GCSE MATHEMATICS

S21-C300

Non-Calculator Assessment Resource P

Higher Tier

Formula list

Area and volume formulae

Where r is the radius of the sphere or cone, l is the slant height of a cone and h is the perpendicular height of a cone:

Curved surface area of a cone = $\pi r l$

Surface area of a sphere = $4\pi r^2$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Kinematics formulae

Where a is constant acceleration, u is initial velocity, v is final velocity, s is displacement from the position when $t = 0$ and t is time taken:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

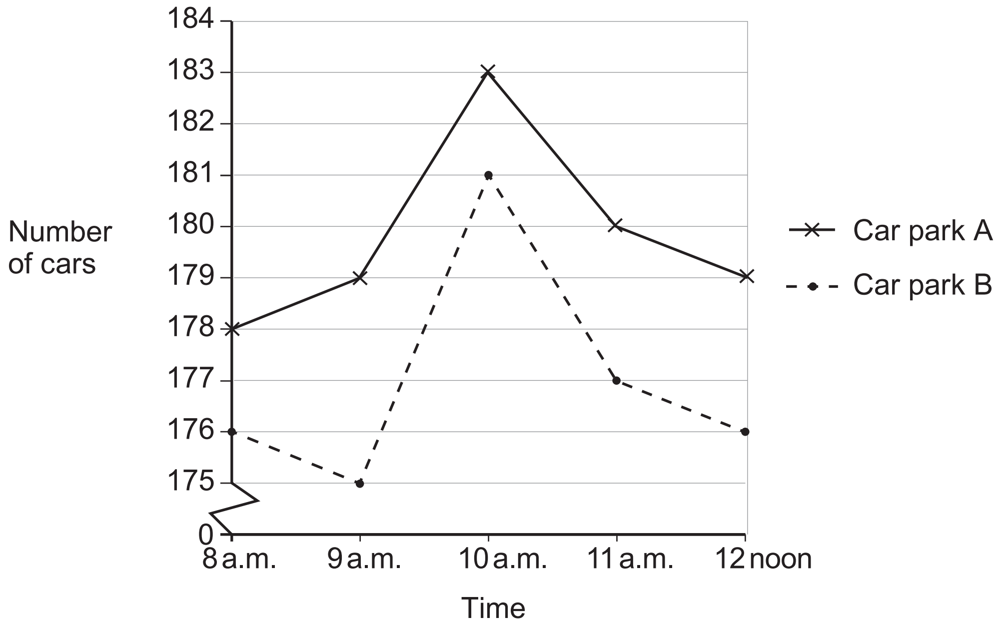
$$v^2 = u^2 + 2as$$

1. Peter and Paula record the number of cars in each of two airport car parks, A and B, between 8 a.m. and 12 noon one Saturday morning.
This was done to find out if there was a peak time for parking during that period.

The table shows the data they collected.

Time	8 a.m.	9 a.m.	10 a.m.	11 a.m.	12 noon
Number of cars in car park A	178	179	183	180	179
Number of cars in car park B	176	175	181	177	176

Paula draws this graph to represent the data.



Peter says,

"This graph is not sensible as it does not show the data fairly."

(a) What has been done in the drawing of the graph that has made Peter think this? [1]

It jumps from 0 to 175, making the differences between number of cars look bigger

(b) What error might this lead to, for people who do not look carefully at the graph? [1]

That 10 am is the peak when actually 11 am only has a difference of 3-4 cars.

2. Ivan is part of a team making bags of free items to give away at a college open evening.

He has:

- 140 discount vouchers,
- 56 pencils,
- 280 sweets

to share between all his bags.

He uses **all** the vouchers, **all** the pencils and **all** the sweets.
He makes as many bags as possible.
The contents of each bag are the same.

How many bags does Ivan make and what does each bag contain? [5]

56 : 140 : 280
28 : 70 : 140
4 : 10 : 20
2 : 5 : 10

$56 \div 2 = 28$ bags

↑

$7 \times 2 \times 2$

7	140	56	280
2	20	8	40
2	10	4	20
	5	2	10

Ivan makes 28 bags containing

5 vouchers, 2 pencils, 10 sweets.

3. *(a)* Simplify $\dfrac{x^2 \times x^7}{x^3}$. [2]

$\dfrac{x^9}{x^3} = x^6$

(b) (i) Find the positive value of $16^{\frac{1}{4}}$. [1]

$\sqrt[4]{16} = \left(16^{\frac{1}{2}}\right)^{\frac{1}{2}} = \sqrt{\left(\sqrt{16}\right)} = \sqrt{4} = \underline{\underline{2}}$

(ii) Find the value of $27^{\frac{4}{3}}$. [2]

$\sqrt[3]{27^4} = \left(27^{\frac{1}{3}}\right)^4 = \left(\sqrt[3]{27}\right)^4$

$= 3^4 = \underline{\underline{81}}$

$3 \times 3 = 9$

$9 \times 3 = 27$

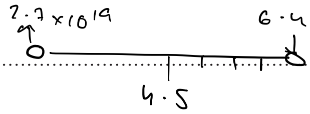
$27 \times 3 = 81$

(c) **Estimate** the value of $(3{\cdot}9\times10^6)^3$.
Give your answer in standard form. [3]

$(3\cdot9\times10^6)^3$ | $4\times4=16$, $16\times4=64$

$(4\times10^6)^3 = 4^3\times10^{6\times3} = 64\times10^{18}$
$= 6.4\times10^{19}$

$(3\times10^{16})^3 = 27\times10^{18} = 2.7\times10^{19}$



Estimate 6×10^{19}

(d) Write $\dfrac{42}{\sqrt6}$ in the form $a\sqrt6$ where a is an integer. [2]

$\dfrac{42}{\sqrt6} = \dfrac{42\sqrt6}{\sqrt6\times\sqrt6} = \dfrac{42\sqrt6}{6} = 7\sqrt6$

$a=7$

4. A tennis club has 240 members.
They each played a senior, main or junior event in one of three competitions, *A*, *B* or *C*.

