Biomedical Admissions Test (BMAT)

Section 2: Mathematics

Questions by Topic

M4: Algebra

M4: Algebra - Questions by Topic

Mark scheme and explanations at the end

1 If x>10000 then what is the closest value of $\frac{x}{(4x+1)}$?

A ∞

B $\frac{1}{4}$

C $\frac{1}{2}$

D $\frac{5000}{10,001}$

E $\frac{5}{3}$

2 If $x = 0.04$and $n = 10000$, calculate the value of the following equation:

$$\sqrt{\frac{x(0.2-x)}{n}}$$

A 0.8

B 0.08

C 0.008

D 0.0008

E 0.00008

3 In Quebec, the average daily rainfall was $X cm$ for the month of April. If the average for the first 14 days of the month was $(X - 12)\ cm$, what is the average rainfall, in cm, for the remaining 16 days of the month.

A $X + 12$

B $\frac{X}{2}$

C $\frac{2X+21}{2}$

D $\frac{21X+2}{21}$

E $\frac{X+6}{2}$

4 The mean time for this year's Birmingham annual snail race for a group of 15 snails was 25 minutes. The time for a second group of snails were added and the value of the mean went up to 36 minutes.

Which formula represents the relationship between the number of snails in the second group, S,and the mean time, in minutes, of the second group, T?

A $S = \frac{11}{T-15}$

B $S = \frac{11}{T-36}$

C $S = \frac{165}{T-36}$

D $S = \frac{165T}{T-15}$

E $S = \frac{165}{T-15}$

5 Given that $-4 = y + 2$, solve the following equation for x:

$$\frac{4x+1}{5}+\frac{2x-4}{2y} = 8$$

A $-\frac{224}{19}$

B $\frac{224}{19}$

C $\frac{20}{19}$

D $-\frac{20}{19}$

E $\frac{244}{29}$

6 Solve $5x^2 + 4x = -5$

A $x = -0.4$

B $x = 0.4$

C $x = -0.4$ and $\frac{3}{7}$

D $x = 0.4$ and $\frac{3}{7}$

E No real solutions

7 Express $\frac{4}{x(2-x)} - \frac{2}{x}$ as a single fraction in its simplest form

A $\frac{2}{3x-x^2}$

B $\frac{-2}{2-x}$

C $\frac{-2}{3x-x^2}$

D $\frac{2}{2-x}$

E $\frac{-x-x^2}{x^2(2-x)}$

8 Simplify the following expression: $5x\left(\frac{2x^6}{x^{\frac{2}{3}}}\right)^{-6}$

A $10x^{\frac{19}{3}}$

B $320x^{32}$

C $\frac{5}{64x^{31}}$

D $\frac{10}{x^{32}}$

E $5x^{\frac{2}{3}}$

9 Solve the following expression for x:

$$6x + 3 = \frac{5(x^2 + 1)}{x}$$

A $x = -\frac{3}{2} \pm \frac{\sqrt{29}}{2}$

B $x = -\frac{9}{2} \pm \frac{\sqrt{23}}{2}$

C $x = \frac{9}{2} \pm \sqrt{23}$

D $x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$

E $x = -\frac{3}{2} \pm \sqrt{29}$

10 Rearrange $x = \frac{\sqrt{27y^{3}}}{4}$ to make y the subject

A $y = \frac{2^{\frac{4}{3}}x^{\frac{2}{3}}}{3}$

B $y = \frac{2^{3}x^{2}}{4}$

C $y = \frac{2^{3}x}{4}$

D $y = \left(\frac{2^{\frac{4}{3}}x^{\frac{2}{3}}}{3}\right)^{3}$

E $y = \left(\frac{2^{\frac{4}{3}}x^{\frac{2}{3}}}{4}\right)^{\frac{1}{3}}$

11 Solve $-x + 10 = \frac{3(x^{2}+1)}{x}$

A $x = -\frac{5 \pm \sqrt{13}}{4}$

B $x = -\frac{25 \pm \sqrt{13}}{4}$

C $x = -25 \pm \frac{\sqrt{13}}{2}$

D $x = 5 \pm \frac{\sqrt{13}}{2}$

E $x = \frac{5 \pm \sqrt{13}}{4}$

12 Rearrange the following expression to make y the subject:

$$y^5 - 5y^4 + 10y^3 - 10y^2 + 5y + 2 = x^4 + 5$$

A $y = 1 + (x^4 + 5)^{\frac{1}{5}}$

B $y = -1 + (x^4 + 5)^{\frac{1}{5}}$

C $y = 1 + (x^4 + 2)^{\frac{1}{5}}$

D $y = -1 + (x^4 + 2)^{\frac{1}{5}}$

E $y = -3 + (x^4 + 5)^{\frac{1}{5}}$

13 Rearrange the following expression to make h the subject:

$$x = \frac{4g^2h}{g - 2h}$$

A $h = \frac{2(2g^2 + x)}{gx}$

B $h = \frac{gx}{2(2g^2 + x)}$

C $h = \frac{x}{12}$

D $h = \frac{gx}{4g^2 - 2x}$

E $h = \frac{12}{x}$

14 I have one brother and one sister. If I square root my brother's age, I get my sister's age. In three years, my sister will be half my brother's age.

How old are my brother and sister respectively?

A 3 and 9

B 5 and 25

C 4 and 16

D 16 and 4
E 9 and 3

15 If $x.y = (x^2 y) + (x - y)$, calculate the value of $(2.3).4$

A 19.46
B 44
C 121.46
D 334
E 491

16 The curve $6y - 12 = 6x^2 - 18x$ is intersected at two points, A and B, by the straight line

$y + x = 5.$

Find the coordinates of points A and B

A (2,3) and (-1,8)
B (-4,5) and (-5,4)
C (2,8) and (-1,3)
D (-1,6) and (3,2)
E (4,5) and (-4,-5)

17 Solve $-4 \leq \frac{x+2}{3} \leq 6$

A $-14 \geq x \geq 16$
B $-14 \leq x \leq 6$
C $-4 \leq x \leq 16$
D $-4 \geq x \geq 16$
E $-14 \leq x \leq 16$

18 Solve $\frac{-4x+1}{7} < \frac{2x+4}{8}$

A $x > -\frac{10}{23}$

B $x < \frac{10}{23}$

C $x < -\frac{10}{23}$

D $-x < -\frac{10}{23}$

E $-x > -\frac{10}{23}$

19 Factorise fully $(3x-2)^2 - 49$

A $(3x+9)(x-6)$

B $3(3x+5)(x-3)$

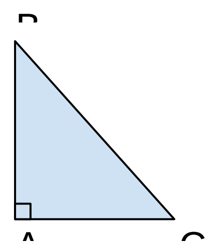
C $(9x+1)(x-6)$

D $(3x+5)(3x-9)$

E $(3x+7)(3x-7)$

20 In the triangle below, length $AB = \sqrt{x+2}$, length $AC = \sqrt{y}$ and length $BC = 2\sqrt{2}$. The area of this triangle is 2 cm^2.

