$$Circumference\ of\ full\ circle = 2\pi r = 2\pi \times 5 = 10\pi$$

$$\Rightarrow Arc\ length = \frac{x^\circ}{360} x\ 10\pi \qquad \Rightarrow 10 = \frac{x^\circ}{360} \times 10\pi \qquad \Rightarrow x^\circ = \frac{360}{\pi}$$

Next, find the area of the full circle:

$$Circle\ Area = \pi r^2 \qquad \Rightarrow Circle\ Area = \pi \times 5^2 \qquad \Rightarrow Circle\ Area = 25\pi$$

Now that we know the angle x and the area of the full circle:

$$Area\ of\ Sector = \frac{x^\circ}{360} \times Area\ of\ full\ Circle = \frac{360}{\pi} \times \frac{1}{360} \times 25\pi$$

$$\Rightarrow Area\ of\ Sector = \frac{1}{\pi} \times 25\pi \qquad \Rightarrow \qquad Area\ of\ sector = 25$$

2 **D is the answer**

To find the point at which the curve and line intersect, the equations for the two lines need to be solved simultaneously to find their common points.

Notice that all the terms in the equation of the curve $6y - 12 = 6x^2 - 18x$ are a multiple of 6. Divide this equation through by 6 to make the terms smaller, this will save time when doing calculations.

$$y - 2 = x^2 - 3x \qquad (1)$$

$$y + x = 5 \qquad (2)$$

Rearrange *(2)* to make *y* the subject:

$$y = 5 - x \qquad (3)$$

There are several methods to solve the pair of simultaneous equations. Here are two possible methods.

Method 1: Solve by substitution	**Method 2**: Solve by subtraction
Substitute *(3)* into *(1)* $5 - x - 2 = x^2 - 3x$ $\Rightarrow 3 - x = x^2 - 3x$ $\Rightarrow 0 = x^2 - 2x - 3$	Rearrange *(1)* to make y the subject $y = x^2 - 3x + 2 \quad (4)$ Calculate *(4)*-*(3)* $y = x^2 - 3x + 2$ - $y = -x + 5$ $0 = x^2 - 2x - 3$

Using both methods, the same quadratic $x^2 - 2x - 3 = 0$ is obtained.

Solve the quadratic to obtain solutions for x. The easiest way to solve this quadratic is by factorising, but the quadratic equation can also be used.

$$(x-3)(x+1) = 0 \qquad \Rightarrow x = 3 \text{ and } x = -1$$

Use equation *(3)* to find the y values for each of the x values:

When $x = 3,\ y = 5 - 3 \qquad \Rightarrow x = 3,\ y = 2$

When $x = -1,\ y = 5 - -1 \qquad \Rightarrow x = -1,\ y = 6$

3 **A is the answer**

Shaded area = Circle area - Unshaded area

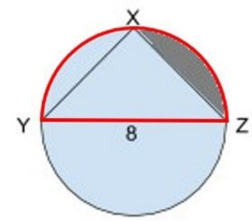
Area of a circle = πr^2

Since YZ = Diameter = 8 $\Rightarrow$ Radius $= 4$

Area = $\pi(4)^2$ $\Rightarrow$ Area of the circle = 16π

The line YZ is the diameter, which splits the circle into **two semi-circles.**

Area of 1 semicircle = Area of whole circle $\div 2 =$ $16\pi \div 2 = 8\pi$
The area of the circle outlined by the red line $= 8\pi$



Find the area of the triangle and subtract this value from the area of the semicircle to find the area of the two segments either side of the triangle.

Area of a triangle = $\frac{1}{2}\,base \times perpendicular\ height$

The perpendicular height of the triangle will be from point X to the midpoint of YZ. We know that YZ is the diameter of the circle and thus the midpoint of YZ will be the centre of the circle. This means the perpendicular height of the triangle is from point X to the centre of the circle, which is the radius.

