BioMedical Admissions Test (BMAT)

Section 2: Mathematics

Topic M2: Numbers

Topic M2: Numbers

Symbols

There are many symbols which are commonly used in mathematics. The most well known are the basic operators, those being: $+ \quad - \quad \times \quad \div \quad =$. However, there are other symbols which you need to be familiar with.

Not Equal ≠

The equal symbol (**=**) in maths is used very frequently to show that two sides of an equation hold the same value. The not equal symbol (**≠**) is less commonly used but still holds importance.

- ➔ For example we know that 0 does not equal 1. This is written as $0 \neq 1$ using symbols.

Less-than < , Greater-than >

The less-than or greater-than symbols are used to show **inequalities**: when one side of an equation is bigger than another.

- ➔ In the case of less-than, we know that 4 is less-than 5. This is written as $4 < 5$.
- ➔ Alternatively we know that 5 is greater-than 4 and this is written as $5 > 4$.

Less-than or Equal to ≤ , Greater-than or Equal to ≥

Similar to, less-than and greater-than symbols this is again used to show inequalities however, in this case the **inequalities may be equal**.

- ➔ For example if the answer to an equation is either $x = 4 \ or \ x = 5$ then it would be be incorrect to say $x < 5$ as it is possible for x to equal 5. In this instance we use the less-than or equal to symbol instead. Thus: $x \leq 5$.

Likewise with the greater-than or equal to symbol.

Exponential Numbers

Exponential numbers, or **indices**, are numbers that are **raised to a power**. They are written in the form x^a with a being the value it is raised to. I

n simple cases this value refers to the **number of times it is multiplied by itself**. The most common values for a include 2 (squared numbers) and 3 (cubed numbers).

Squares

Square numbers are numbers **raised to the power two**. This means that the number is multiplied by itself twice. For example, $5^2 = 5 \times 5 = 25$.

It is essential that you are familiar with the most common **square numbers**.

$1^2 = 1$	$7^2 = 49$
$2^2 = 4$	$8^2 = 64$
$3^2 = 9$	$9^2 = 81$
$4^2 = 16$	$10^2 = 100$
$5^2 = 25$	$11^2 = 121$
$6^2 = 36$	$12^2 = 144$

The inverse of a square number is called a **square root** and it looks like this: $\sqrt{x}$

When taking the square root of a number there will always be two possible answers, a positive and negative. For example, if $x = \sqrt{25}$, then $x = +5 \text{ or } -5$. This is because all numbers that are squared become positive.

This means when working to find an answer to an equation both values have to be taken into account and calculated individually. Therefore, it would be more accurate to write: $x_1 = 5 \text{ and } x_2 = -5$ as it is not possible to know which value was used.

Up to now, it is important to note that it is likely that you have only studied **positive square roots**. These are where the value in the square root sign is positive: $\sqrt{10}$

However, there is a whole section of maths applied to **negative square roots**. This is when the number is negative: $\sqrt{-16}$. If you have ever tried a negative square root in a calculator then you will know that it becomes a maths error. This is true as there is no real answer, the answer is **complex** and so we use $i = \sqrt{-1}$ the imaginary number.

Cubes

Cubed numbers are numbers **raised to the power three**. This means that the number is multiplied by itself three times. For example, $5^3 = 5 \times 5 \times 5 = 125$.

These are the most common **cube numbers** you are likely to come across.

$1^3=1$	$4^3=64$
$2^3=8$	$5^3=125$
$3^3=27$	$10^3=1000$

The inverse of cubed number is a **cubed root** and it looks like this: $\sqrt[3]{x}$
Unlike square roots, cubed roots do not have a positive and negative root.
For example, $(-3)^3 = -27$ but $3^3 = 27$.

> ***Tip***: remember that square numbers means to the power of 2, as a square is only 2D, whereas cube numbers are to the power of 3, as cubes are 3D.

The four operations

The main operations you will need in basic maths are **addition**, **subtraction**, **multiplication** and **division**. You need to know how to use all four of these operations for both positive and negative values of the following types of numbers: integers, decimals, simple fractions and mixed numbers.