Find the value of x.



A 4

B $\sqrt{2}$

C 2

D $2\sqrt{2}$

E -2

21 Solve the following simultaneous equations

$$9^x(27)^y = 1$$

$$8^y \div (\sqrt{2})^x = 16\sqrt{2}$$

Find the values of x and y

A $x = 1.8, y = -1.2$
B $x = -1.8, y = 1.2$
C $x = -2, y = -1.33$
D $x = 2, y = 1.33$
E $x = 0.6, y = 1.6$

22 Sarah is older than her brother Edward. Their ages in years are such that twice the square of Edward's age subtracted from the square of Sarah's age gives a number equal to 6 times the difference of their ages. The sum of their ages is equal to 5 times the difference of their ages.

Find the age in years of each of the siblings.

A Sarah is 24 years old, Edward is 18 years old.
B Sarah is 6 years old, Edward is 9 years old.
C Sarah is 18 years old, Edward is 12 years old.
D Sarah is 3 years old, Edward is 6 years old.
E Sarah is 12 years old, Edward is 18 years old.

23 Solve the following simultaneous equations

$$x^2 + 2xy + 4y^2 = 28$$
$$x + 2y = 6$$

Find all the possible values of x and y.

A $x = 2, y = 1; x = 4, y = 2$
B $x = 3, y = 1.5; x = 5, y = 0.5$
C $x = 7, y = -0.5; x = 1, y = 2.5$
D $x = 2, y = 2; x = 4, y = 1$
E $x = 3, y = 3; x = 5, y = 0.5$

24 Solve the equations $\frac{1}{x^2}+\frac{1}{y^2}=13$ and $\frac{1}{x}+\frac{1}{y}=5$.

Find all the possible values of $x$and y.

A $x=\frac{1}{6}, y=\frac{1}{4}; x=\frac{1}{4}, y=\frac{1}{6}$

B $x=\frac{1}{8}, y=\frac{1}{5}; x=\frac{1}{5}, y=\frac{1}{8}$

C $x=\frac{1}{3}, y=\frac{1}{2}; x=\frac{1}{2}, y=\frac{1}{3}$

D $x=\frac{1}{\sqrt{26}}, y=\frac{1}{\sqrt{13}}; x=\frac{1}{\sqrt{13}}, y=\frac{1}{\sqrt{26}}$

E $x=\frac{1}{\sqrt{6}}, y=\frac{1}{\sqrt{11}}; x=\frac{1}{\sqrt{11}}, y=\frac{1}{\sqrt{6}}$

25 A man travels a distance of 196 km by train and returns the same distance in a car which travels at an average speed of 21 km/h faster than the train. The total journey takes 11 hours. Find the average speeds of the train and the car respectively.

A Speed of train is 13.36 km/h, speed of car is 34.36 km/h
B Speed of train is 21 km/h, speed of car is 28 km/h
C Speed of train is 26.36 km/h, speed of car is 68.36 km/h
D Speed of train is 42 km/h, speed of car is 56 km/h
E Speed of train is 28 km/h, speed of car is 49 km/h

26 Find two consecutive positive odd numbers given that the difference between their reciprocals is $\frac{2}{63}$.

A -9, -7
B 7, 9
C 21, 23
D -23, -21
E 31, 33

27 The equations of two graphs are given below:

$$x - y + 2 = 0$$
$$\frac{y}{x} + \frac{8}{x} = x - 2$$

Which of the following are the points of intersection of the two graphs?

1 $(-2,0)$
2 $(-7,-5)$
3 $(2,-4)$
4 $(5,7)$

A 1 and 4 only
B 2 and 3 only
C 1 and 3 only
D 2 and 4 only
E 1 and 2 only

28 A straight line passes through the points A and B with the coordinates $(4,4)$ and $(1,8)$, respectively.

Which one of the following is an equation of a straight line which is parallel to the line AB?

A $y = -\frac{3}{4}x + 12$
B $2y = 8x + 4$
C $y = -\frac{4}{3}x - 5$
D $y = 4x - 1$
E $2y = -\frac{4}{3}x$

29 The two equations given below define an area on a graph.

$$y \geq x^3 + 3$$

$$y < 3^x + 2$$

Which one of the following points lie within the area defined?

1 $(1,4.5)$
2 $(0.5,2.7)$
3 $(0.8,2.9)$
4 $(1.5,6.5)$

A 1 and 4 only
B 1 and 2 only
C 2 and 3 only
D 1, 2 and 3 only
E 2,3 and 4 only

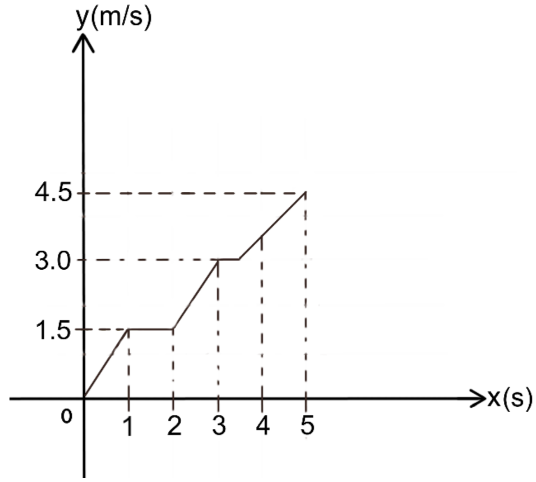
30 A mobile phone company devised the following structure of charges for its users as follows:

First 40 minutes at 18 pence per minute
Next 40 minutes at 12 pence per minute
Any additional minutes at 6 pence per minute

Find the number of minutes used when the total cost is £16.02.