Perpendicular height of triangle = 4 $\Rightarrow$ Area of triangle $= \frac{1}{2} \times 8 \times 4 = 16$

Area of two segments = Area of semicircle - Area of triangle $\Rightarrow 8\pi - 16$

Area of one segment (shaded segment) = $\frac{8\pi - 16}{2}$ $\Rightarrow 4\pi - 8$

4 **A is the answer**

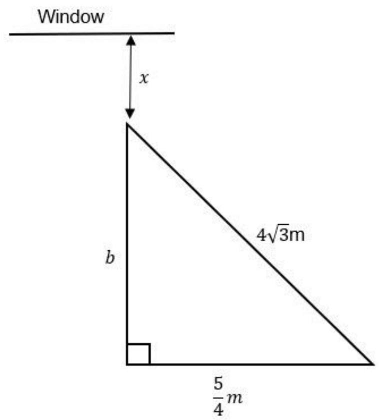
The formula for the volume of a cone with base radius r and height h is $\frac{1}{3}\pi r^2 h$.

Volume of Frustum = Volume of Original cone - Volume of Removed Cone
$= \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h$

where H = height of original cone, h = height of removed cone
R = base radius of original cone, r = base radius of removed cone

Volume of original cone: $\frac{1}{3}\pi \times 8^2 \times 60 = 1280\pi$
Volume of removed cone: $\frac{1}{3}\pi \times 4^2 \times 15 = 80\pi$
Volume of Frustum: $1280\pi - 80\pi = 1200\pi$

5 **A is the answer**



We need to find the distance labelled x. To do so, first find the value of b which is how far up the wall the ladder reaches.

Notice, 1.25 has been converted into fraction $\frac{5}{4}$ for ease of use in calculations.

Find the value of b by using Pythagoras' Theorem: $a^2 + b^2 = c^2$

$$\Rightarrow c^2 - a^2 = b^2 \qquad \Rightarrow (4\sqrt{3})^2 - (\tfrac{5}{4})^2 = b^2 \qquad \Rightarrow 48 - \tfrac{25}{16} = b^2$$

Convert 48 into a fraction with denominator 16:

$$b^2 = \frac{48\times16}{16} - \frac{25}{16} \quad \Rightarrow \quad b^2 = \frac{768}{16} - \frac{25}{16} \quad \Rightarrow \quad b^2 = \frac{743}{16}$$

$$\Rightarrow \quad b = \sqrt{\frac{743}{16}} \quad \Rightarrow \quad b = \frac{\sqrt{743}}{4}$$

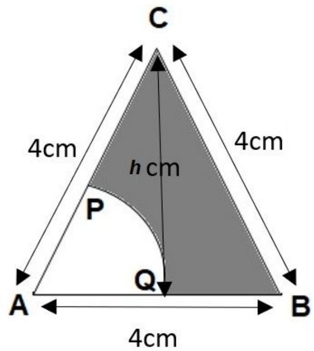
Remember the window is $19m$ high so,

$$x = 19 - b \quad \Rightarrow \quad x = 19 - \frac{\sqrt{743}}{4}$$

6 **D is the answer**

ABC is an equilateral triangle. This means that all the sides will be of the same length and all the angles of the equilateral triangle will be 60°.

Since ABC is an equilateral triangle sitting within square $ABED$, the side lengths of the equilateral triangle ABC will be the same as the side lengths of the square $ABED$:

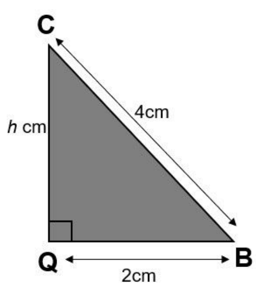


From the diagram it can be seen that:

$$Area\ of\ shaded\ region = Area\ of\ triangle - Area\ of\ sector$$

$$Area\ of\ triangle = \frac{1}{2} \times base \times height = \frac{1}{2} \times 4 \times h$$

Use Pythagoras' Theorem to calculate h:



$$a^2 + b^2 = c^2$$

$$\Rightarrow a^2 + 2^2 = 4^2 \qquad \Rightarrow a^2 = 12$$

$$\Rightarrow a = \sqrt{12} \qquad \Rightarrow h = \sqrt{12}$$

$$Area\ of\ triangle = \frac{1}{2} \times 4 \times \sqrt{12} = 4\sqrt{3}$$

$$Area\ of\ Sector = \frac{x^\circ}{360} \times Area\ of\ full\ circle$$

Since it is an equilateral triangle, all the angles of the triangle will be 60° and so $x = 60^\circ$

$$\Rightarrow Area\ of\ Sector = \frac{60}{360} \times Area\ of\ full\ circle$$

Now we need to find the area of the circle. A is the centre of the circle. This means $AQ\ (or\ AP)$ is the radius of the circle. Q is the midpoint of AB and so $AQ = 2cm$.

