GCSE MATHEMATICS

S21-C300

Non-Calculator Assessment Resource O

Higher Tier

Formula list

Area and volume formulae

Where r is the radius of the sphere or cone, l is the slant height of a cone and h is the perpendicular height of a cone:

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Kinematics formulae

Where a is constant acceleration, u is initial velocity, v is final velocity, s is displacement from the position when $t = 0$ and t is time taken:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

1. Gita is carrying out a survey to find out what people think of a proposed new road for Redville.

(a) Gita decides to ask the first 20 people she meets at Redville bus station between 8 a.m. and 9 a.m. on a Monday morning.

Give **two** reasons why this plan is unlikely to produce reliable results. [2]

Reason 1:

The people she wants to ask take the bus so might not drive on the road (so excludes non-bus riders from survey)

Reason 2:

The people won't be selected randomly, as the time range chosen is very short - only in the early morning thus consider only people going to work / school between 8 - 9 am.

(b) Here is a question from Gita's survey:

How often do you use your car?

1 – 2 ☐ 3 – 4 ☐ 4 – 5 ☐ 6 + ☐

Make **two** criticisms of Gita's question. [2]

Criticism 1:

There is no option for people who don't use their car

Criticism 2:

There is no time period

ie. use the car in a week? in a month?

2. *(a)* Solve $5x - 1 = 3x + 4$. [2]

$2x = 5$

$x = \frac{5}{2}$

(b) Solve the following simultaneous equations. [2]

$2x + y = 8$ (1)
$x - y = 1$ (2)

$2x + y = 8$
$+ \quad x - y = 1$
$3x = 9$
$x = 3$

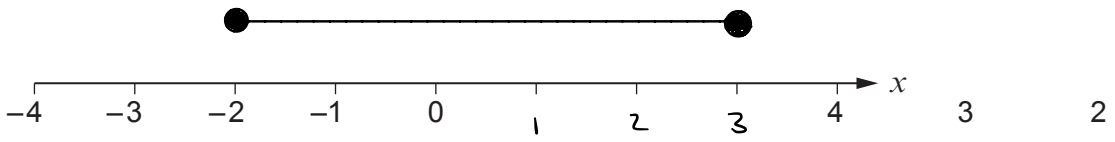
$3 - y = 1$
$y = 2$

CHECKING

$2(3) + 2) = 6 + 2 = 8$

$x = 3$
$\therefore \quad y = 2$

(c) Represent the inequality $-2 \leqslant x \leqslant 3$ on the number line below. [1]

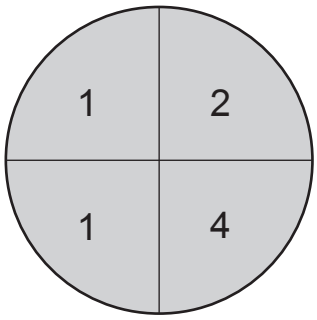


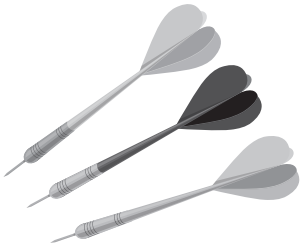
(d) Solve $\frac{2x}{3} < 4$. [2]

$2x < 12$

$x < 6$

3. The diagram shows a dartboard with 4 sectors of equal size.





Sanjeev throws 3 darts which all hit this dart board.
Each dart is equally likely to hit any sector of the dart board. → $P = \frac{1}{4}$

He **multiplies** his three numbers to find his score.

Work out the probability that his score is an odd number. [2]

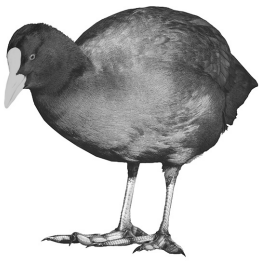
only way it is odd is if he hits the one 3 times

$P(1) = \frac{1}{2}$

$\left(\frac{1}{2}\right)^3 = \frac{1}{8}$

$P(\text{score} = \text{odd}) = \frac{1}{8}$

4. A scientist wants to find out how many coots there are on a lake.



One Monday morning, she captures a random sample of 48 coots and tags them.
She then releases them back onto the lake.