A 67
B 107
C 147
D 187
E 227

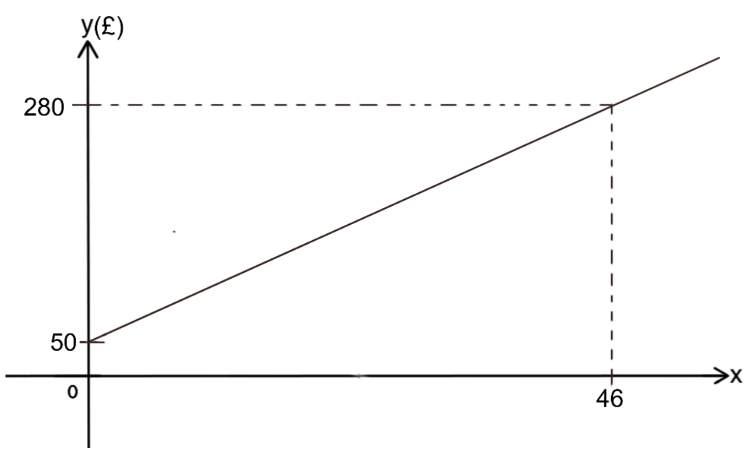
31 The graph below shows the speed of a body over a period.



Find the total distance travelled during the first 5 seconds.

A 13.875 m
B 15.875 m
C 12.625 m
D 14.375 m
E 11.625 m

32 Angelica, the president of a school's student council, wishes to order some T-shirts with the council's logo and design on them for the members. She goes to Mr Murni's shop to find out the cost of making the T-shirts. Mr Murni shows her a graph displaying the cost (£C) of making N number of T-shirts.



The photography club has 88 members including Angelica and she wishes to order a T-Shirt for each member.

Calculate the cost of making 88 T-shirts.

A £536
B £490
C £416
D £572
E £438

Solutions

1 **B is the answer**

Divide the numerator and denominator of the fraction by x. (Remember, as long as you are dividing both top and bottom by the same thing, you are not changing the value of the fraction, simply rewriting it in a different form).

$$\frac{x \div x}{(4x+1) \div x} \qquad \Rightarrow \frac{1}{4+\frac{1}{x}}$$

With the fraction rewritten like this, we can see clearly that as x becomes larger, i.e. $> 10{,}000$, the $\frac{1}{x}$ becomes very small and approaches 0, just leaving $\frac{1}{4}$.

2 **D is the answer**

Method 1: Substitute the values for x and $n$into the given equation

$$\sqrt{\frac{x(0.2-x)}{n}} \quad \Rightarrow \sqrt{\frac{0.04(0.2-0.04)}{10000}} \quad \Rightarrow \sqrt{\frac{0.04(0.16)}{10000}} \quad \Rightarrow \sqrt{\frac{0.0064}{10000}} = \quad \frac{\sqrt{0.0064}}{\sqrt{10000}}$$

To calculate $\sqrt{10000}$:

$$\sqrt{10000} \equiv \sqrt{100 \times 100} = 10 \times 10 = 100$$

To calculate $\sqrt{0.0064}$, multiply$\sqrt{0.0064}$ by$\sqrt{10000}$ to get $\sqrt{64} = 8$. But remember, $\sqrt{0.0064}$ was multiplied by $\sqrt{10000}$ therefore you must UNDO this action and divide 8 by $\sqrt{10000}$ $(= 100)$ to get 0.08

The equation is now $\frac{0.08}{100} = 0.0008$

Method 2: Substitute the values for x and $n$into the given equation

$$\sqrt{\frac{x(0.2-x)}{n}} \quad \Rightarrow \sqrt{\frac{0.04(0.2-0.04)}{10000}} \quad \Rightarrow \sqrt{\frac{0.04(0.16)}{10000}}$$

Eliminate the decimals in the numerator by multiplying **both** the numerator and denominator by 1000. Then, continue to solve the equation.

$$\sqrt{\frac{0.04(0.16)}{10000}} = \sqrt{\frac{0.04(0.16)\times 10000}{10000\times 10000}} = \sqrt{\frac{4(16)}{100000000}} = \sqrt{\frac{64}{100000000}}$$

$$= \frac{8}{10000} = 0.0008$$

3 **C is the answer**

Let the average rainfall over the remaining 16 days = Y.Thus the following equation can be written:

$$X = \frac{14(X-12)+16Y}{30} \Rightarrow\ 30X = 14X - 168 + 16Y$$

Rearrange to make $Y$the subject because we want to find the expression for the average rainfall over the remaining 16days:

$$30X - 14X + 168 = 16Y \Rightarrow \frac{16X+168}{16} = Y$$

$$Y = \frac{2X+21}{2}$$

4 **C is the answer**

$$Total\ mean\ of\ both\ groups$$
$$= \frac{(Total\ time\ Group\ 1\ +\ Total\ time\ Group\ 2)}{(No.of\ snails\ Group\ 1\ +\ No.of\ snails\ Group\ 2)}$$

Total time of snails in Group 1:

$$Mean\ = \frac{Total\ time}{No.of\ snails} \qquad \Rightarrow 25 = \frac{Total\ time}{15} \qquad \Rightarrow Total\ time\ =\ 25\times 15$$

$$Total\ time\ of\ snails\ in\ Group\ 1\ =\ 375$$

Total time of snails in Group 2:

$$Mean\ = \frac{Total\ time}{No.of\ snails} \qquad \Rightarrow \qquad T = \frac{Total\ time}{S}$$

(where T = mean time of second group, S = no. of snails in second group)

$$Total\ time\ of\ snails\ in\ Group\ 2\ =\ TS$$

Total mean time of both groups = 36

$$\Rightarrow 36 = \frac{375 + TS}{15 + S} \Rightarrow 36(15 + S) \ = \ 375 + TS \Rightarrow 540 + 36S = 375 + TS$$

$$\Rightarrow 165 = TS - 36S \ \Rightarrow 165 = S(T - 36) \Rightarrow S = \frac{165}{T-36}$$

5 **B is the answer**

This equation has **2 variables** in, x and y: $\frac{4x+1}{5}+\frac{2x-4}{2y} = 8$.

We have been told $-4 = y + 2$. Rearrange this to make y **the subject** and then **substitute** this into the algebraic fraction so that we are only working with **one variable**.

$$-4 = y + 2 \qquad \Rightarrow y = \ -6$$

$$\Rightarrow \frac{4x+1}{5}+\frac{2x-4}{2(-6)} = 8 \qquad \Rightarrow \frac{4x+1}{5}+\frac{2x-4}{-12} = 8$$

Now that we are working with only one variable, x,**eliminate the fractions** by multiplying through by the denominators.

$$-12(4x + 1)+5(2x - 4) = 8(-12)(5)$$

So here we are multiplying everything by 5 and then by -12 to eliminate both denominators.

