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GCSE MATHEMATICS

S21-C300

With Calculator Assessment Resource L

Higher Tier

Formula list

Area and volume formulae

Where r is the radius of the sphere or cone, l is the slant height of a cone and h is the perpendicular height of a cone:

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Kinematics formulae

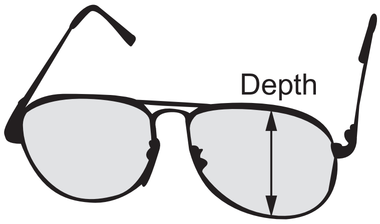
Where a is constant acceleration, u is initial velocity, v is final velocity, s is displacement from the position when $t = 0$ and t is time taken:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

1. Marie works for an optician.
She records the depth of a lens in each of the 100 pairs of glasses on display.



Her results are summarised in the table.

Depth of lens, x mm, to the nearest mm	(x) mid point	(f) Number of pairs of glasses	(f x)
$10 \leqslant x < 20$	15	5	75
$20 \leqslant x < 30$	25	20	500
$30 \leqslant x < 40$	35	23	805
$40 \leqslant x < 50$	45	52	2340
		total = 100	total = 3720

(a) (i) Calculate an estimate for the mean depth of a lens. [4]

$\bar{x} = \frac{3720}{100} = 37.20$

mean depth of a lens is 37.2 mm

(ii) In which group does the median lie? [1]

median is 50 glasses so in $40 \leq x < 50$

(b) In the display of 100 pairs of glasses at *Davy's Opticians*, the mean depth of a lens is exactly the same as Marie's opticians.

Marie says,

"Considering only the mean depth of a lens, our display is **certain** to be very similar to the display in *Davy's Opticians*."

Explain why Marie is incorrect. [1]

As Davy's opticians could have higher depth of lens classes overall which could cause the same mean depth but different displays

2. *(a)* Expand and simplify $(x + 6y)(3x + 5y)$. [3]

$(x + 6y)(3x + 5y)$

$3x^2 + 5xy + 18xy + 30y^2$

$= 3x^2 + 23xy + 30y^2$

(b) Factorise $x^2 - 13x + 36$. [2]

$(x - 9)(x - 4)$

36
-4 -9
-13

(c) Solve $w^2 + 7w - 18 = 0$. [3]

$\left(w + \frac{7}{2}\right)^2 - 12.25 - 18 = 0$

$\left(w + \frac{7}{2}\right)^2 = 30.25$

$\left(w + \frac{7}{2}\right) = 5.5$

$w = 2.$

$w + 7/2 = -5.5$

$w = -9$

or

$w = \frac{-7 \pm \sqrt{49 - 4 \times -18}}{2}$

$= \frac{-7 \pm 11}{2} = 2$ or -9

(d) Factorise $y^2 - 121$. [1]

$(y + 11)(y - 11)$

(e) You are given that:

- $y = x^2 + bx + c$
- $y = 16$ when $x = 0$
- $y = 0$ when $x = -2$

Find the values of b and c. [4]

$y = x^2 + bx + c$

$\boxed{16 = c}$

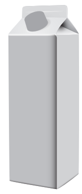
$y = x^2 + bx + 16$

$0 = 4 - 2b + 16$

$2b = 20$

$\boxed{b = 10}$

3. 7 cartons of apple juice and 2 cartons of grapefruit juice cost £6.15 altogether.
5 cartons of apple juice and 8 cartons of grapefruit juice cost £9.19 altogether.



Use an algebraic method to calculate the **total** cost of 2 cartons of apple juice and 5 cartons of grapefruit juice.

[5]

$7a + 2g = 6.15$

$5a + 8g = 9.19$

Calculate $2a + 5g = ?$

$$\begin{array}{rl} 28a + 8g &= 24.6 \\ -\; 5a + 8g &= 9.19 \\ \hline 23a &= 15.41 \\ a &= 0.67 \end{array}$$

$7(0.67) + 2g = 6.15$

$2g = 1.46$

$g = 0.73$

$\therefore \; 2a + 5g = 2(0.67) + 5(0.73)$

$= 1.84 + 3.65 = £4.99$

Total cost of 2 cartons of apple juice and 5 cartons of grapefruit juice is £ 4.99

4. Find the nth term of the following sequence. [2]

$a =$ −7, −4, 1, 8, 17, ...