$$Area\ of\ full\ circle = \pi r^2 \qquad \Rightarrow Area\ of\ Circle = \pi \times 2^2 = 4\pi$$

$$\Rightarrow Area\ of\ Sector = \frac{60}{360} \times 4\pi = \frac{2}{3}\pi$$

Now,

$$\text{Area of shaded region} = \text{Area of triangle} - \text{Area of sector}$$

$$\Rightarrow \text{Area of Shaded region} = 4\sqrt{3} - \tfrac{2}{3}\pi$$

A is incorrect because the area of the shaded region has been found by subtracting the area of the triangle from the area of the sector, which is incorrect.

C is incorrect because the area of the triangle has been calculated using $h = 4$. You cannot assume that the perpendicular height of the triangle is the same as the side length. You must work out the perpendicular height using Pythagoras' Theorem.

7 **D is the answer**

$\text{Area of circle} = \pi r^2$ $\Rightarrow 18\pi = \pi r^2$ $\Rightarrow 18 = r^2$

$\Rightarrow r = \sqrt{18}$ $\Rightarrow r = \sqrt{9 \times 2}$ $\Rightarrow r = 3\sqrt{2}$

$\text{Circumference of circle} = \pi D = 2\pi r$

$\Rightarrow \text{Circumference} = 2\pi \times 3\sqrt{2}$ $\Rightarrow \text{Circumference} = 6\pi\sqrt{2}$

Note, r and D indicate the radius and circumference, respectively.

8 **C is the answer**

$\text{Surface area of a CYLINDER} = 2\pi rh + 2\pi r^2$

The question tells you $h = 8$ and that the $\text{area of the circle} = 10\pi$.

Calculate r using the area of the circle:

$\text{Area of Circle} = \pi r^2$ $\Rightarrow 10\pi = \pi r^2$ $\Rightarrow 10 = r^2$ $\Rightarrow$ $r = \sqrt{10}$

$\text{Surface area of a CYLINDER} = 2\pi rh + 2\pi r^2 = (2\pi \times \sqrt{10} \times 8) + (2\pi \times (\sqrt{10})^2)$

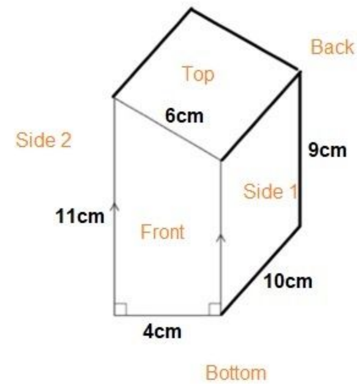
$\Rightarrow \text{Surface area} = 16\pi\sqrt{10} + (2\pi \times 10) = 16\pi\sqrt{10} + 20\pi$

$\Rightarrow \text{Surface area} = 4\pi(4\sqrt{10} + 5)$

D is incorrect because the surface area has been calculated using $2\pi rh + \pi r^2$ which is incorrect. A cylinder's surface area has **two** circles, one at the top of the cylinder and one at the bottom.

9 **D is the answer**

Be careful when working out the sides to this shape, it is important you don't miss any.



Area of front :

$$Area\ of\ trapezium = \frac{a+b}{2} \times h \quad \Rightarrow Area = \frac{9+11}{2} \times 4 \quad \Rightarrow Area = 40$$

$\Rightarrow Area\ of\ front = 40\ cm^2$

The back is the same as the front, so $Area\ of\ back = 40\ cm^2$

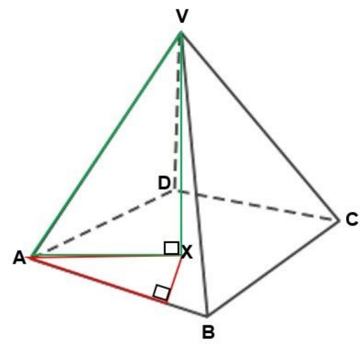
$Area\ of\ side\ one = 10 \times 90 = 90\ cm^2$
$Area\ of\ side\ two = 11 \times 10 = 110\ cm^2$
$Area\ of\ top = 6 \times 10 = 60\ cm^2$
$Aea\ of\ bottom = 4 \times 10 = 40\ cm^2$