You should already be very confident with working with integers and decimals, so the 'fractions' section later on will go over these operations..

Priority of operations

In a calculation, different operations must be done in the following order, commonly referred to as BIDMAS:

1. **B**rackets
2. **I**ndices (includes roots and reciprocals)
    - Reciprocals means the inverse of a number, e.g. the reciprocal of 6 is $\frac{1}{6}$
3. **D**ivision
4. **M**ultiply
5. **A**ddition
6. **S**ubtraction

Example:

(2 + 2) x 2^2 + 4 ÷ 2

Brackets: (4) x 2^2 + 4 ÷ 2

Indices: 4 x 4 + 4 ÷ 2

Division: 4 x 4 + 2

Multiply: 16 + 2

Addition: 18

Multiples, Factors and Prime Factors

Multiples and Factors

Multiples of a number, x, refer to x multiplied by any integer. This is often referred to as the times table of the number.

For example: List the first four multiples of 2 → 2, 4, 6, 8 (Remember that 2 in itself is a multiple of 2 as 2 x 1 = 2!)

Factors of a number, x, are all the numbers that x can be divided by and still give an integer answer.

For example: List all the factors of 24 → 1, 2, 3, 4, 6, 8, 12, 24

1 x 24
2 x 12
3 x 8
4 x 6

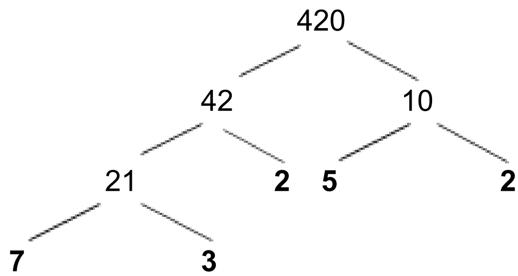
Finding Prime Factors

A prime number is a number that is only divisible by itself and 1. Note that **1 is not a prime number**.

Examples: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43…

In order to find the prime factors of a number, you must break down the given number into its factors and then break these factors down into prime factors. The easiest way to do this is to make a **prime factor tree**.

Example: Express 420 as a product of prime factors.



$420 = 2^2 \times 3 \times 5 \times 7$

You can use **prime factorisations** to solve more complex questions. For example, you can find the square root of large numbers:

Example: What is $\sqrt{129600}$? Use the fact that $129600 = 2^6 \times 3^4 \times 5^2$

$129600 = 2^6 \times 3^4 \times 5^2$

$129600 = (2^3)^2 \times (3^2)^2 \times (5)^2$

Therefore $\sqrt{129600} = 2^3 \times 3^2 \times 5 = 8 \times 9 \times 5 = \mathbf{360}$

Lowest Common Multiple

The lowest common multiple of two numbers (x, y) is the **smallest** possible number that is divisible by **both** x and y.

To find the lowest common multiple of a number:

1. List the multiples of x and y.
2. Find the smallest multiple that is common to both x and y.

Example: Find the lowest common multiple of 9 and 12.

Multiples of 9 are 9, 18, 27, <u>36</u>, 45, 54…

Multiples of 12 are 12, 24, <u>36</u>…

⇒ Lowest common multiple = **36**.

Highest Common Factor

Highest common factor is the **largest** number that all numbers (x, y and z) are divisible by.

To find the highest common factor:

1. List the factors of all the numbers (x, y and z).
2. Find the largest factor that is common to all numbers x, y, z.

Example: Find the highest common factor of 36, 54, and 72.
Factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36
Factors of 54 are 1, 2, 3, 6, 9, 18, 27, 54
Factors of 72 are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

Systematic listing

Systematic listing means that if there are m ways of doing a task, then for each of those m tasks there are n ways to do another task. The total number of ways for both of the tasks to be done are $m \times n$.

Example 1:
If I pack 5 T-shirts, 3 pairs of jeans and 2 pairs of shoes on my holiday, how many different combinations of outfits can I wear?

5 x 3 x 2 = 30 outfits

Example 2:
You are trying to guess my 4 digit passcode for my phone. Each digit is between 0 and 9 inclusive. How many different codes are possible if I tell you that each digit is different?