$d =$ +3 +5 +7 +11

2 2 2 $= c$

$a + (n-1)d + \frac{1}{2}(n-1)(n-2)c$

$-7 + (n-1)3 + \frac{1}{2}(n-1)(n-2)2$

$-7 + \cancel{3n} - 3 + n^2 - \cancel{2n} - n + 2$

$= n^2 - 8$

$a =$ first term

$d =$ difference between first two numbers

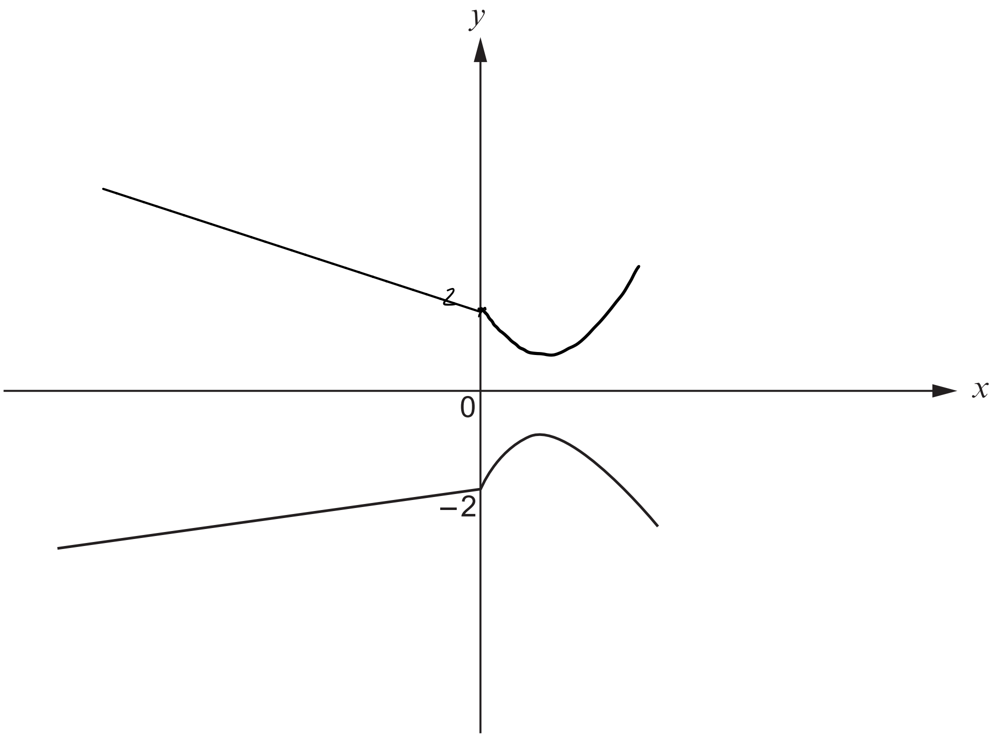
$c =$ second difference

or

$$\begin{array}{c|ccccc|c}
 & 1 & 2 & 3 & 4 \ldots & n & \\
\frac{2n^2}{2} & 1 & 4 & 9 & 16 & n^2 & n^2 \\
-8 & -7 & -4 & 1 & 8 & n^2-8 & -8
\end{array}$$