$\Rightarrow Total\ Surface\ Area\ of\ shape = 40 + 40 + 90 + 110 + 60 + 40 = 380\ cm^2$

A is incorrect because the bottom has been calculated to have the same area as the top. The top and bottom are in fact different shapes and so will have different areas.
B is incorrect because side one and side two have been taken to have the same area.
C is incorrect because the perpendicular height $(4\ cm)$ has not been used to calculate the area of the trapezium (front side). Instead, the slanting height ($6\ cm$) has been used, which is incorrect.
E is incorrect because the base area has not been added.

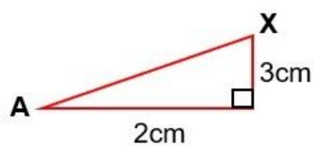
10 **A is the answer**

This question tests your ability to use 3D Pythagoras.



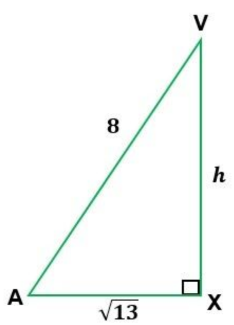
Let X be the centre of the base. The vertical height is the distance from the centre X to the highest point of the pyramid, the vertex V.

To find length VX, first find length AX.



$$AX^2 = 2^2 + 3^2 = 13$$
$$AX = \sqrt{13}$$

Equivalently, you could find the length of the diagonal AC and then divide the result by 2. Next, use the value AX to find $h = VX$:



$$8^2 = (\sqrt{13})^2 + h^2 \qquad \Rightarrow h^2 = 8^2 - \sqrt{13}^{\,2}$$
$$\Rightarrow h^2 = 64 - 13 \qquad \Rightarrow h^2 = 51$$

$$\Rightarrow h = \sqrt{51}$$

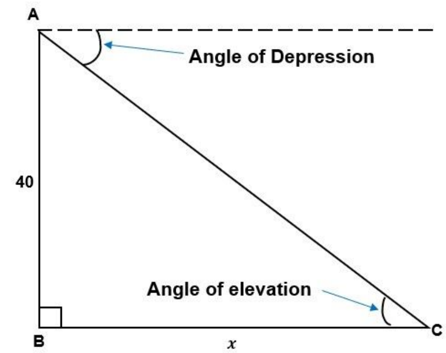
11 **A is the answer**

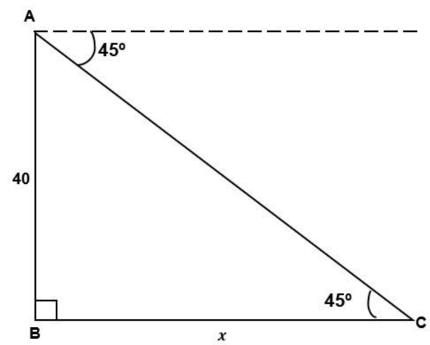
This question tests your knowledge of trigonometry and knowing what the angle of depression is. Draw a diagram to help you.

Let A be the top of the lighthouse and B the base of the lighthouse. We know that the lighthouse is $40\ m$ tall and so $AB = 40$.

Let C be the position where Ben's sister is standing. To find the distance his sister is from the base of the lighthouse, we need to find the distance labelled x

The angle of depression is the angle downwards from the horizontal.
The angle of elevation is the angle upwards from the horizontal.
Angle of depression = Angle of elevation





We know the length of the "opposite side" to the angle is 40.
We want to find the length of the "adjacent side" (x). This calculation requires the use of $tan\theta = \frac{opposite}{adjacent}$.

$$tan(45) = \frac{40}{x} \quad \Rightarrow \quad x = \frac{40}{tan(45)}$$

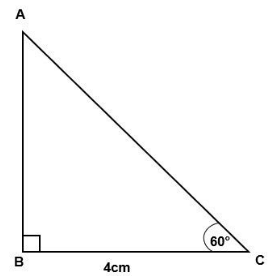
In the question we are told $tan(45) = 1$,

$$\Rightarrow x = \frac{40}{1} \quad \Rightarrow x = 40$$

B is incorrect, because the angle of depression has been labelled incorrectly and therefore $cos(45)$ has been used to find the length x which is incorrect.
C is incorrect because $sin(45)$ has been used.
D and E are incorrect.

