# GCSE MATHEMATICS

S21-C300

## Non-Calculator Assessment Resource K

Higher Tier

**Formula list**

*Area and volume formulae*

Where $r$ is the radius of the sphere or cone, $l$ is the slant height of a cone and $h$ is the perpendicular height of a cone:

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

*Kinematics formulae*

Where $a$ is constant acceleration, $u$ is initial velocity, $v$ is final velocity, $s$ is displacement from the position when $t = 0$ and $t$ is time taken:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

**1.** *(a)* Solve $19 - 4x = 11$. [2]

$19 - 4x = 11$

$19 - 11 = 4x$

$8 = 4x$

$2 = x$

*(b)* Solve $\dfrac{2x-3}{4} = 3x$. [3]

$2x - 3 = 12x$

$-10x = 3$

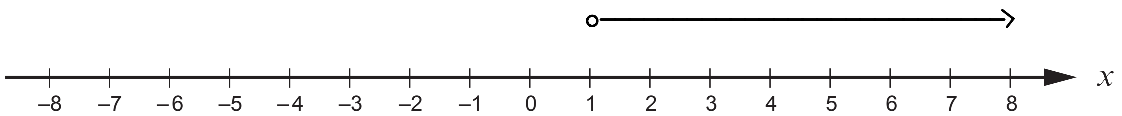
$x = \dfrac{-3}{10}$

*(c)* (i) Solve $3x + 2 > 5$. [2]

$3x > 3$

$x > 1$

(ii) Represent your answer to part *(c)*(i) on the number line below. [1]



**2.** The table shows some of the values of $y = x^2 + x - 1$ for $-2 \leqslant x \leqslant 1$.

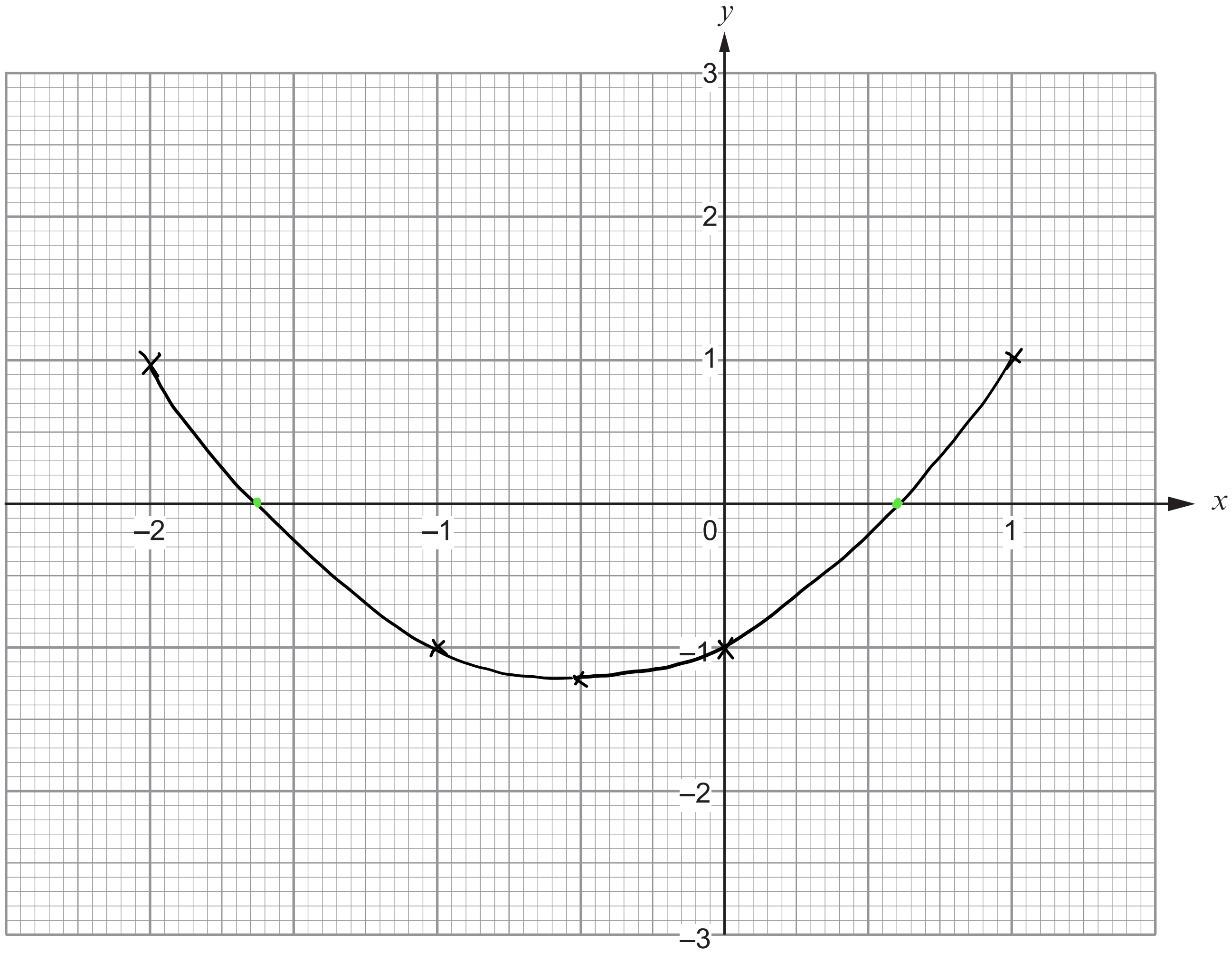
| $x$ | $-2$ | $-1$ | $-0{\cdot}5$ | $0$ | $1$ |
|---|---|---|---|---|---|
| $y = x^2 + x - 1$ | 1 | $-1$ | $-1{\cdot}25$ | $-1$ | 1 |