10 x 9 x 8 x 7 = 5040

There are 10 choices for the first digit (as it is between 0 and 9 inclusive). Therefore once that digit has been used, there are only 9 digits left. This continues until the 4th digit.

Index laws

Multiplication

To multiply powers of the same base number, you simply **add the indices**.
Example 1: $a^x \times a^y = a^{x+y}$
Example 2: $(ab)^n = a^n b^n$

Division

To divide powers, you **subtract the indices**.
Example: $a^7 \div a^3 = a^{7-3} = a^4$

Raising a power to another power

Multiply the powers in this case
Example: $(a^b)^n = a^{b \times n}$

Fractions

The power applies to both the **numerator and denominator** of the given fraction.

Example: $(\frac{1}{a})^y = \frac{1^y}{a^y}$

Negatives

To rearrange this you can form the **positive reciprocal** of the base number.

Example: $a^{-b} = \frac{1}{a^b}$

> ***Tips:***
> Any number to the power of 1 is the number given, e.g. $3^1 = 3$
> Any number raised to the power of 0 equals 1, e.g. $2^0 = 1$
> The power $\frac{1}{2}$ is the same as the square root and the power $\frac{1}{3}$ is the same as cube root

Standard Form

Expressing in standard form

Standard form refers to a number represented in the form: $A \times 10^n$ where A is between 1 and 10, n is how far the decimal point moves.

n is positive for numbers greater than 1, meaning that the decimal shifts right, and negative for numbers smaller than 1, meaning that the decimal shifts right

.

Example 1: Express 45500 in standard form.
45500 = 4.55×10^4

Example 2: Express 0.00000256 in standard form.
0.00000256 = 2.56×10^{-6}

Example 3: Express 8.45×10^{-4} as an ordinary number.
8.45×10^{-4} = 0.000845

Example 4: What is 48.9 million in standard form?
48.9 million = 48.9×10^6 = 4.89×10^7

Important to note: million = 1×10^6, billion = 1×10^9

Multiplying and Dividing

Group the front numbers together and the powers of 10 together.

Example 1: Calculate $(5.64 \times 10^6) \times (4.91 \times 10^{-4})$. Give your answer in standard form.

$(5.64 \times 10^6) \times (4.91 \times 10^{-4})$
$= (5.64 \times 4.91) \times (10^6 \times 10^{-4})$
$= 27.6924 \times 10^{6+(-4)}$
$= 27.6924 \times 10^2$
$= 2.76924 \times 10^3$

Example 2: Calculate $1950 \div (3.9 \times 10^8)$. Give your answer in standard form.

$1950 \div (3.9 \times 10^8)$
$= 1.95 \times 10^3 \div (3.9 \times 10^8)$
$= \frac{1.95 \times 10^3}{3.9 \times 10^8}$
$= 0.5 \times 10^{3-8}$
$= 5 \times 10^{-1} \times 10^{-5}$
$= 5 \times 10^{-1+(-5)}$
$= 5 \times 10^{-6}$

Adding and Subtracting

Rewrite to make sure the powers of 10 are the same.

Example: Calculate $(7.5 \times 10^6) + (1.2 \times 10^4)$. Give your answer in standard form.

$(7.5 \times 10^6) + (1.2 \times 10^4)$
$= (7.5 \times 10^6) + (0.012 \times 10^6)$
$= 7.512 \times 10^6$

Fractions

Simplification

To simplify a fraction, **divide both** the numerator and the denominator by the **same number** until they can no longer be divided any further.

Example: Simplify $\frac{24}{36}$

$\frac{24}{36} = \frac{12}{18} = \frac{6}{9} = \frac{2}{3}$

Common Denominators

To compare fractions or for addition and subtraction, all fractions need to be converted to have the **same denominator**.

This can be done by finding the **lowest common multiple** of the denominators.

Example 1: Rearrange the fractions given in ascending order. ($2\frac{3}{4}$, $\frac{13}{6}$, $\frac{7}{8}$)

The lowest common multiple of 4, 6, 8 is 24.