The following Monday morning, she captures a second random sample of 30 coots and counts the number that are tagged.

The scientist finds that 20 of the coots in the second sample are tagged.

Assume that the number of coots on the lake remains constant.

How many coots are there likely to be on the lake?
Show calculations to justify your answer. [3]

1st time | tagged 2nd time

$\frac{48}{x} = \frac{20}{30} = \frac{2}{3}$

or

$\frac{48}{x} = \frac{2}{3}$

$48 \times 3 = 2x$

$24 \times 3 = x$

$72 = x$

or

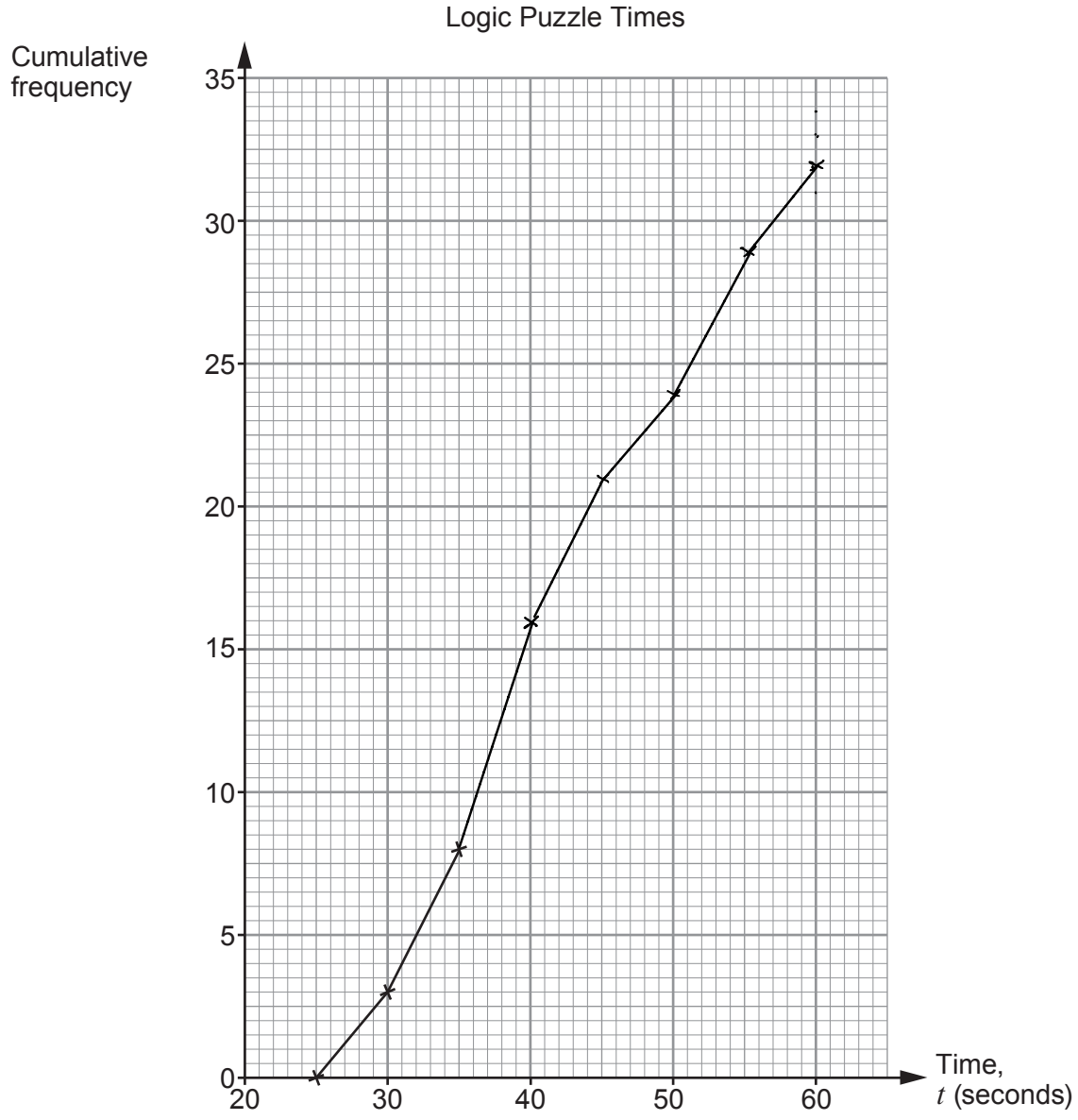
$\frac{48 \times 30}{20} = 72$

$\frac{\text{1st sample} \times \text{2nd sample}}{\text{number of recaptured}} = \text{estimated population}$

5. The table shows a summary of the time, in seconds, it takes each of 32 people to complete a logic puzzle.

Time, t (seconds)	$t \leqslant 25$	$t \leqslant 30$	$t \leqslant 35$	$t \leqslant 40$	$t \leqslant 45$	$t \leqslant 50$	$t \leqslant 55$	$t \leqslant 60$
Cumulative frequency	0	3	8	16	21	24	29	32