Multiply out the brackets first before rearranging:

$$-48x - 12 \ + \ 10x \ - 20 \ = \ -480$$

Rearrange the equation so that all the $x's$ are on one side and all the numbers are on the other side. **Combine terms** where possible:

$$-38x \ - \ 32 \ = \ -480 \qquad \Rightarrow \ -38x \ = \ -448$$

Multiply through by -1 to get a positive x:

$$38x \ = \ 448 \qquad \Rightarrow \qquad x \ = \ \frac{448}{38}$$

Simplify by dividing both numerator and denominator by 2 to get:

$$x \ = \ \frac{224}{19}$$

6 E is the answer

One of the 5 options for the correct answer is "**no solution**". When this is given to you as an option, a good time-saving trick is to **use the discriminant** to quickly determine if the quadratic equation has any solutions.
Doing this will allow you to determine if there are no **solutions, one solution or two solutions** and this will help you be confident you have found the right answer(s) when solving the equation further, if necessary.
First rearrange the equation given so that is in the form **$ax^2 + bx + c = 0$**

$$5x^2 + 4x = -5 \qquad \Rightarrow 5x^2 + 4x + 5 = 0$$

Use the discriminant $b^2 - 4ac$ to determine how many solutions there are:

$$b^2 - 4ac \qquad \Rightarrow (4)^2 - (4 \times 5 \times 5) = -84$$

Because $b^2 - 4ac = -84 < 0$ this tells you there are **no solutions** to this quadratic equation and so **E** is the correct answer.

This is a quicker method to reach the correct answer than directly attempting to solve the equation.

> ***Exam Tip*** - If "no solution" appears as an answer option for solving quadratic equations, use the discriminant to quickly check if there are in fact any real solutions before attempting to solve the equation. This will **save valuable time** if in fact there are no solutions, such as in the question above.
>
> $b^2 - 4ac < 0 :$ *No Solutions*
>
> *If* $b^2 - 4ac = 0 :$ 1 *Solution*
>
> *If* $b^2 - 4ac > 0 :$ 2 *Solutions*

7 D is the answer

Notice how **x is a common term in both denominators**. $x$is a factor of $x(2-x)$ and so the common denominator will be $x(2-x)$.
The first fraction is already over the common denominator and so nothing needs to be done to it.
Multiply the top and bottom of the 2nd fraction by $(2-x)$ so that the denominator becomes :

$$\frac{4}{x(2-x)} - \frac{2(2-x)}{x(2-x)}$$

Expand and simplify:

$$\frac{4-2(2-x)}{x(2-x)} \Rightarrow \frac{4-(4-2x)}{x(2-x)} \Rightarrow \frac{2x}{x(2-x)}$$ (Be careful! $4-(4-2x)=2x$)

Cancel the $x's$ to get $\frac{2}{2-x}$

B is incorrect. Remember a **negative multiplied by a negative gives a positive**, i.e. $-(-2x)=2x$

8 **C is the answer**

When **raising** one power to another power, you **multiply** them. Leave the terms inside the bracket raised to negative one for now to make things easier, and just raise the terms inside the bracket to the power of 6 by multiplying the powers.

$$5x\left(\frac{2x^{6}}{x^{\frac{2}{3}}}\right)^{-6} \Rightarrow 5x\left(\frac{2^{6}x^{6\times6}}{x^{\frac{2\times6}{3}}}\right)^{-1} \Rightarrow 5x\left(\frac{64x^{36}}{x^{4}}\right)^{-1}$$

Negative powers cause the reciprocal of the terms raised to the value of the negative power. The -1 causes the terms inside the bracket $\left(\frac{64x^{36}}{x^{4}}\right)$ which are being raised to the power of -1 to be turned upside down, i.e. the reciprocal of these terms.

$$5x\left(\frac{64x^{36}}{x^{4}}\right)^{-1} \Rightarrow 5x\left(\frac{x^{4}}{64x^{36}}\right)$$

When **dividing indices, subtract the powers**:

$$5x\left(\frac{x^{4}}{64x^{36}}\right) \Rightarrow 5x\left(\frac{x^{4-36}}{64}\right) \Rightarrow 5x\left(\frac{x^{-32}}{64}\right)$$

Finish off by expanding the brackets. When **multiplying powers** of the same term, i.e. $5x$ multiplied by $\frac{x^{-32}}{64}$, **add the powers**. Remember the $x$term in $5x$ is being raised to the power of 1

$$5x\left(\frac{x^{-32}}{64}\right) \Rightarrow \frac{5x^{1+-32}}{64} \Rightarrow \frac{5x^{-31}}{64} \Rightarrow \frac{5}{64x^{31}}$$

9 **A is the answer**

This question requires you to be able to use the **quadratic formula:**

The solutions for $ax^{2}+bx+c=0$ are given by $x=\frac{-b\pm\sqrt{b^{2}-4ac}}{2a}$

First rearrange the equation given to you so that it is in the form $ax^{2}+bx+c=0$

$$6x + 3 = \frac{5(x^2 + 1)}{x} \Rightarrow x(6x+3) = 5x^2 + 5 \Rightarrow 6x^2 + 3x = 5x^2 + 5 \Rightarrow x^2 + 3x - 5 = 0$$

Substitute the values for a, b and c into the quadratic formula to find the values for x:

$$x = \frac{-3 \pm \sqrt{3^2 - (4 \times 1 \times -5)}}{2 \times 1} \Rightarrow \frac{-3 \pm \sqrt{29}}{2} \Rightarrow -\frac{3}{2} \pm \frac{\sqrt{29}}{2}$$

Exam Tip - You will **NOT** be given the quadratic formula in your BMAT exam and therefore you must learn it and know how to use it correctly.

10 **A is the answer**

When rearranging formula, a good first step is to try an isolate the term containing the subject on one side, in this case to try and isolate y^3

Multiply both sides by 4

$$4x = \sqrt{27y^3}$$

Square both sides

$$(4x)^2 = 27y^3 \qquad \Rightarrow 16x^2 = 27y^3$$

Divide both sides by 27

$$\frac{16x^2}{27} = y^3$$

Cube root both sides to make y the subject of the equation

$$y = \sqrt[3]{\frac{16x^2}{27}}$$

Simplify and tidy up

$$y = \sqrt[3]{\frac{8 \times 2 \times x^2}{27}} \Rightarrow y = \frac{2\sqrt[3]{2x^2}}{3} \Rightarrow y = \frac{2^1 \times 2^{\frac{1}{3}} \times x^{\frac{2}{3}}}{3} \Rightarrow y = \frac{2^{\frac{4}{3}} x^{\frac{2}{3}}}{3}$$

Exam Tip - When dealing with **fractional powers** such as the power $\frac{2}{3}$, **split the fraction into a root and a power** and do them in that order, root first, then power. I.e. the power $\frac{2}{3}$ means cube root and then raise to the power of 2 (square)