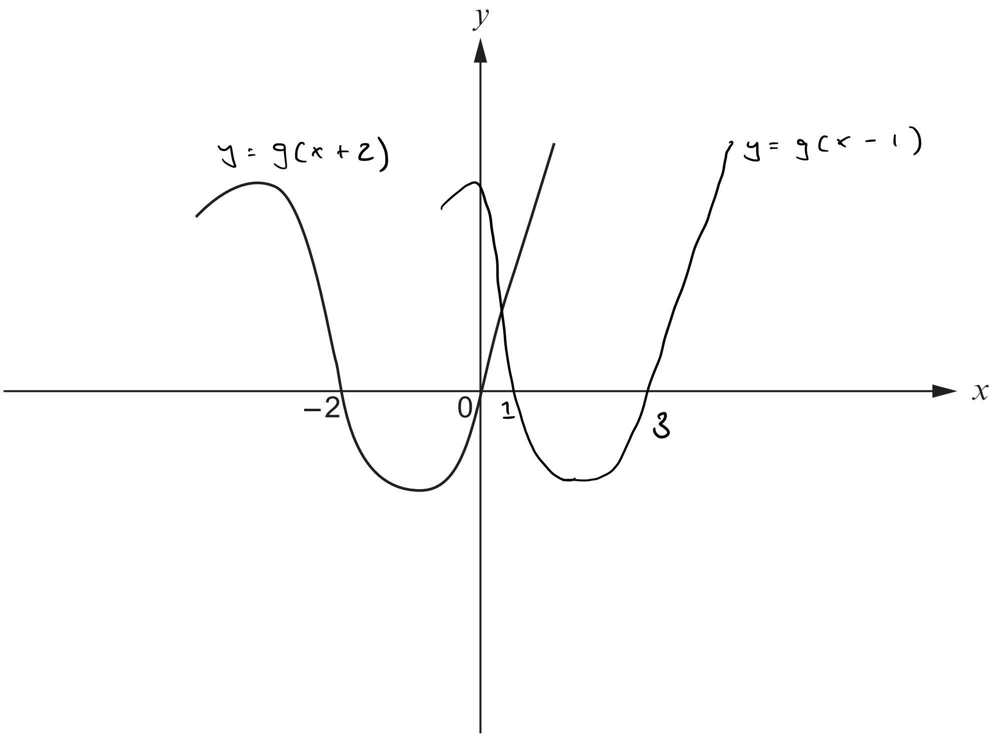
5. *(a)* The diagram shows a sketch of $y = f(x)$.

On the same diagram, sketch the curve $y = -f(x)$.
Mark clearly the coordinates of any point where this curve crosses an axis. [2]