12 **E is the answer**

The questions tells you $cos(60) = 0.5$. To find length AB you first need to calculate length AC and then use Pythagoras' Theorem to find AB. AB cannot be found directly using trigonometry in this question because we have only been given the trigonomic ratio for $cos(60)$.



To find length AC :

$$cos\theta = \frac{adjacent}{hypotenuse}$$

Hypotenuse = side opposite right angle = longest side = AC

$$\Rightarrow cos(60) = \frac{4}{AC} \Rightarrow 0.5 = \frac{4}{AC} \Rightarrow AC = \frac{4}{0.5}$$

$$AC = 8$$

To find length AB :

$$AC^2 = AB^2 + BC^2 \quad \Rightarrow AB^2 = AC^2 - BC^2 \quad \Rightarrow AB^2 = 8^2 - 4^2$$

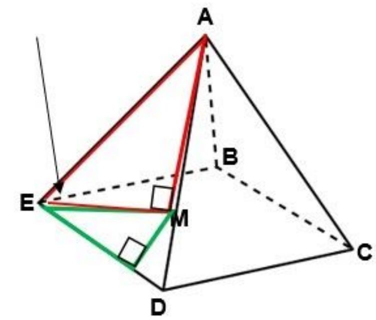
$$\Rightarrow AB^2 = 64 - 16 \quad \Rightarrow AB^2 = 48 \quad \Rightarrow AB = \sqrt{48}$$

$$\Rightarrow AB = \sqrt{16 \times 3} \quad \Rightarrow AB = 4\sqrt{3}$$

13 **B is the answer**

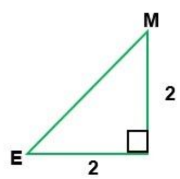
This question involves 3D trigonometry and Pythagoras' Theorem.

The angle to be found is the angle side AE makes with the base, marked by the black arrow in the following diagram.



To be able to use trigonometry to find $angle\ AEM$, you need to know the lengths of two sides. We know length AM and the length of side EM can be found using Pythagoras' Theorem on the green triangle.

Finding length EM :



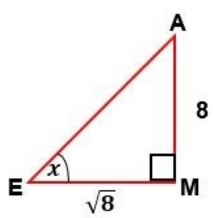
$EM^2 = 2^2 + 2^2 \quad \Rightarrow 4 + 4 = 8 \quad \Rightarrow EM^2 = 8$

$EM = \sqrt{8}$

Finding angle AEM :

$Length\ EM = Adjacent$

$Length\ AM = Opposite$



We know the adjacent and opposite lengths therefore to find angle x use:

$tan\ x = \frac{Opposite}{Adjacent} \quad \Rightarrow tan\ x = \frac{8}{\sqrt{8}}$

Simplify:

$tan\ x = \frac{8}{\sqrt{4 \times 2}} \quad \Rightarrow tan\ x = \frac{8}{2\sqrt{2}} \quad \Rightarrow tan\ x = \frac{4}{\sqrt{2}}$

$tan\ x = \frac{2^2}{2^{\frac{1}{2}}} \quad \Rightarrow tan\ x = 2^{2-\frac{1}{2}} \quad \Rightarrow tan\ x = 2^{\frac{3}{2}} \Rightarrow (\sqrt{2})^3$

To find x, inverse the trigonometric function:

$\Rightarrow$ Angle $AEM = tan^{-1}(2\sqrt{2})$

A and C are incorrect. They have been put in as solutions to trick the candidate. Many candidates will reach the stage of $tan\, x = \frac{8}{\sqrt{8}}$ and notice $\frac{8}{\sqrt{8}}$ appears many times in the range of solutions offered. Rather than continue their working and simplify the surd, they will assume they have made a mistake using tan and select the answer containing either cos or sin; this could be due to time pressures or due to not thinking to simplify their own working.

D and E are incorrect because the inverse trigonometric function has not been used - to find an angle you must use the inverse function.