11 **E is the answer**

This question requires you to be able to use the **quadratic formula:**

The solutions for $ax^2 + bx + c = 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

First rearrange the equation given to you so that it is in the form $ax^2 + bx + c = 0$

$$-x + 10 = \frac{3(x^2+1)}{x} \Rightarrow x(-x+10) = 3x^2 + 3 \Rightarrow -x^2 + 10x = 3x^2 + 3 \Rightarrow -4x^2 + 10x - 3 = 0$$

Substitute the values for a, b and c into the quadratic formula to find the values for x

$$x = \frac{-10 \pm \sqrt{10^2 - (4 \times -4 \times -3)}}{2 \times -4} \Rightarrow \frac{-10 \pm \sqrt{100-48}}{-8} \Rightarrow \frac{-10 \pm \sqrt{52}}{-8}$$

Simplify the terms:

$$\frac{-10 \pm \sqrt{52}}{-8} \Rightarrow \frac{-10 \pm \sqrt{4 \times 13}}{-8} \Rightarrow \frac{-10 \pm 2\sqrt{13}}{-8}$$

Divide each of the terms by 2 (the common factor) to reach the simplified answer:

$$x = \frac{5 \pm \sqrt{13}}{4}$$

12 **C is the answer**

This is a **fifth order polynomial,** which you are **not** expected to be able to factorise for BMAT. Instead of spending precious time trying to factorise this polynomial, **the trick to this question is to look at the five answer options provided to you.**
In all of the answer options, expanding the bracket on the right side of the equation, either expanding $(x^4 + 5)^{\frac{1}{5}}$ or expanding $(x^4 + 2)^{\frac{1}{5}}$ involves making $(y \pm 1)^5$ on the left side. From this you should **deduce that** $(y \pm 1)^5$ **is likely to be the solution** to the fifth order polynomial.

Looking further at the fifth order polynomial $y^5 - 5y^4 + 10y^3 - 10y^2 + 5y + 2$ you can see that there are **negative terms** in the equation, $-5y^4$ and $-10y^2$. This means that the factorised bracket must be $(y-1)^5$ and not $(y+1)^5$ because expanding $(y+1)^5$ would not produce the negative terms $-5y^4$ and $-10y^2$.

Working with $(y-1)^5$, calculate that the constant produced by expanding this bracket will be $(-1)^5 \Rightarrow -1$

In the polynomial provided in the question, the constant *c* is $+2$. Therefore:

$$(y-1)^5+3 = y^5-5y^4+10y^3-10y^2+5y+2$$

and so

$$(y-1)^5+3 = x^4+5$$

Rearrange to make y the subject:

$$(y-1)^5 = x^4+2 \Rightarrow y-1 = \sqrt[5]{(x^4+2)}$$

$$\Rightarrow y-1 = (x^4+2)^{\frac{1}{5}} \qquad \Rightarrow y = 1+(x^4+2)^{\frac{1}{5}}$$

13 **B is the answer**

Multiply both sides by $g-2h$:

$$x = \frac{4g^2h}{g-2h} \quad \Rightarrow \quad x(g-2h) = 4g^2h$$

Expand brackets:

$$gx-2xh = 4g^2h$$

Add $2xh$ to both sides so that all the terms containing h are on the same side:

$$gx = 4g^2h + 2xh$$

Factorise the right hand side to isolate h:

$$gx = h(4g^2+2x) \quad \Rightarrow \quad h = \frac{gx}{4g^2+2x} \quad \Rightarrow \quad h = \frac{gx}{2(2g^2+x)}$$

14 **A is the answer**

Set up two **simultaneous equations** from the information provided. Let B stand for brother and S stand for sister.

"If I square root my brother's age, I get my sisters age" $\Rightarrow \sqrt{B} = S \quad (1)$

"In three years, my sister will be half my brother's age" $\Rightarrow\ S+3\ =\ \frac{B+3}{2}$ *(2)*

Since we are dealing with algebraic terms and fractions and not just whole numbers, to make the equations easier to work with, square both sides of equation 1 to **eliminate the square root** function.

$$B = S^{2}\ (3)$$

Once you have set up the two simultaneous equations, there are several different methods to solve this. In this method, we will **solve by substitution**.

Rearrange *(2)* to make B the subject:

$$2(S+3) = B+3 \quad \Rightarrow \quad 2S+6 = B+3 \quad \Rightarrow \quad B\ = 2s+3\ (4)$$

Substitute equation *(4)* into *(3):*

$$2s+3 = S^{2}$$

Solve the quadratic equation to find a value for S:

$$S^{2} - 2s - 3 = 0 \qquad \Rightarrow (S+1)(S-3) = 0$$
$$\Rightarrow S =\ -1, S = 3$$

Remember, we are dealing with age here and therefore $S \neq -1$ because age cannot be negative. So the sister's age is 3.
Substitute $S\ = 3$ into equation *(3)* to find B:

$$B\ = 3^{2} \quad \Rightarrow \quad B\ = 9 \quad \Rightarrow \quad S\ = 3, B\ = 9$$

Therefore, the sister is 3 and the brother is 9.

15 **E is the answer**

$$(2.3)\ =\ (2^{2} \times 3)\ +\ (2-3) \qquad \Rightarrow (2.3) = 11$$
$$(11.4) = (11^{2} \times 4)\ +\ (11-4) \qquad \Rightarrow (2.3).4 = (11.4) = 491$$

Exam Tip - It is important to be aware of the **common notations** used. The '.' represents a multiplication operation and it may confuse students into thinking 2.3 refers to a decimal rather than the multiply function. An easy way to distinguish this is to look for **consistent usage** of notation within a single question. Since the question $(2.3).4$ has two '.', they have to have the same mathematical meaning, and since we know that the second '.' represents multiply, the first '.' should hence also mean multiply.

16 **D is the answer**

To find the point at which the curve and line intersect, the equations for the two lines need to be solved simultaneously to find their common points.

Notice that all the terms in the equation of the curve $6y - 12 = 6x^2 - 18x$ are a multiple of 6. Divide this equation through by 6 to make the terms smaller, this will save time when doing calculations.

$$y - 2 = x^2 - 3x \qquad (1)$$
$$y + x = 5 \qquad (2)$$

Rearrange *(2)* to make *y* the subject:

$$y = 5 - x \qquad (3)$$

There are several methods to solve the pair of simultaneous equations. Here are two possible methods.