Of the club members:
- 110 played in *A*,
- 30 played in a junior event,
- 25 played in the senior event in *B*,
- no junior played in *C*,
- 40 of those who played in *C* were in the main event.

The number of members who played in a senior event was 150% more than those who played in a junior event.
The ratio of members who played in *B* and *C* was *B* : *C* = 6 : 7.
The probability that a member played in the junior event in *A* was 0·1.

A member is selected at random from the club.

Use the table to help you to find the probability that this member played in a Main event or played in *B* but not both.
You must show all your working. [6]

	Senior	Main	Junior	Totals	
A			24	110	
B	25	29	6	60	6
C	30	40	0	70	7
Totals	45	165	30	240	

senior = 1·5 junior
= 1·5 × 30
= 45

240 − 110 = 130
B + C = 130
6x + 7x = 130
13x = 130
x = 10

10 × 6 = 60 = B
10 × 7 = 70 = C
Main = 165 B = 60
overlap = 29

$$p = \frac{165 + 60 - 29}{240} = \frac{196}{240} = \frac{49}{60}$$

Probability = $\frac{49}{60}$

5. Write $7 \cdot \dot{3}4\dot{1}$ as a fraction. [2]

$x = 7.341\,341\,341\ldots$

$1000x = 7341.341341$

$1000x - x = 7341.341341 - 7.341341$

$999x = 7334$

$\boxed{x = \dfrac{7334}{999}}$

$$\begin{array}{r} 7341 \\ -\quad 7 \\ \hline 7334 \end{array}$$

6. Alys has 10 different-coloured tokens.
Each day, she chooses 3 of her tokens at random and places them in a row on her desk.

(a) Find the number of different ways in which this can be done. [2]

10 tokens
3 tokens at random. $\Big\}$ so → $10 \times 9 \times 8 = 720$

or $\Big\{ \rightarrow \dfrac{10!}{(10-3)!} = \dfrac{10!}{7!} = \dfrac{7! \times 8 \times 9 \times 10}{7!} = 8 \times 9 \times 10 = 72 \times 10 = 720$

(b) One of her tokens is pink and another is green.

Find the number of arrangements where the middle token is pink or green. [2]

p g x x p g x could be any of the
g p x x g p   8 remaining tokens

only 4 possibilities + 8 possibilities for x
$\therefore$ $4 \times 8 = 32$ different arrangements is possible

7. *(a)* $f(x)=\sqrt{x-1}$ for $x \geqslant 1$.

Show that $f^{-1}(x)<1$ has no solutions. [3]

$f(x) = \sqrt{x-1}$

$y = \sqrt{x-1}$

$y^2 = x-1$

$y^2+1 = x$

swap x and y

$x^2+1 = y$

$x^2+1 = f^{-1}(x)$

$x^2+1<1$

$x^2<0$

$\therefore x<0$

which is not possible as x must be $\geqslant 1$, so there are no solutions

(b) $g(x)=5^x$

$h(x)=x+3$

Solve $gh(x)=\dfrac{1}{25}$. [4]

$gh(x) = g(x+3) = 5^{x+3}$

$5^{x+3} = \dfrac{1}{25}$

$5^{x+3} = \dfrac{1}{5^2}$

$5^{x+3} = 5^{-2}$

$x+3 = -2$

$x = -5$

8. In this question, all lengths are in centimetres.

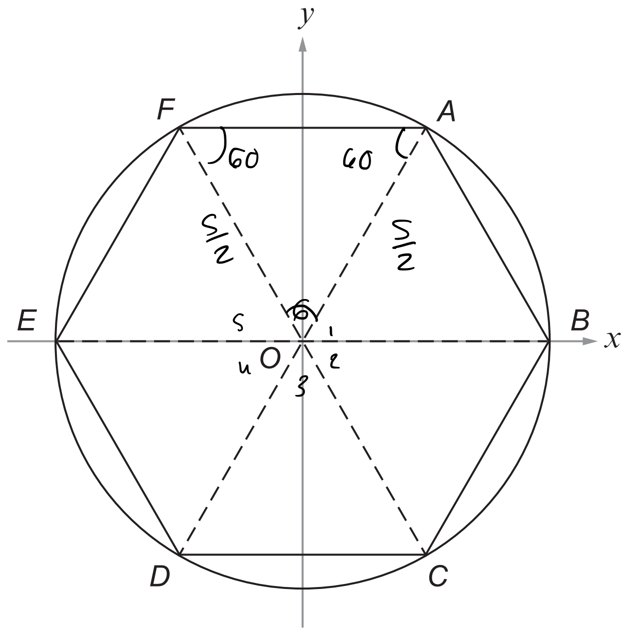


Diagram not drawn to scale

The diagram shows a sketch of a circle, centre *O*.
Points *A*, *B*, *C*, *D*, *E* and *F* lie on the circumference of the circle.
Triangles *AOB*, *BOC*, *COD*, *DOE*, *EOF* and *FOA* are congruent.

The circle has equation $x^2 + y^2 = \dfrac{25}{4}$.

Calculate the perimeter of the hexagon *ABCDEF*.
You must justify any decisions that you make. [4]

$x^2 + y^2 = \frac{25}{4} \rightsquigarrow r = \frac{5}{2}$

$\frac{360}{6} = 60°$

$\frac{180 - 60}{2} = 60°$

all the angles in a triangle are equal, ∴ equilateral triangle

∴ perimeter $= \frac{5}{2} \times 6$

$= 5 \times 3 = 15$