GCSE MATHEMATICS

S21-C300

Non-Calculator Assessment Resource M

Higher Tier

Formula list

Area and volume formulae

Where r is the radius of the sphere or cone, l is the slant height of a cone and h is the perpendicular height of a cone:

Curved surface area of a cone = $\pi r l$

Surface area of a sphere = $4\pi r^2$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Kinematics formulae

Where a is constant acceleration, u is initial velocity, v is final velocity, s is displacement from the position when $t = 0$ and t is time taken:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

1. The diagram shows a cylinder.

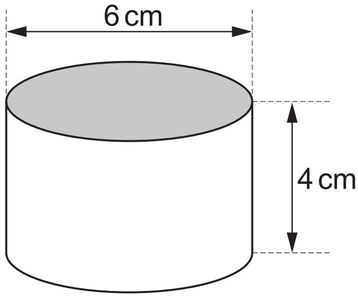
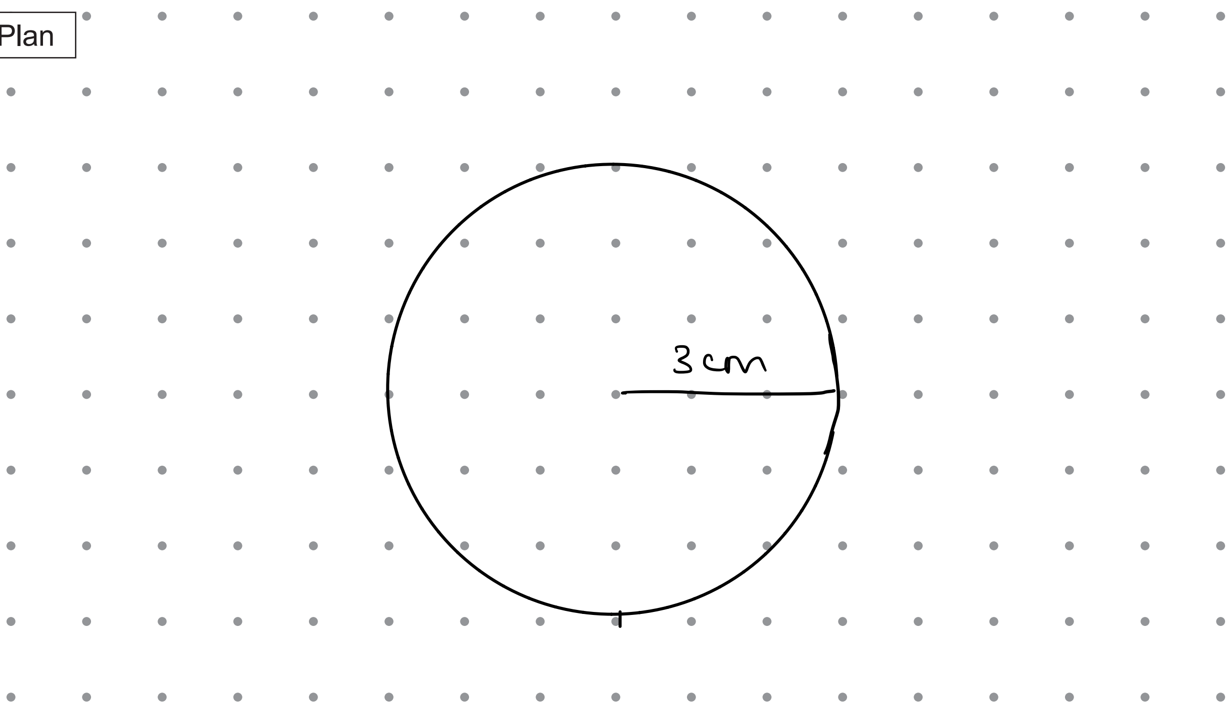