Method 1: Solve by substitution	Method 2: Solve by subtraction
Substitute *(3)* into *(1)* $5 - x - 2 = x^2 - 3x$ $\Rightarrow 3 - x = x^2 - 3x$ $\Rightarrow 0 - x^2 - 2x - 3$	Rearrange *(1)* to make y the subject $y = x^2 - 3x + 2$ *(4)* Calculate *(4)-(3)* $y = x^2 - 3x + 2$ - $y = -x + 5$ $0 = x^2 - 2x - 3$

Using both methods, the same quadratic $x^2 - 2x - 3 = 0$ is obtained.

Solve the quadratic to obtain solutions for x. The easiest way to solve this quadratic is by factorising, but the quadratic equation can also be used.

$$(x-3)(x+1) = 0 \qquad \Rightarrow x = 3 \; and \; x = -1$$

Use equation *(3)* to find the y values for each of the x values:

When $x = 3, y = 5 - 3$ $\Rightarrow x = 3, y = 2$

When $x = -1, y = 5 - -1$ $\Rightarrow x = -1, y = 6$

17 **E is the answer**

When there are two inequality signs in an equation, the same function must be done to each part of the inequality.

$(\times 3)$ $\quad -4 \times 3 \leq x + 2 \leq 6 \times 3 \qquad \Rightarrow -12 \leq x + 2 \leq 18$

(-2) $\quad -12 - 2 \leq x \leq 6 - 2 \qquad \Rightarrow -14 \leq x \leq 16$

18 **A is the answer**

Multiply both sides by 7 and 8 to eliminate the fractions in the inequality

$$8(-4x+1) < 7(2x+4) \qquad \Rightarrow -32x + 8 < 14x + 28$$

$(-14x)$ $-46x + 8 < 28$

(-8) $\quad -46x < 20$

$(\div 46)$ $\quad -x < \frac{20}{46} \qquad \Rightarrow -x < \frac{10}{23}$

$(\times -1)$ $\quad x > -\frac{10}{23}$

Exam Tip - Whenever you **multiply** or **divide** by a **negative number**, the inequality sign must be **reversed:**

< becomes >

$\leq$ becomes $\geq$

19 **B is the answer**

The key to this question is to notice this is Difference Of Two Squares (DOTS)

$$\text{DOTS: } x^2 - a^2 = (x+a)(x-a)$$

$$(3x-2)^2 - 49 \quad \Rightarrow \quad [(3x-2)+7]\,[(3x-2)-7]$$
$$\Rightarrow \quad (3x-2+7)\,(3x-2-7) \quad \Rightarrow \quad 3(3x+5)(x-3)$$

20 **C is the answer**

Using the formulas for area of a triangle and pythagoras' theorem, two simultaneous equations can be set up to solve for x and y.

Area of a triangle $= \frac{1}{2} \times base \times height$

$$2 = \frac{1}{2} \times \sqrt{y} \times \sqrt{x+2} \;\Rightarrow 2 = \frac{\sqrt{y}\sqrt{x+2}}{2} \;\Rightarrow 4 = \sqrt{y}\sqrt{x+2} \quad \Rightarrow \sqrt{y} = \frac{4}{\sqrt{x+2}} \; (1)$$

Pythagoras' Theorem: $a^2 + b^2 = c^2$

Let $AC^2 = a^2, AB^2 = b^2 \text{ and } AC^2 = c^2$

$$(\sqrt{y})^2 + (\sqrt{x+2})^2 = (2\sqrt{2})^2 \quad \Rightarrow (\sqrt{y})(\sqrt{y}) + (\sqrt{x+2})((\sqrt{x+2}) = (2\sqrt{2})(2\sqrt{2})$$

$$y + x + 2 = 4\sqrt{4} \quad \Rightarrow y + x + 2 = 8 \quad \Rightarrow y = 6 - x \; (2)$$

Substitute *(2)* into *(1)* and solve for x

$$\sqrt{6-x} = \frac{4}{\sqrt{x+2}}$$

Square both sides

$$(\sqrt{6-x})^2 = \left(\frac{4}{\sqrt{x+2}}\right)^2 \quad \Rightarrow (\sqrt{6-x})(\sqrt{6-x}) = \left(\frac{4}{\sqrt{x+2}}\right)\left(\frac{4}{\sqrt{x+2}}\right) \quad \Rightarrow 6 - x = \frac{16}{x+2}$$

Solve for x

$$(6-x)(x+2) = 16 \quad \Rightarrow 6x - x^2 + 12 - 2x = 16 \;\Rightarrow 4x - x^2 + 12 = 16$$

$$\Rightarrow x^2 - 4x + 4 = 0 \Rightarrow \qquad (x-2)(x-2) = 0 \qquad \Rightarrow x = 2$$

Because the question only asks you to find the value of x there is no need to find the value of y. However if you have time spare and wish to check you have solved for $x$correctly, use the value of x to find the value of y and substitute the values of $x$and y into one of the original equations. If the equation holds true with your values of $x$and y you can be confident you have obtained the right answer.

21 **B is the answer**

This question involves manipulating the powers in each equation to form two new simultaneous equations.

$$9^x(27)^y = 1 \qquad (1)$$

$$8^y \div (\sqrt{2})^x = 16\sqrt{2} \qquad (2)$$

Rearrange equation (1) so that each number is represented as a power of 3, collect terms using the rules of indices:

$$9^x(27)^y = 1$$

$$3^{2x} \cdot 3^{3y} = 3^0$$

$$3^{2x+3y} = 3^0$$

Looking at the powers in the above equation, we obtain equation (3):

$$2x + 3y = 0 \quad (3)$$

Rearrange equation 2 so that each number is represented as a power of 2, collect terms using the rules of indices:

$$8^y \div (\sqrt{2})^x = 16\sqrt{2}$$

$$2^{3y} \div 2^{\frac{x}{2}} = 2^4 \cdot 2^{\frac{1}{2}}$$

$$2^{3y-\frac{x}{2}} = 2^{4\frac{1}{2}}$$

Looking at the powers in the above equation, we obtain equation (4):

$$3y - \frac{x}{2} = 4\frac{1}{2} \quad \Leftrightarrow \quad 6y - x = 9$$

$$x = 6y - 9 \quad (4)$$

Substitute equation (4) into equation (3):

$$2(6y-9)+3y=0$$
$$12y-18+3y=0$$
$$15y=18$$

$$y=1.2$$

Substitute $y=1.2$ into equation (4):

$$x=6(1.2)-9 \Rightarrow x=-1.8$$

A is incorrect
C is incorrect
D is incorrect
E is incorrect because $15y=18$ was solved as $y=1.6$.

22 **C is the answer**

To answer this question you need to form two simultaneous equations from the information given in the question.

Let Sarah's age be S.
Let Edward's age be E.