14 **C is the answer**

The path distance along the two straight edges $X \rightarrow Y \rightarrow Z = 8 + 6 = 14\, m.$

Distance of $X \rightarrow Z$ via the circular path:

$$Circumference\ of\ circle = \pi d \Rightarrow 4\pi$$

BUT to get Z from X via this path, only half the circle is walked around, therefore the length of the circle path walked on is $\frac{4\pi}{2} = 2\pi$.

Use Pythagoras' Theorem to find length of straight line XZ:

$$XZ^2 = XY^2 + YZ^2 \quad \Rightarrow 8^2 + 6^2 = 100 \quad \Rightarrow XZ^2 = 100 \quad \Rightarrow XZ = 10$$

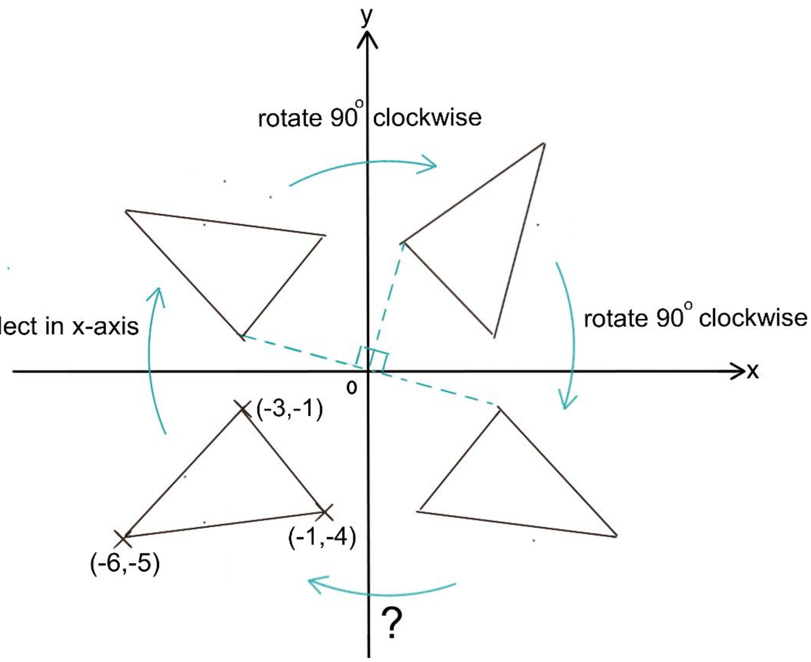
The straight line XZ has the circle within it so the actual distance of the straight part of the path XZ is:

$$XZ\ straight\ path = 10 - diameter\ of\ circle \Rightarrow 10 - 4 = 6$$

Therefore, distance of $X \rightarrow Z$ via the centre circular path is $6 + 2\pi$. Clearly this path is shorter than the 14 m path calculated by the two straight edges, so $6 + 2\pi$ is the final solution.

15 **C is the answer**

Draw out the transformations stated in the question.



From the diagram, we can see that **reflecting in the y-axis** can return the triangle to its original orientation.

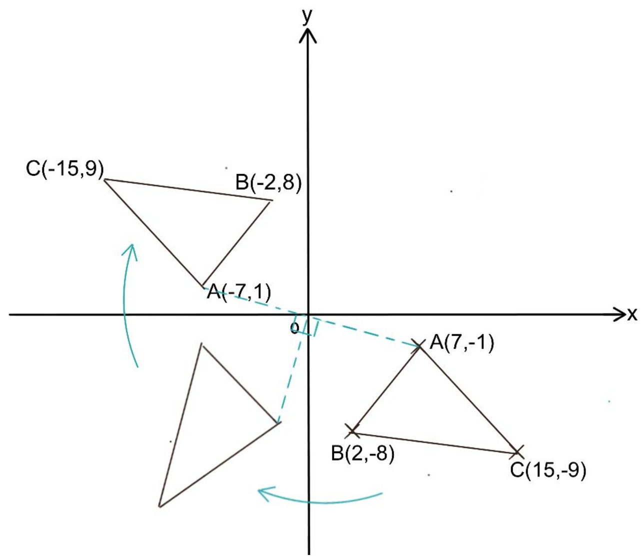
A **table** might help you track the position of the coordinate points. The original coordinates in the final row of the table is achieved by **reflecting the previous points in the y axis** since each x coordinate is **negated**.