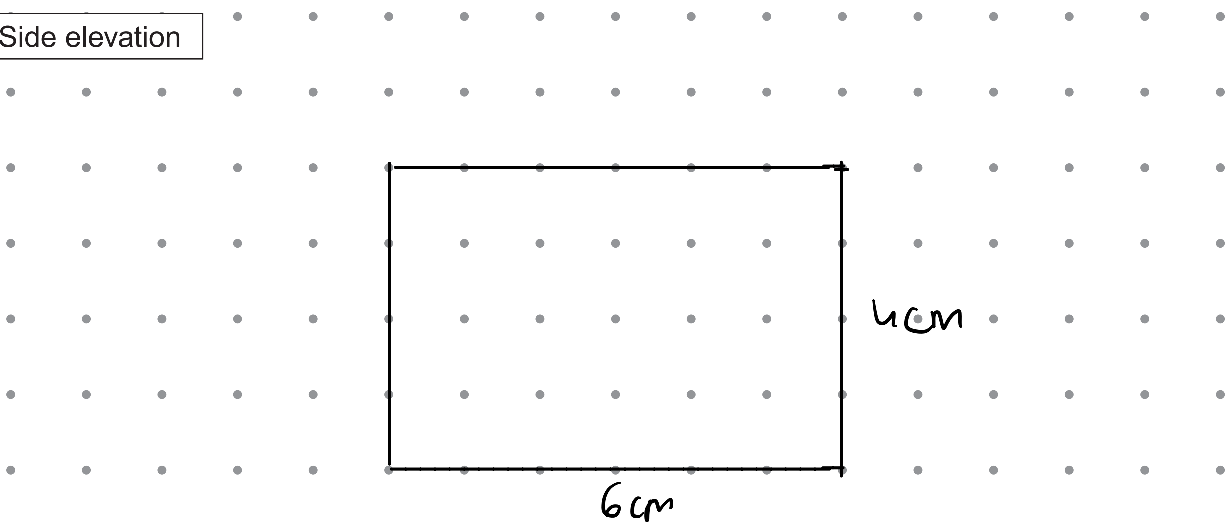
Diagram not drawn to scale

On the 1 centimetre grid below, draw accurately:
- the plan of the cylinder,
- the side elevation of the cylinder. [3]

Plan



Side elevation



2.

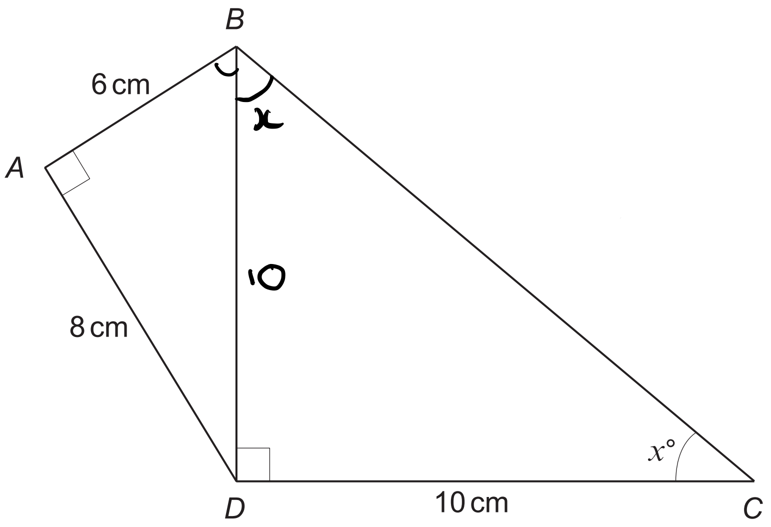


Diagram not drawn to scale

Find the value of x.
You must show all your working. [3]

$\sqrt{6^2+8^2} = 10$

$\triangle BCD$ is a right angle isoceles triangle so...

$180 - 90 = 90$

$90 \div 2 = \boxed{45° = x}$

3.

