GCE PHYSICS

S21-A420QS

Assessment Resource number 3

Comprehension Resource C

Answer **all** *questions.*

Read through the following article carefully.

Conduction in more detail
by Iustiniano Luigi Maurizio

Paragraph

Georg Ohm is credited with establishing the most famous of electrical laws. His law is used in the design of electrical devices that enhance our 21st century living. Can we learn more about the law by looking at the conduction of electricity on the atomic level? 1

As a starting point, one can try to apply the laws of classical physics to the motion of fast-moving electrons as they negotiate pathways through a metal lattice. An applied potential difference provides an electric field which will accelerate electrons until they collide, a fraction of a pico-second later, with an unsuspecting metal ion. This collision then has the effect of randomising the motion of the electron once again and all the good work of the electric field is lost. However, the acceleration commences again immediately and the process repeats over and over again. 2

The question is "Can we model this behaviour using A level physics?" The answer is "Yes we can!" 3

The first thing to do is to obtain a value for the rms speed of an electron (mass, m_e) at room temperature. This can be done quite easily by applying the kinetic theory of gases to "free" electrons in a metal. 4

$$\frac{1}{2}m_e\overline{c^2} = \frac{3}{2}kT \qquad \text{Equation 1}$$

It is relatively straightforward to show that the rms speed of electrons at room temperature is over 100 km s^{-1}. An estimate can also be made of the distance travelled by an electron in between collisions – let's say this distance is 10 atomic diameters which is around 3 nm. This then leads to a time between collisions of a few tens of femtoseconds. 5

Let's apply some more physics to the motion of the electrons. Although electrons have a mean **speed** of 100 km s^{-1} their motion is random and there is no preferred direction of motion. However, all this changes as soon as an electric field, E, is applied. It is easy to show that the acceleration experienced by the electrons is 6

$$a = \frac{eE}{m_e} \qquad \text{Equation 2}$$

in the usual physics notation. By applying $v = u + at$, with $u = 0$ you can obtain

$$v = \frac{eE}{m_e}\tau \qquad \text{Equation 3}$$

where τ is the mean time spent by electrons in between collisions. This gives a final mean drift velocity of

$$\bar{v}_{\text{drift}} = \frac{eE}{2m_e}\tau \qquad \text{Equation 4}$$

Paragraph

When this equation is combined with the drift velocity equation that students learn to derive for A level, you get:

$$I = nA\bar{v}_{\text{drift}}e = nA\left(\frac{eE}{2m_e}\right)\tau e = \frac{nAe^2\tau E}{2m_e} \qquad \text{Equation 5}$$

But if you remember that, for a uniform electric field:

$$E = \frac{V}{d} \qquad \text{Equation 6}$$

and substitute this into Equation 5, you will find you've just derived Ohm's Law,

$$I = \frac{nAe^2\tau}{2m_e d}V \qquad \text{Equation 7}$$

with the resistance of the metal given by:

$$R = \frac{2m_e d}{nAe^2\tau} \qquad \text{Equation 8}$$

You should also be able to see that the last equation for the resistance leads to the equation for resistivity, (ρ):

$$\rho = \frac{2m_e}{ne^2\tau} \qquad \text{Equation 9}$$

These are all very powerful equations and enhance the meaning of Ohm's law. For instance, the condition for Ohm's law is that the temperature must remain constant. We can now explain this in detail – the temperature affects the mean speed of the electrons, which in turn affects their time between collisions which will then change the resistance. 7

However, there is a very important limit to this theory – the laws of physics at the atomic level are governed by quantum theory. In our simplified theory, the distance between collisions is a constant. This leads to a resistivity which is proportional to the square root of temperature and, unfortunately, this is not what happens in experiments. 8

Another important disagreement between this theory and practice is superconductivity. This theory would suggest that resistivity decreases gradually and eventually arrives at zero at a 9 temperature of absolute zero. Again, this doesn't happen in practice and the superconductors lose their resistance suddenly at the superconducting transition temperature. Even more spectacular is the behaviour of electrons below this temperature – they almost cease to be individual electrons but team up in pairs held together by a lattice vibration. These pairs of electrons feel an attractive force due to the lattice vibration between them and travel freely through the lattice without ever experiencing collisions. This means that the mean time between collisions tends to infinity.

To answer the original question about Georg Ohm's spectacular law, it seems that there is a lot 10 to be learned from looking at conduction of electricity at the atomic level. When combined with quantum physics the possibilities seem to be endless or perhaps even infinite.

1

Answer the following questions in your own words. Direct quotes from the original article will not be awarded marks.

(a) In your own words, describe the motion of electrons both with and without the application of an electric field (see paragraphs 2 and 6). [4]

(b) Calculate the temperature at which electrons are expected to have an rms speed of 100 km s^{-1} (see Equation 1). [2]

(c) Show that the acceleration of an electron in an electric field, of strength E, is:

$$a = \frac{eE}{m_e} \qquad \text{(Equation 2)} \qquad [2]$$

(d) Use a mean time of 40 fs to calculate the electric field strength, E, when the mean drift velocity of electrons is 0.25 mm s^{-1} (see Equation 4). [2]

(e) Explain briefly how the author can derive Equation 9 from Equation 8. [2]

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(f) Copper has a resistivity of $1.68 \times 10^{-8}\ \Omega\ \mathrm{m}$ and has 8.5×10^{28} free electrons per m^3. Use equation 9 to calculate the mean time between collisions for copper. [2]

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(g) Explain why resistivity should be proportional to the square root of temperature (see paragraph 8 and Equation 9). [4]

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(h) State and explain what charge you would expect a lattice vibration to carry (see paragraph 9). [2]

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