'Twice the square of Edward's age subtracted from the square of Sarah's age gives a number equal to 6 times the difference of their ages.'

$$S^2-2E^2=6(S-E) \quad (1)$$

The sum of their ages is equal to 5 times the difference of their ages.

$$S+E=5(S-E) \quad (2)$$

Next you need to rearrange equation (2) to find E in terms of S:

$$S+E=5(S-E)$$
$$S+E=5S-5E$$
$$6E=4S$$

$$E=\frac{2}{3}S \quad (3)$$

Substitute equation (3) into equation (1):

$$S^2-2(\frac{2}{3}S)^2=6(S-\frac{2}{3}S)$$

$$S^2 - 2(\frac{4}{9}S^2) = 2S$$
$$9S^2 - 8S^2 = 18S$$
$$S^2 - 18S = 0$$
$$S(S - 18) = 0$$
$$S = 0 \text{ or } S = 18$$

Since we are looking at age, we must have $S = 18$.

Substitute $S = 18$into equation (3):

$$E = \frac{2}{3}(18) = 12$$

Therefore, we have that Sarah is **18** and Edward is **12**.

23 **D is the answer**

You are given the following two equations:

$$x^2 + 2xy + 4y^2 = 28 \quad (1)$$
$$x + 2y = 6 \quad (2)$$

Rearrange equation (2) to express x in terms of y:

$$x + 2y = 6$$
$$x = 6 - 2y \quad (3)$$

Substitute equation (3) into equation (1):

$$(6 - 2y)^2 + 2(6 - 2y)y + 4y^2 = 28$$
$$36 - 24y + 4y^2 + 12y - 4y^2 + 4y^2 = 28$$
$$4y^2 - 12y + 8 = 0$$
$$y^2 - 3y + 2 = 0$$
$$(y - 1)(y - 2) = 0$$
$$y = 1 \text{or } y = 2$$

Substitute $y = 1$and then $y = 2$ back into equation (3) to obtain the corresponding x values:

When $y = 1, \quad x = 6 - 2(1) = 4$

When $y = 2, \quad x = 6 - 2(2) = 2$

Therefore, we have the following two sets of solutions:

$$x = 2, y = 2; x = 4, y = 1$$

An **alternative method** of finding the solution to this question is to test each of the possible answers by inputting the various x and y values into the equations. This allows you to then choose the answer in which the equations are satisfied.

> ***Exam Tip*** - Check your answer by substituting the final answer back into the original equations!

24 **C is the answer**

You are required to solve the following equations:

$$\frac{1}{x^2} + \frac{1}{y^2} = 13 \quad (1)$$
$$\frac{1}{x} + \frac{1}{y} = 5 \quad (2)$$

To make these equations easier to handle, first make some substitutions so that they are in a more recognisable form. Rewrite the equations with the substitutions:

Let $\frac{1}{x}$ be A, $\frac{1}{y}$ be B. Then,

$$A^2 + B^2 = 13 \quad (3)$$
$$A + B = 5 \quad (4)$$

Rearrange equation (4) to obtain an expression for A in terms of B:

$$A = 5 - B \quad (5)$$

Substitute equation (5) into equation (1):

$$(5 - B)^2 + B^2 = 13$$
$$25 - 10B + B^2 + B^2 - 13 = 0$$
$$2B^2 - 10B + 12 = 0$$
$$B^2 - 5B + 6 = 0$$
$$(B - 2)(B - 3) = 0$$
$$B = 2, B = 3$$

$$B = \frac{1}{y} = 2 \Rightarrow y = \frac{1}{2}$$
$$B = \frac{1}{y} = 3 \Rightarrow y = \frac{1}{3}$$

Substitute $B = 2$ and $B = 3$into equation (4) to obtain the corresponding values of A. This will allow you to find the corresponding x values:

$$A = 5 - 2 = 3$$
$$A = 5 - 3 = 2$$
$$\frac{1}{x} = 3 \ \Rightarrow \ x = \frac{1}{3}$$
$$\frac{1}{x} = 2 \ \Rightarrow x = \frac{1}{2}$$

Therefore, we have the pairs of solutions: $x = \frac{1}{3}, y = \frac{1}{2}$ and $x = \frac{1}{2}, y = \frac{1}{3}$.

25 **E is the answer**

First you need to form two **simultaneous equations** from the information given in question. The first equation can be formed from the information that the car is 21 km/h faster than the train, and the second equation can be formed from the knowledge that the total travelling time is 11 hours.

Remember, $Speed\ (km/h) = \frac{Distance\ (km)}{Time\ (h)}$.

Let speed of train be T and speed of car be C.

$$C = T + 21 (1)$$
$$\frac{196}{T} + \frac{196}{C} = 11 (2)$$

Substitute equation (1) into equation (2):

$$\frac{196}{T} + \frac{196}{T + 21} = 11$$
$$\frac{196(T + 21) + 196T}{T(T + 21)} = 11$$
$$196(T + 21) + 196T = 11T(T + 21)$$
$$196T + 4116 + 196T = 11T^2 + 231T$$
$$11T^2 - 161T - 4116 = 0$$

$$T = \frac{-(-161) \pm \sqrt{(-161)^2 - 4(11)(-4116)}}{2(11)}$$

$$T = \frac{161+455}{22} \text{ or } T = \frac{161-455}{22}$$

In this case, speed must be positive so we have:

$$T = 28$$
$$C = 28 + 21 \Rightarrow C = 49$$

Therefore, the **train travels at 28 km/h** and the **car travels at 49 km/h**.

> ***Exam Tip*** - If the calculations involving simultaneous equations look too tricky, find the correct answer by substitution the possible answers you are given into the required equations. Choose the answer in which both equations are satisfied!

26 **B is the answer**

Let x be the smaller positive odd number. Then $x + 2$ is the next consecutive odd number. To answer this question, you need to recall that the **reciprocal** of a number is 1 divided by that number. The question looks at the **difference**, so you need to subtract the smaller reciprocal value from the larger reciprocal value. Since x is less than $x + 2$, the reciprocal of x will be greater than the reciprocal of $x + 2$.

$$\frac{1}{x} - \frac{1}{x+2} = \frac{2}{63}$$

$$\frac{(x+2) - x}{x(x+2)} = \frac{2}{63}$$

$$63(x + 2 - x) = 2x(x + 2)$$

$$126 = 2x^2 + 4x$$

$$2x^2 + 4x - 126 = 0$$

$$x^2 + 2x - 63 = 0$$

$$(x - 7)(x + 9) = 0$$

$$x = 7, x = -9$$

Since the question asks for positive odd numbers, we must have $x = 7$. Then, $x + 2 = 9$so we have the required consecutive positive odd numbers to be **7** and **9**.