Original	**(-3,-1)**	**(-1,-4)**	**(-6,-5)**
Reflected in x axis	(-3,1)	(-1,4)	(-6,5)
Rotated 180° clockwise	(3,-1)	(1,-4)	(6,-5)
?	(-3,-1)	(-1,-4)	(-6,-5)

A,B, D, E are incorrect.

16 **E is the answer**

It helps to draw a diagram to understand how the coordinates change under the transformation.



When they are rotated 180° you can see from the diagram that each **coordinate value is negated**. For example, (7,-1) becomes (-7,1), where the sign of each value has changed.

This gives the following coordinate values for the new figure: **(-7,1), (-2,8), (-15,9)**

A is incorrect.
C is incorrect.
D is incorrect.
E is incorrect.

17 **A is the answer**

The formula required to answer this question is the formula which links **density** to mass and volume:

$$Density = \frac{Mass}{Volume} \qquad (1)$$

To determine the mass of the silver ball, we first need to find its **volume**. You must make sure the radius of the ball is converted into cm from mm, before using it in the equation for volume - as we want volume in cm^3.

Note, $1\ cm = 10\ mm$.

Radius $5\ mm \Leftrightarrow Radius\ 0.5\ cm$

Therefore,

$$V = \frac{4}{3}\pi r^3 \Rightarrow V = \frac{4}{3}\pi(0.5)^3 \Rightarrow V = \frac{4}{3}\pi(0.125) \Rightarrow V = \frac{4}{3}\pi(\frac{125}{1000}) = \frac{\pi}{6}$$

Then, to find the mass, equation (1) needs rearranging into the following form:

$$Mass = Density \times Volume$$

Therefore,

$$Mass = 10.5 \times \frac{\pi}{6} = \frac{21}{2} \times \frac{\pi}{6} = \frac{21}{12}\pi = \frac{7}{4}\pi$$

So, the mass of the silver ball is $\frac{7}{4}\pi\ grams$.

B is incorrect because the radius has not been converted into cm.
C is incorrect because the volume equation was incorrectly used - the radius was squared rather than being cubed.
D is incorrect because the radius was halved for use in the formula for volume.
E is incorrect because the radius has not been converted into cm and the volume equation was incorrectly used - the radius was squared rather than being cubed.

18 **A is the answer**

One side of a rectangle is $(16 - \sqrt{7})\ cm$. The rectangle has an area of $996\ cm^2$.

First, calculate the length of the other side of the rectangle. Use complementary (conjugate) surds to remove the surd from the denominator:

$$\frac{996}{16-\sqrt{7}} = \frac{996}{16-\sqrt{7}} \times \frac{16+\sqrt{7}}{16+\sqrt{7}} = \frac{996(16+\sqrt{7})}{16^2-7} = \frac{996(16+\sqrt{7})}{249} = 4(16+\sqrt{7})\ cm$$

Calculate the perimeter of the rectangle by adding up each of the sides:

$$Perimeter = (16-\sqrt{7}) + (16-\sqrt{7}) + 4(16+\sqrt{7}) + 4(16+\sqrt{7}) = 160 + 6\sqrt{7}\ \ cm$$

Therefore, the **perimeter** of the rectangle is $160 + 6\sqrt{7}\ \ cm$.

B is incorrect because this is the calculated length of the missing side of the rectangle.
C is incorrect because this is half the required area.
D is incorrect because this is the perimeter of the square with sides $16 - \sqrt{7}$.
E is incorrect because it is the sum of only three of the sides of the rectangle.

19 **A is the answer**

First calculate the total area of the field. Use pythagoras' theorem to calculate the width of the field:

$$Width\ of\ field = \sqrt{(65)^2 - (35)^2} = \sqrt{4225 - 1225} = \sqrt{3000} = 10\sqrt{30}$$

$$Area\ of\ field = 10\sqrt{30} \times 35 = 350\sqrt{30}$$

Since corn is only going to be grown in 25% of the field, calculate 25% of the total area to obtain the final answer:

$$Area\ for\ corn = \frac{350\sqrt{30}}{100} \times 25 = \frac{35\sqrt{30}}{10} \times 25 = \frac{35\sqrt{30}}{2} \times 5 = \frac{175\sqrt{30}}{2}$$

So, the total area of the field is $350\sqrt{30}\ m^2$ and the area used to grow corn is $\frac{175\sqrt{30}}{2}\ m^2$.