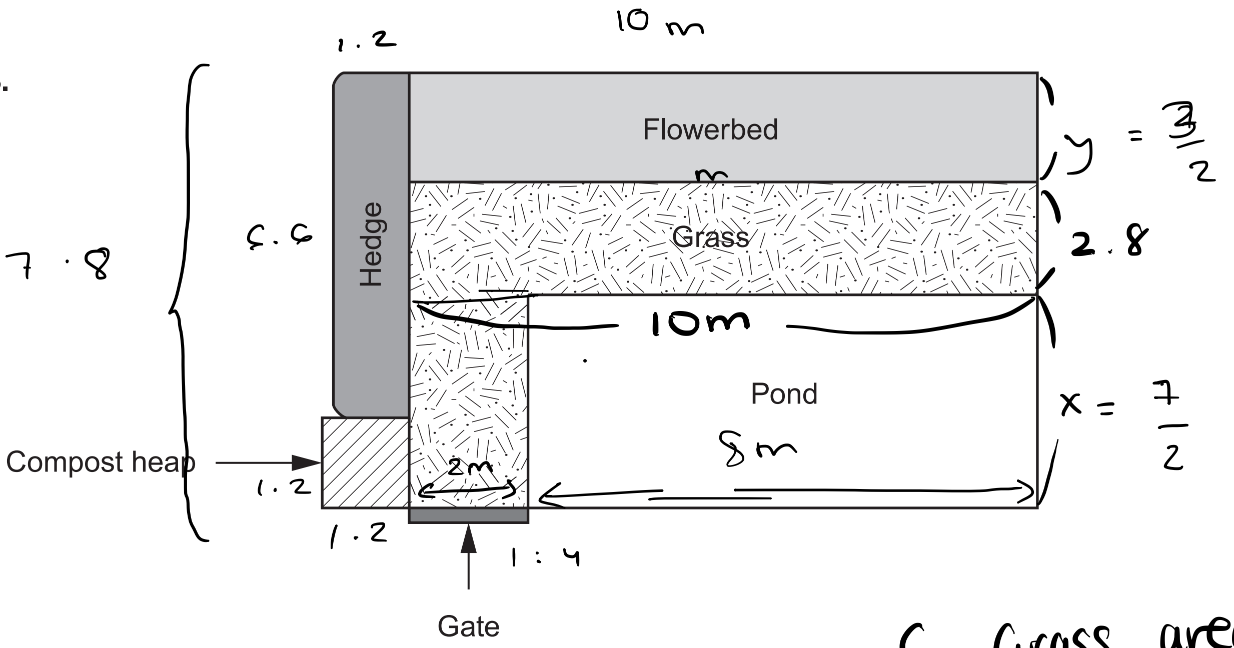


Diagram not drawn to scale

Grass area:

$2.8 \times 10 + \frac{7}{\cancel{2}} \times \cancel{2}$

$= 28 + 7$

$= \boxed{35\,m^2}$

The diagram shows a garden which has:

- an L shaped area of grass,
- a rectangular flowerbed and pond,
- a square compost heap.

The length of each side of the compost heap is 1·2 m.
The ratio of the length of the compost heap to the length of the hedge is 2 : 11.

The length of the gate is 2 m.
The length of the gate is $\frac{1}{4}$ of the length of the pond.

The area of the pond is 28 m^2.
The perimeter of the flowerbed is the same as the perimeter of the pond.