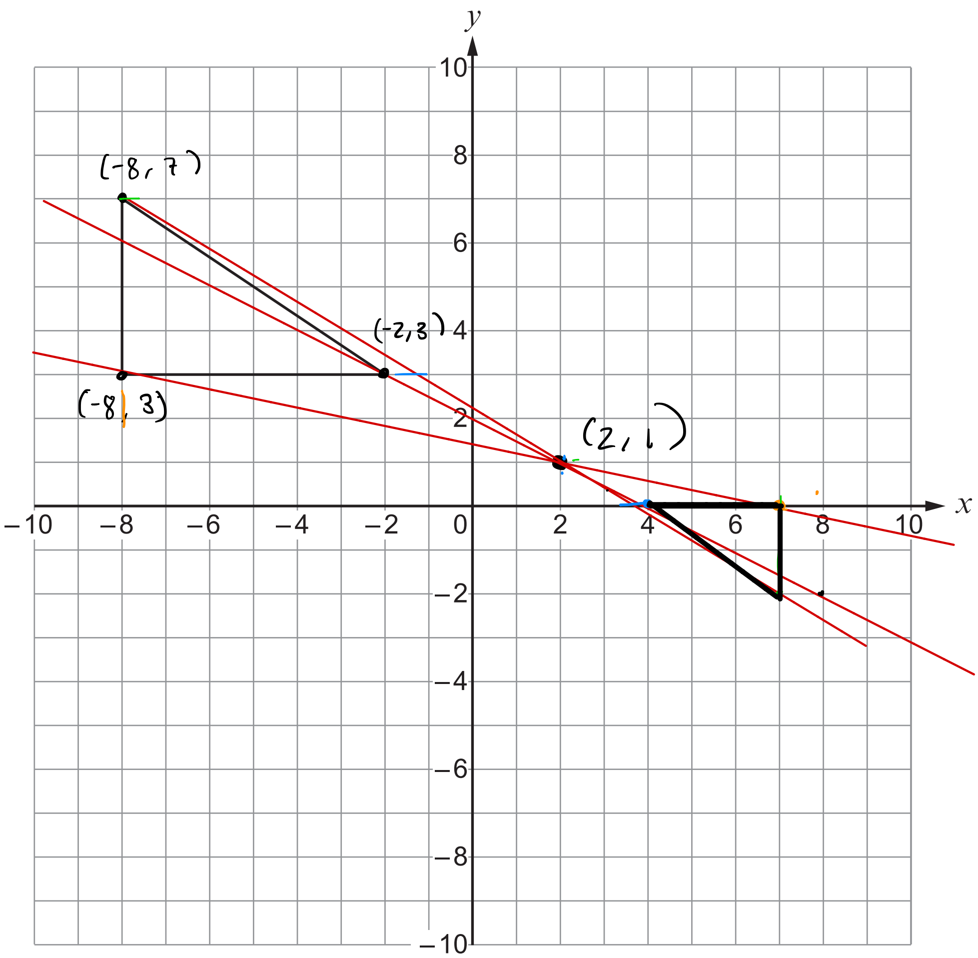


(b) The diagram shows a sketch of $y = g(x + 2)$.

On the same diagram, sketch the curve $y = g(x - 1)$.
Mark clearly the coordinates of the points where this curve crosses the x-axis. [3]



(c) Enlarge the triangle, shown on the grid below, by a scale factor of $-\frac{1}{2}$ with (2, 1) as the centre of the enlargement. [2]



6. A cone has a radius x cm, a perpendicular height $(x + 2)$ cm and a slant height 16·4 cm.

$x^2 + (x+2)^2 = 16.4^2$

$x^2 + x^2 + 4x + 4 = 16.4^2$

$2x^2 + 4x - 264.96 = 0$

$\downarrow \div 2$

$x^2 + 2x - 132.48 = 0$

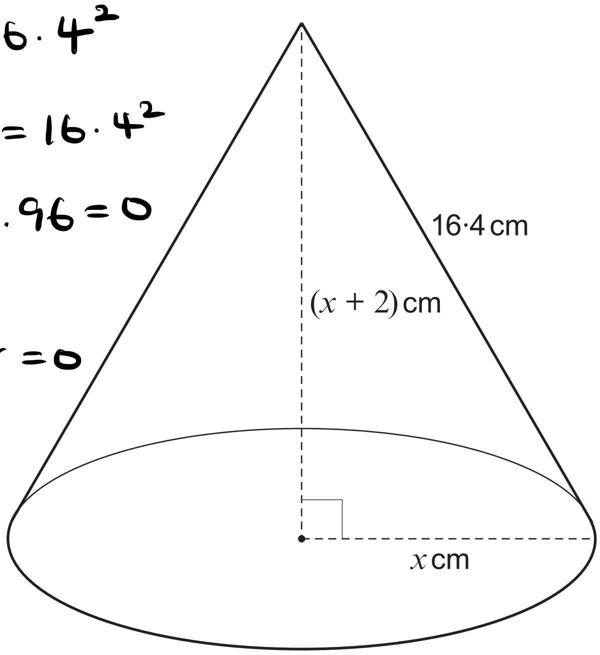


Diagram not drawn to scale

Show that x satisfies the equation $x^2 + 2x - 132{\cdot}48 = 0$ and calculate the volume of the cone. You must show all your working. [7]

Pythagoras

$\sqrt{x^2 + (x+2)^2} = 16.4 \leftarrow \left(\sqrt{a^2 + b^2} = c\right)$

or

$x^2 + x^2 + 4x + 4 = 268.96$

$2x^2 + 4x + 4 = 268.96$

$2x^2 + 4x - 264.96 = 0 \quad (\div 2)$

$x^2 + 2x - 132.48 = 0$

$\longrightarrow x = 10.55335449$ or $x = -12.553\ldots$

$\therefore r = 10.553\ldots$ cm

not possible

Volume of a cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times (10.553\ldots)^2 \times (10.553\ldots + 2)$

$= 1464.095695 \approx \boxed{1460 \text{ cm}^3}$ (3sf)