B is incorrect because this is the area of the whole field.
C is incorrect because the diagonal of the field was used as the width of the field.
D is incorrect because the wrong form of pythagoras's theorem was used.
E is incorrect because the wrong percentage of the total field area was used.

20 **E is the answer**

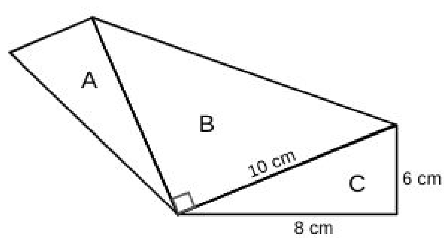
This question requires knowledge of the definition of an **isosceles** triangle and the meaning of **similar triangles**.

First, determine the length of the sides of triangle C using the information given.

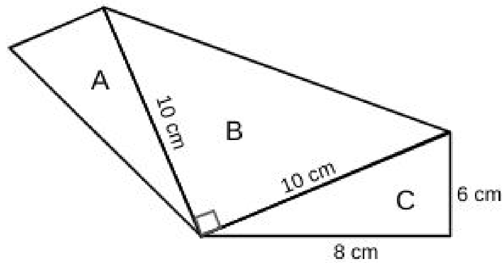
$$Area\ of\ C = \tfrac{1}{2} \times b \times h \quad \Rightarrow 24 = \tfrac{1}{2} \times b \times 6 \quad \Rightarrow b = \tfrac{24\times 2}{6} \quad \Rightarrow b = 8\ cm$$

Use Pythagoras's theorem to determine the length of the hypotenuse of C:

$$Hypotenuse\ of\ C = \sqrt{(8)^2 + (6)^2} = \sqrt{64 + 36} = \sqrt{100} = 10\ cm$$



Since B is an **isosceles** triangle it has two equal sides. It is also a right angles triangle so the two equal lengths must be the two lengths which are each not the hypotenuse. Therefore, we obtain that the other side of B (not the hypotenuse) is also 10 cm:



Triangle A is **similar** to triangle C. this means that we can obtain the lengths of triangle A by determining the **scale factor enlargement** of triangle C.
The 8 cm base of triangle C is matched to the 10 cm base of triangle A. Calculate the scale factor enlargement:

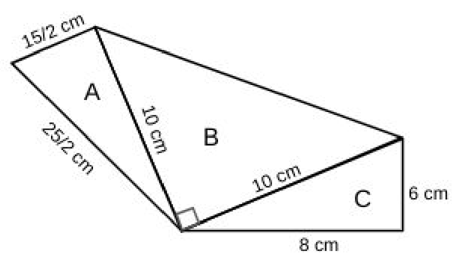
$$Scale\,factor\ enlargement = \frac{10}{8} = \frac{5}{4}$$

Therefore, each of the sides of triangle A are $\frac{5}{4}$ times larger than the sides of triangle C.

Method 1:
Calculate the length of triangle A sides:

$$Triangle\ A\ hypotenuse = 10 \times \frac{5}{4} = \frac{25}{2}$$

$$Triangle\ A\ shortest\ side = 6 \times \frac{5}{4} = \frac{15}{2}$$



Calculate the perimeter of triangle A:

$$Perimeter\ of\ A = 10 + \frac{15}{2} + \frac{25}{2} = 30\ cm$$

Method 2:
Calculate the perimeter of triangle C and then multiply it by the scale factor enlargement to obtain the perimeter of triangle A:

$$Perimeter\ of\ A\ = Perimeter\ of\ C \times \frac{5}{4}$$
$$\Rightarrow \quad Perimeter\ of\ A\ = (10 + 6 + 8) \times \frac{5}{4} = 24 \times \frac{5}{4} = 30\ cm$$

Therefore, triangle A has a **30 cm perimeter**.

A is incorrect because this is the area of triangle C.

B is incorrect because this is the perimeter of triangle C.

C is incorrect.

D is incorrect.