(a) Complete the cumulative frequency diagram below to show these results. [2]



(b) (i) How many people took more than 40 but not more than 50 seconds to complete the puzzle? [1]

$24 - 16 = 8$

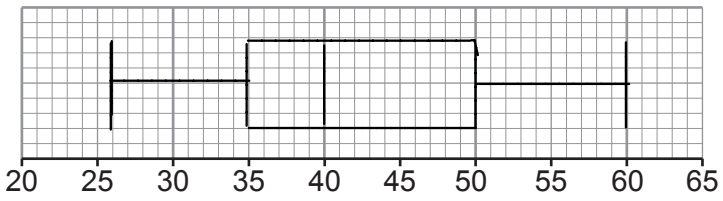
(ii) Complete the inequality to show the modal class. [1]

$35 < t \leqslant 40$

(c) Eddie uses the data from part *(a)* to obtain estimates and draw a box plot.

He also knows that the fastest time is 26 seconds.
Eddie also **assumes** that the slowest time is 60 seconds.

(i) Draw Eddie's box plot. [4]



Logic Puzzle Time, t (seconds)

$\frac{32}{4} = 8$, $32 - 8 = 24$

(ii) Explain why Eddie's assumption may not be correct. [1]

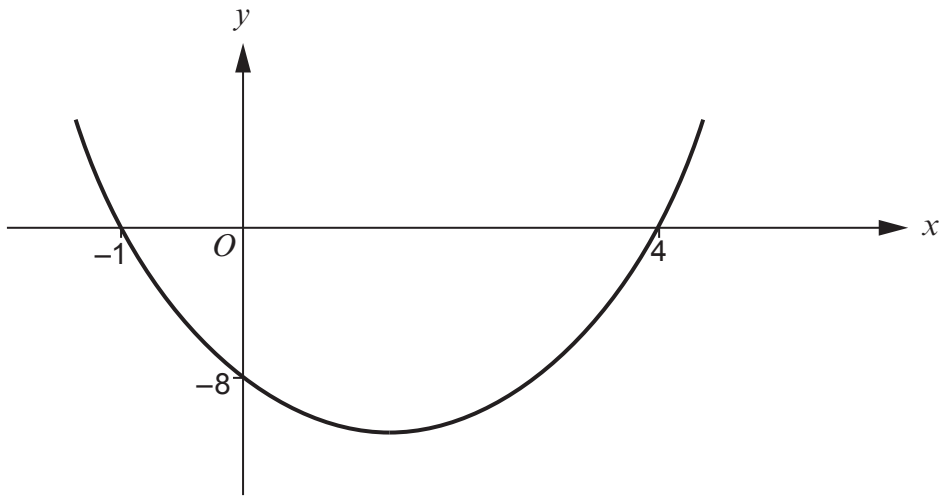
as the slowest person could be anywhere from 26 to 60 seconds

(iii) Eddie's assumption is not actually correct.
What effect does this have on each of the range and the interquartile range? [2]

Effect on the range: it makes the range smaller

Effect on the interquartile range: no effect

6. *(a)*



The diagram shows a sketch graph of a quadratic function.