$2\frac{3}{4} = \frac{11}{4} = \frac{66}{24}$

$\frac{13}{6} = \frac{52}{24}$

$\frac{7}{8} = \frac{21}{24}$

So the correct order is $\frac{7}{8}$, $\frac{13}{6}$, $2\frac{3}{4}$.

Example 2: Calculate $3\frac{2}{5} - \frac{13}{6}$.

$3\frac{2}{5} - \frac{13}{6} = \frac{17}{5} - \frac{13}{6} = \frac{102}{30} - \frac{65}{30} = \frac{37}{30} = 1\frac{7}{30}$

Operations

To **add or subtract fractions**, you must ensure that all fractions are placed over a common denominator (bottom half of fraction).

For example: $\frac{1}{4} + \frac{2}{3} = \frac{3}{12} + \frac{8}{12} = \frac{11}{12}$

To **multiply a fraction**, you can simply multiply the numerators (top half) and then multiply the denominators (bottom half). Remember if there are any mixed numbers, then change these into top-heavy fractions first.

For example: $\frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$

To **divide a fraction**, you must invert the second fraction and then change the division sign to a multiplication sign and do as above. Remember if there are any mixed numbers, then change these into top-heavy fractions first.

For example: $2\frac{3}{4} \div \frac{1}{6}$

$= \frac{11}{4} \div \frac{1}{6}$

$= \frac{11}{4} \times \frac{6}{1}$

$= \frac{66}{4} = \frac{33}{2} = 16\frac{1}{2}$

Mixed Numbers and Improper Fractions

Mixed numbers consist of an integer part and a fraction part e.g. $4\frac{1}{3}$

Improper fractions $\frac{x}{y}$ are fractions where $x > y$ e.g. $\frac{23}{4}$

Example 1: Write $4\frac{1}{3}$ as an improper fraction.

Use formula $a\frac{b}{c} = \frac{c \times a + b}{c}$

$4\frac{1}{3} = \frac{3\times4+1}{3} = \frac{13}{3}$

Example 2: Write $\frac{23}{4}$ as a mixed number.

Divide the numerator by the denominator.

23 ÷ 4 = 5 remainder 3 so $\frac{23}{4} = 5\frac{3}{4}$.

Fractions, Decimals and Percentages

Common Conversions

The most important conversions to know off the top of your head:

Fraction	Decimal	Percentage
$\frac{1}{2}$	0.5	50%
$\frac{1}{4}$	0.25	25%
$\frac{3}{4}$	0.75	75%
$\frac{1}{3}$	0.333333...	33 $\frac{1}{3}$ %
$\frac{2}{3}$	0.666666...	66 $\frac{2}{3}$ %
$\frac{1}{10}$	0.1	10%
$\frac{2}{10}$	0.2	20%
$\frac{1}{5}$	0.2	20%
$\frac{2}{5}$	0.4	40%

Decimals

Converting terminating decimals to fractions

The **numerator** will be the digits after the decimal point, the **denominator** will be a power of 10 with the same number of zeros as there were decimal places.

Example: 0.5= $\frac{5}{10}$, 0.75= $\frac{75}{100}$, 0.028= $\frac{28}{1000}$

Converting recurring decimals to fractions

1. Let decimal be *r*.
2. Multiply *r* by a power of 10 to move it past decimal point by one full repetition.
3. Subtract to get rid of the decimal part.
4. Divide to leave *r* and cancel if possible.

Example: Write 0.1666666… as a fraction.

Let $r = 0.1666666\ldots$

$100r = 16.6\ldots$

$-\ \ 10r = \ 1.666666\ldots.$

$90r = 15$

$r = \frac{15}{90} = \frac{1}{6}$

Surds and multiples

A **surd** is an expression that uses square or cube roots when the numbers cannot be simplified into an integer or a fraction. They look like these: $\sqrt{12}\ or\ 3\sqrt{7}$

The rules of operation are as such:

Multiplication: $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

Division: $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$

Addition and **subtraction**: $\sqrt{a} + \sqrt{b}$ must be left like this as cannot simply add or subtract

Simplification: split the surd apart, e.g. $\sqrt{300} = \sqrt{3 \times 100} = 10\sqrt{3}$

Rationalising the denominator

This is a way of removing surds from the denominator of a fraction to make the fraction look tidier.