*(a)* Complete the table above. [2]

$-2: \; 4 - 2 - 1 = 1$ $\quad 1: \; 1 + 1 - 1 = 1$

$0: \; 0 + 0 - 1 = -1$

*(b)* On the graph paper below, draw the graph of $y = x^2 + x - 1$ for $-2 \leqslant x \leqslant 1$. [2]




*(c)* State the equation of the line of symmetry of the curve $y = x^2 + x - 1$. [1]

equation of the line of symmetry is

$x = -0.5$

*(d)* Use your graph to solve $x^2 + x - 1 = 0$. [2]

$y = 0$, so look for where the graph crosses the $x$-axis $\therefore$ $x = 0{\cdot}6$

$x = -1{\cdot}625$

**3.** Rearrange $6(x + y) = 8x - 5$ to make $x$ the subject. [3]

$6x + 6y = 8x - 5.$

$6y + 5 = 2x$

$\frac{6y + 5}{2} = x$

$3y + \frac{5}{2} = x$

**4.** *(a)* Find the value of each of the following.

(i) $0{\cdot}8^{-1}$ [1]

$\frac{1}{0\cdot 8} = \frac{1}{8/10} = \frac{10}{8} = \frac{5}{4} = 1\cdot 25$

(ii) $625^{\frac{1}{4}}$ [1]

$\sqrt[4]{625} = 5$

(iii) $\left(\frac{1}{64}\right)^{\frac{2}{3}}$ [2]

$\sqrt[3]{\left(\frac{1}{64}\right)^2} = \sqrt[3]{\frac{1}{4096}} = \frac{1}{16}$

*(b)* Write $81 \times \frac{3^0}{27^2}$ as a power of 3. [2]

$3^4 \times \frac{1}{(3^3)^2} = \frac{3^4}{3^6} = \frac{1}{3^2} = \frac{1}{9}$

*(c)* Simplify $\frac{(5ab^4)^3}{a^2}$. [3]

$\frac{5^3 a^3 b^{12}}{a^2} = \frac{5^3 ab^{12}}{1} = 5^3 ab^{12} = 125ab^{12}$

**5.** The functions $f(x)$ and $g(x)$ are defined for $x > 0$ by

$$f(x) = \frac{8}{x},$$

$$g(x) = x + 5.$$

*(a)* Find and simplify an expression for $ff(x)$. [2]

$ff(x) = \frac{8}{8/x} = \frac{8x}{8} = x.$

*(b)* Using your answer to part *(a)*, or otherwise, explain the relationship between $f(x)$ and $f^{-1}(x)$. [1]

$y = \frac{8}{x} \rightarrow x = \frac{8}{y} \therefore f^{-1}(x) = \frac{8}{x}.$

The function is its own inverse.

$f(x) = f^{-1}(x)$

*(c)* Solve $g^{-1}f(x) = 11$. [4]

$g(x) = y = x + 5$

$y - 5 = x$

$x - 5 = y = g^{-1}(x)$

$g^{-1}f(x) = \frac{8}{x} - 5. = 11$

$\frac{8}{x} = 16$

$\frac{8}{16} = x = \frac{1}{2}$

**6.**

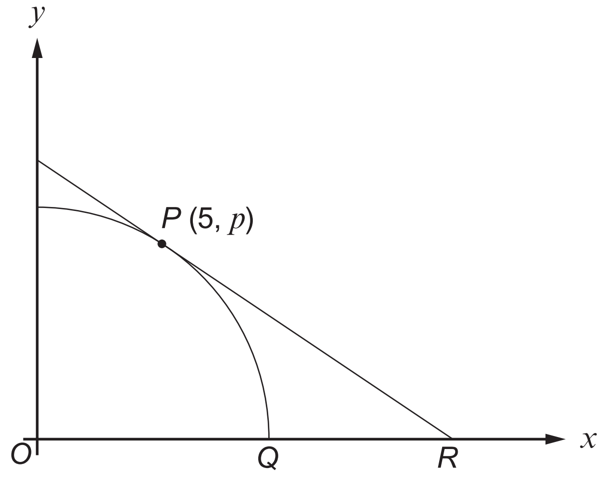



*Diagram not drawn to scale*

The diagram shows an arc of a circle with centre *O* and equation $x^2 + y^2 = 50$.
*P* $(5, p)$ lies on the circle.
The arc meets the $x$-axis at *Q*.
The tangent to the circle at *P* meets the $x$-axis at *R*.

(a) Find the value of $p$, the $y$-coordinate of *P*. [1]

$x^2 + y^2 = 50$ $\therefore p = 5$

$5^2 + y^2 = 50$ $P\ (5, 5)$

$y^2 = 25, \quad y = 5$

(b) Show that the equation of the tangent to the circle at *P* is $y = mx + 10$, where $m$ is a constant. [4]

$x^2 + y^2 = 50$

gradient of radius $= \frac{5}{5} = 1$, gradient of tangent $= -1$

$y = -x + c$.

$P \Rightarrow$ $5 = -5 + c$

$10 = c$

$y = -x + 10, \quad m = -1$

(c) Find the exact length of *QR*. [2]

Find the exact length of QR

@R, $0 = -x + 10$ $QR = 10 - 5\sqrt{2}$

$x_R = 10$

@Q, $x^2 + 0^2 = 50$ exact form

$x^2 = 50$

$x = \sqrt{50} = 5\sqrt{2}$