Find the equation of this curve. [3]

y intercept $= -8$

$y = ax^2 + bx - 8$

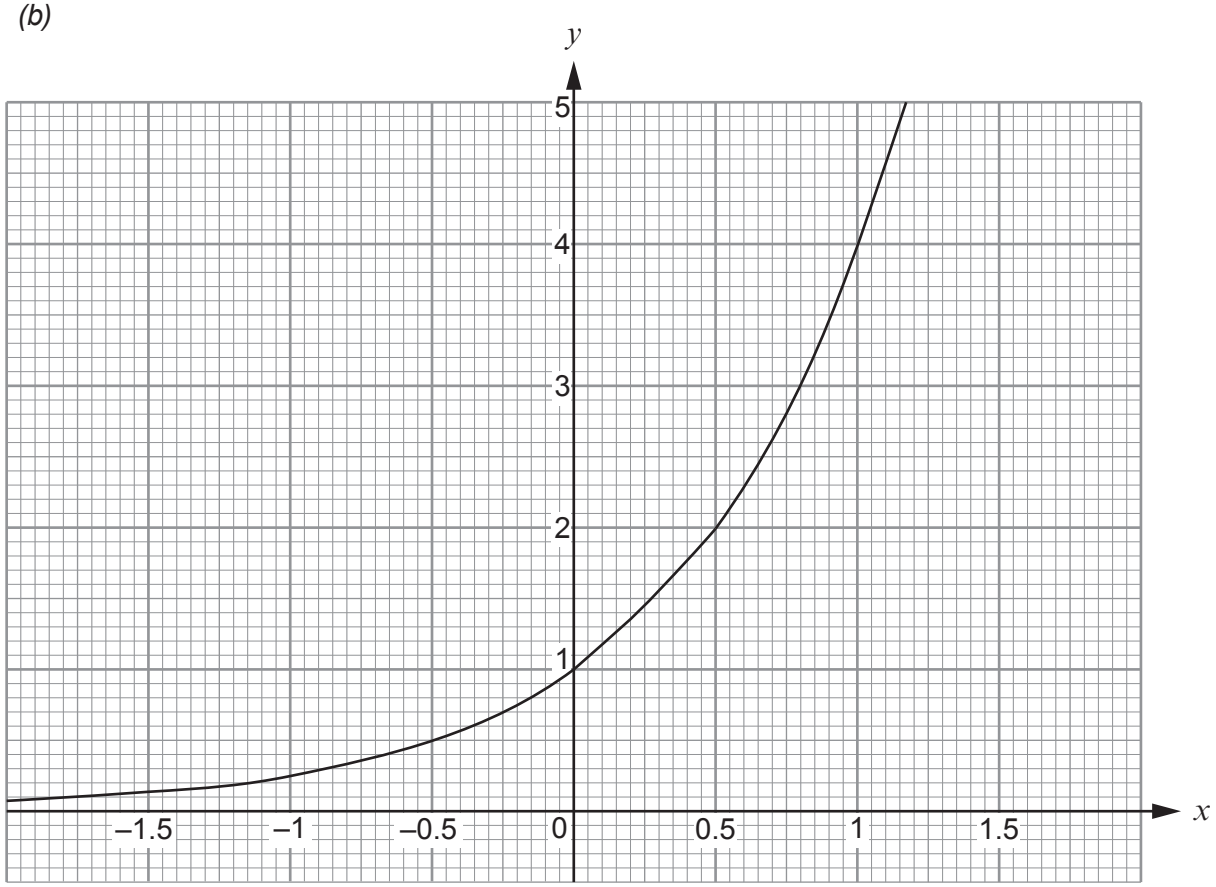
$= (2x + 2)(x - 4)$ ← got them from x-intercepts

$= 2x^2 - 8x + 2x - 8$

$= 2x^2 - 6x - 8$

$y = 2x^2 - 6x - 8$

(b)



The diagram shows the graph of the curve $y = k^x$.