Example: $\frac{2}{\sqrt{3}} = \frac{2\ x\ \sqrt{3}}{\sqrt{3}\ x\ \sqrt{3}} = \frac{2\sqrt{3}}{3}$

If the denominator is more complicated, then you may have to make the denominator the **difference of two squares**.

- If the denominator is in the form of $x + \sqrt{y}$ then you must multiple top and bottom of the fraction by $x - \sqrt{y}$
- If the denominator is in the form of $x - \sqrt{y}$ then you must multiple top and bottom of the fraction by $x + \sqrt{y}$.

Therefore, for both cases the denominator will become:

$$(x + \sqrt{y})(x - \sqrt{y}) = x^2 - (\sqrt{y})^2 = x^2 - y$$

Example: $72 - \sqrt{3} = \frac{7(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{7(2+\sqrt{3})}{2^2 - \sqrt{3}^2} = \frac{7(2+\sqrt{3})}{4-3} = 7(2 + \sqrt{3})$

Bounds

Upper and lower bounds

For measurement rounded to a given unit, the lower bound would be ½ a unit smaller and the upper bound would be ½ a unit bigger.

Example 1: A tree is 10 m tall to the nearest metre. Find the upper and lower bounds for its height.

Lower bound = 9.5 m
Upper bound = 10.5 m

Example 2: The mass of a chair is given as 5.6 kg to the nearest 0.1 kg. Find the upper and lower bounds for its height.

Lower bound = 5.55 kg
Upper bound = 5.65 kg

Maximum and minimum values for Calculations

Example 1: A curtain is measured as being 3.8m by 5.4m, to the nearest 0.1m. Calculate the minimum and maximum possible values for the area of the curtain.

Minimum possible area = 3.75×5.35
= $20.0625\ m^2$

Maximum possible area = 3.85×5.45
= $20.9825\ m^2$

Example 2: x = 4.2 and y = 5.5, both given to 1 d.p. What are the minimum and maximum values of $x \div y$?

Minimum possible value = smallest possible x ÷ largest possible y = $4.15 \div 5.55$ = 0.761 (3 d.p.)

Maximum possible value = largest possible x ÷ smallest possible y = $4.25 \div 5.45$ = 0.780 (3 d.p.)

Rounding Numbers

Decimal Places (d.p.)

Example 1: Give 8.5464 to 2 decimal places.
8.5464 = 8.55 (2 d.p.)

Example 2: Give 9.895 to 2 decimal places.
9.895 = 9.90 (2 d.p.)

Note that the zero must still be written!

Significant Figures (s.f.)

Example 1: Give 0.0059748 to 3 significant figures.
0.0059748 = 0.00597 (3 s.f.)

Example 2: Give 1.2549 to 2 significant figures.
1.2549 = 1.3 (2 s.f.)

Estimating

Example: Estimate the value of $\frac{39.8 + 73.4}{66.5 \times 2.3}$.
Round all numbers individually first before proceeding.
$\frac{39.8 + 73.4}{66.5 \times 2.3} \approx \frac{40 + 70}{70 \times 2} = \frac{110}{140} = \frac{11}{14}$

Inequality notation

Related to rounding:

- If a number is rounded to *n*, then there is a range of values that the original number could be.
- For 2 d.p. if the rounded number is *n*, then the smallest possible value is *n*-0.005 and the largest is *n*+0.005
- This can be written as n-0.005 $\leq x \leq n$ + 0.005
- For example, if *m* is 2.43 when rounded to 2 d.p. then 2.425 $\leq m \leq$ 2.435

Related to truncating:

- Truncating is when the end digits are cut off without rounding, e.g. 1.499 truncated to 2 d.p. is 1.49
- For 2 d.p. if the truncated number is *n* then the smallest possible value is *n* but it must be smaller than *n*+0.01
- This can be written as $n \leq n < n$+0.01
- For example, if *m* is 2.43 when truncated to 2 d.p. then 2.43 ≤ *m* < 2.44