Find the area of the grass. [6]

Area pond

★ $28m^2 = 8x$

$x = \frac{28}{8} = \frac{7}{2}$

★ Finding y:

$16 + 7 = 20 + 2y$

$23 = 20 + 2y$

$\frac{3}{2} = y$

★ Total Area

7.8×10

$= 78$

Grass dimensions: $l = 10m$

$w = (6.6 + 1.2) - (\frac{3}{2} + \frac{7}{2})$

$= 7.8 - 5 = 2.8m$

or

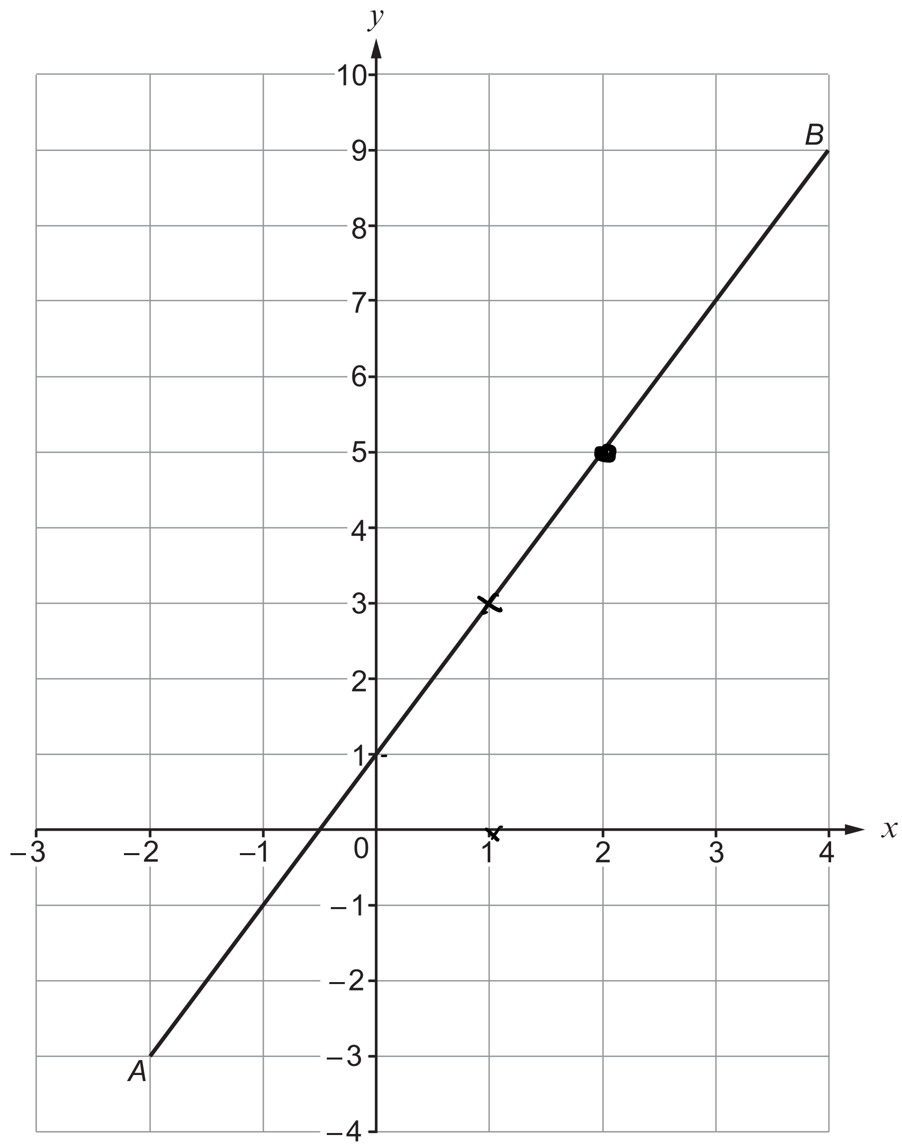
★ AREA OF GRASS.

$78 - 28 = \left(10 \times \frac{3}{2}\right) +$ Area Grass

$50 = 15 +$ Area Grass

$35 =$ area grass

4.



The diagram shows the graph of a straight line, *AB*.