Find the value of the positive integer k. [2]

$y = k^x$

$2 = k^{\frac{1}{2}} \rightarrow 2 = \sqrt{k}$

$4 = k$

$k =$ 4

7. In this question, all lengths are in centimetres.

A circle has equation $x^2 + y^2 = 49$.

Points *A*, *B* and *C* all lie on this circle.
Their co-ordinates are *A* $(a, 0)$, *B* $(b, 0)$ and *C* $(0, c)$, where $a < 0$, $b > 0$ and $c > 0$.

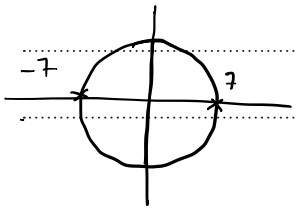
(a) Find the length of the line *AB*. [2]

$x^2 + y^2 = 49$

A $(a, 0)$ $a^2 + 0^2 = 49$

$a = 7$ or $a = -7$ $(-7 < 0)$

B $(b, 0)$ $b^2 + 0^2 = 49$, $b = 7$ $(7 > 0)$

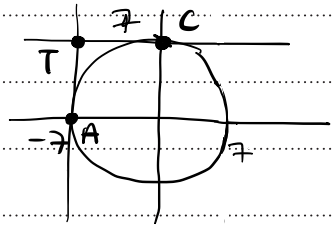


AB = 14 cm

(b) The tangent to the circle at *A* and the tangent to the circle at *C* meet at the point *T*.

Find the coordinates of *T*. [2]

$C = (0, c)$



$c^2 = 49$

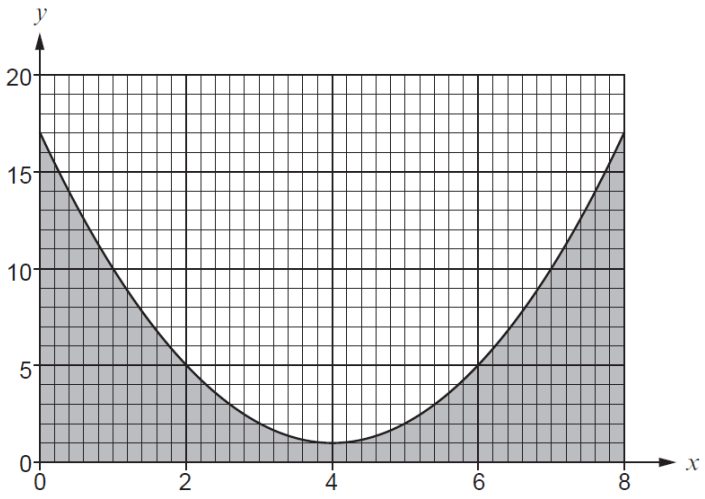
$c = 7$

$y = 7$ and $x = -7$ meet

$\therefore$ at $-7, 7$

T (-7 , 7)

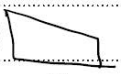
8.



The diagram shows the graph of $y = (x - 4)^2 + 1$ for $0 \leqslant x \leqslant 8$.

(a) Using four vertical strips of equal width, estimate the area of the shaded region. [4]

1.  $= \frac{1}{2}(5+17)2 = 22$



$= \frac{1}{2}(1+5)2 = 6$

and reflected as $x = 4$, so area $= 2(22+6)$

$= 2(28)$

$= 56$

(b) Is your answer to part *(a)* an underestimate or an overestimate?

Underestimate ☐ Overestimate ☑

Explain how you decide. [1]

the graph is curved downwards and not a straight line forming a trapezium. Estimate includes extra area between the straight line and the curve.