A is incorrect because the question asked for positive odd numbers.
C is incorrect.
D is incorrect. This is immediately obvious as the question asked for positive odd numbers.
E is incorrect.

27 **A is the answer**

Although the question involves graphical equations, there is actually **no need to draw out** the individual **equations**. As the question is asking for **points of intersection**, we instead treat the two graphical equations as **simultaneous equations** instead and solve to find the coordinates of the points of intersection.

First, rearrange the equations to make y the subject:

$$x - y + 2 = 0 \Rightarrow \quad y = x + 2 \quad (1)$$
$$\frac{y}{x} + \frac{8}{x} = x - 2 \Rightarrow \ y + 8 = x^2 - 2x \Rightarrow \quad y = x^2 - 2x - 8 \qquad (2)$$

Equate equations (1) and (2) and solve for x:

$$x + 2 = x^2 - 2x - 8$$
$$x^2 - 3x - 10 = 0$$
$$(x - 5)(x + 2) = 0$$
$$x = 5\ ,\ x = \ -2$$

Since there are two **distinct** x values, there will be two points of **intersection**. Substitute the values of x into any one of equations (1) or (2). Choose the simpler equation to save time.

When $x = 5,\ y = (5) + 2 = 7$
When $x = \ -2,\ y = (-2) + 2 = 0$

Hence the points of intersection are $(5{,}7)$and $(-2{,}0)$.

Alternatively, you can **substitute the given options of coordinates into the equations** to check whether they are valid points of intersections. This is not always recommended as it can be quite **time-consuming** but would be a good back-up method to use if you are unable to solve the simultaneous equations. It is also a good way to **check** that you have the correct points of intersection.

B, C, D, E are incorrect.

28 **C is the answer**

To find a line **parallel** to the line AB, you need to identify a line with the **same gradient** as the line AB.

So, find the gradient of the line AB: $Gradient = \frac{Change\ in\ y}{Change\ in\ x}$

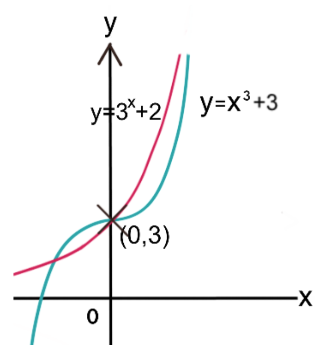
$$Gradient\ of\ AB = \frac{8-4}{1-4} = \frac{4}{-3} = -\frac{4}{3}$$

Any line parallel to AB will have the gradient $-\frac{4}{3}$. Therefore, to find the right answer, all that remains to do is to choose the answer in which the line has the corresponding gradient. This leaves the correct answer $y = -\frac{4}{3}x - 5$.

A is incorrect because the line has gradient $-\frac{3}{4}$.
B is incorrect because the line has gradient 4.
D is incorrect because the line has gradient 4.
E is incorrect because the line has gradient $-\frac{2}{3}$.

29 **A is the answer**

It might help to draw a rough **sketch** of both equations. This will give an idea of which region is satisfied by both inequalities.



Since $y \geq x^3 + 3$, we would be looking at the area of the graph where $x > 0, y > 3$. Immediately, we can **eliminate** option 2 and 3. Of the solutions given, we can assume A is correct as all the other answers involve options 2 or 3.

You can check that coordinates of options 1 and 4 do indeed satisfy the region by inputting them into the two equations and checking that the inequality is satisfied.

B is incorrect.
C is incorrect.
D is incorrect.
E is incorrect.

> ***Exam Tip*** - Sometimes the quickest way to the solution is by eliminating the incorrect answers from the options given!

30 **C is the answer**

If 40 minutes is used, cost is $40 \times 18p = 720p = £7.20$

If 80 minutes is used, cost is $£7.20 + (40 \times 12p) = £7.20 + £4.80 = £12$

Since $£16.02$is charged, that means that after the first two sets of 40 minutes, the cost of the additional minutes is $£4.02$.

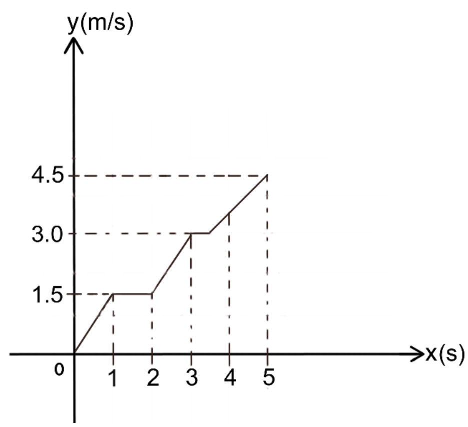
Number of additional minutes used is: $\frac{402}{6} = 67$ additional minutes

Total number of minutes used is: $80 + 67 =$ 147 minutes

31 **E is the answer**

Distance=speed×time

Therefore, the **total distance** travelled will be the **area** under the graph. You need to find the area under the graph by calculating the area of each individual shape.



Divide the area under the graph into a triangle, square, trapezium, square and then finally another trapezium. Denote each area as A1, A2, A3, A4 and A5, respectively.

Area of first triangle (between 0-1 s): $A1 = \frac{1}{2} \times 1 \times 1.5 = \frac{3}{4}$

Area of first square (between 1-2 s): $A2 = 1 \times 1.5 = \frac{3}{2}$

Area of first trapezium (between 2-3 s): $A3 = \frac{1}{2}(1.5 + 3) \times 1 = \frac{9}{4}$

Area of second square (between 3-3.5 s): $A4 = \frac{1}{2} \times 3 = \frac{3}{2}$

Area of second trapezium (between 3.5-5 s): $A5 = \frac{1}{2}(3 + 4.5) \times 1.5 = \frac{45}{8}$

$$Total\ distance\ = A1 + A2 + A3 + A4 + A5 = \frac{3}{4} + \frac{3}{2} + \frac{9}{4} + \frac{3}{2} + \frac{45}{8} = \frac{93}{8} = 11.625\ m$$

A is incorrect
B is incorrect
C is incorrect
D is incorrect

32 **B is the answer**

First, find the equation of the graph. The **y-intercept** is clearly 50, and you can find the **gradient** by dividing the change in y by the change in x:

$$y = \frac{280 - 50}{46}x + 50 \Rightarrow y = \frac{230}{46}x + 50 \Rightarrow y = 5x + 50$$

Once you have the equation, to obtain the cost (y), substitute $x = 88$ into the equation:

$$y = 5(88) + 50 = 490$$

Therefore, the total cost will be **£490**.

A is incorrect
C is incorrect
D is incorrect
E is incorrect