(a) Find the equation of this line.
Give your answer in the form $y = mx + c$. [3]

y - intercept = 1, $\therefore$ c = 1

$y = mx + c$

$\Rightarrow (5, 2)$ $5 = 2m + 1$

$4 = 2m$

$2 = m$

$\therefore y = 2x + 1$

$y =$ $2x + 1$

(b) Find the equation of the perpendicular bisector of the line *AB*. [4]

gradient of AB $= 2$

graden to the normal of AB $= \frac{-1}{2}$

$y = -\frac{1}{2}x + C$.

midpoint of AB $= (1, 3)$

x: $\frac{-2+4}{2} = \frac{2}{2} = 1$

y: $\frac{-3+9}{2} = \frac{6}{2} = 3$

$\Rightarrow \quad 3 = -\frac{1}{2}(1) + c$

$\frac{7}{2} = c$

$\therefore \quad y = -\frac{1}{2}x + \frac{7}{2}$

5. It is known that y varies inversely as the cube root of x and that $y = 2$ when $x = 27$.

(a) Find a formula for y in terms of x. [3]

$y = \frac{k}{\sqrt[3]{x}}$. $\therefore y = \frac{6}{\sqrt[3]{x}}$.

$2 = \frac{k}{\sqrt[3]{27}}$

$2 = \frac{k}{3}$

$6 = k$

(b) Using your answer to part *(a)*, find

(i) y when $x = 1000$, [1]

$y = \frac{6}{\sqrt[3]{1000}} = \frac{6}{10} = 0 \cdot 6$

(ii) x when $y = 3$. [2]

$3 = \frac{6}{\sqrt[3]{x}} \rightarrow \sqrt[3]{x} = 2$

$x = 8$

6. *(a)*

$$V_0 = 10\,000$$

$$V_{n+1} = 0.8\,V_n \quad \text{where} \quad n \geqslant 0$$

This iterative formula can be used to work out the value V_n of a particular type of car when it is n years old.

(i) Show that a car of this type that is 1 year old is worth £8000. [1]

$V_1 = 0 \cdot 8 \times 10000$ $\therefore V_1 = £8000$

$= 8000$

(ii) Use this formula to find the value of a car of this type that is 3 years old. [3]

$V_2 = 8000 \times 0 \cdot 8 = 6400$

$V_3 = 6400 \times 0 \cdot 8 = 5120$

$^{3}640$

$\times\ 8$

5120

$\therefore V_3 = £5120$

Value is £ 5120

(b)

A newly built house is worth £240 000 and is expected to increase in value by 2% each year.

Complete the following iterative formula to show this information. [1]

$V_0 =$ 240 000

$V_{n+1} =$ 1·02 V_n where $n \geqslant 0$

7. The function f is defined, for $x \neq 1$, by $f(x) = \dfrac{7}{x-1}$.

(a) (i) Explain why $x \neq 1$ for this function. [1]

when $x = 1$ the function is equal to $\frac{7}{0}$

which is impossible

(ii) Show that $f^{-1}(x) = \dfrac{a}{x} + b$, where a and b are integers. [2]

1 make x the subject

$y = \dfrac{7}{x-1}$, $y(x-1) = 7$

$(x-1) = \dfrac{7}{y}$

$x = \dfrac{7}{y} + 1$

2 change signs

$y = \dfrac{7}{x} + 1$; $a = 7$, $b = 1$

(b) The functions g and h are defined for all real x by

$$g(x) = \sqrt[3]{x+1} ,$$

$$h(x) = 9x^3 .$$

Solve $hg(x) = f(x)$. [5]

$hg(x) = 9\left(\sqrt[3]{x+1}\right)^3 = 9(x+1) = 9x + 9$

$hg(x) = f(x)$: $9x + 9 = \dfrac{7}{(x-1)}$

$(9x+9)(x-1) = 7$

$9x^2 - \cancel{9x} + \cancel{9x} - 9 = 7$

$9x^2 = 16$

$x^2 = \dfrac{16}{9}$

$x = \pm\sqrt{\dfrac{16}{9}} = \pm\dfrac{4}{3}$