GCSE MATHEMATICS

S21-C300

With Calculator Assessment Resource K

Higher Tier

Formula list

Area and volume formulae

Where r is the radius of the sphere or cone, l is the slant height of a cone and h is the perpendicular height of a cone:

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Kinematics formulae

Where a is constant acceleration, u is initial velocity, v is final velocity, s is displacement from the position when $t = 0$ and t is time taken:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

1.

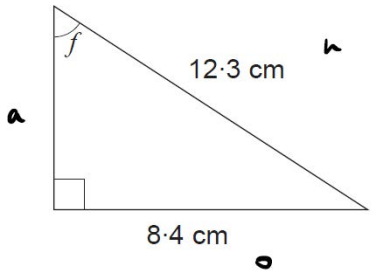


Diagram not drawn to scale

Calculate the size of angle f. [3]

$\sin f = \frac{o}{h}$

$= \frac{8.4}{12.3}$

$= 0.6829...$

$f = 43.07°$

2. *(a)* The volume of a sphere with a radius of 2·7 cm is equal to the volume of a cuboid. The base of the cuboid has an area of 14·2 cm^2.

Calculate the height of the cuboid. [4]

$r = 2.7\,cm$

$82.447\,cm^3$

Volume of a sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 2.7^3$

Volume of cuboid $= (w \times l) \times h = 82.447$

(base area)

$82.447 = 14.2 \times h$

$h = 82.447 \div 14.2 = 5.806\,cm$

$\therefore h = 5.806 \approx 5.8$ (1dp) cm

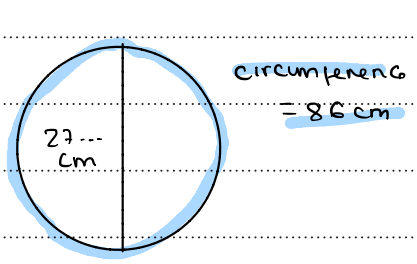
(b) A piece of paper is in the shape of a circle.
The circumference of the circle is 86 cm.

The paper is cut into 2 semi-circles.
Calculate the **perimeter** of one of the semi-circles.

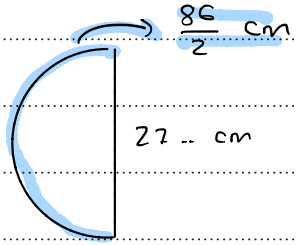
Give your answer correct to the nearest $\frac{1}{10}$ cm. [5]

$\pi d = 86\,cm$

$d = 27.374650\,21\,cm$



circumference = 86 cm



$\frac{86}{2}$ cm

Perimeter $= 27.37... + \frac{86}{2} = 70.3746...\,cm$

$\approx 70.4\,cm$

(to the nearest $\frac{1}{10}$ cm)

3. In scientific reports, temperatures are often given using more than one temperature scale.

Celsius, Fahrenheit and kelvin are all measured on linear scales.

Use the information given below to complete the tables.

(a) [1]

Degrees Celsius	Degrees Fahrenheit
30	86
40	104
50	122
60	140

Celsius to Fahrenheit

$= \left(x^{\circ} \times \frac{9}{5} \right) + 32$

or 1.8

(b) [2]

Kelvin	Degrees Celsius
0	−273.15
100	−173.15
200	−73.15
300	26·85
400	126·85

$400 - 126.85 = 273.15$

$300 - 26.85 = 273.15$

$\therefore 0^{\circ} = 273.15\,K$

(c) [5]

Kelvin	Degrees Celsius	Degrees Fahrenheit
320	46.85	116.33

$320 - 273.15 = 46.85^{\circ}$

$\left(46.85^{\circ} \times \frac{9}{5} \right) + 32 = 116.33^{F}$

4. Ben draws an irregular pentagon.
The interior angles of the pentagon he has drawn are all less than 180°.

Ben attempts to express the interior angles of his pentagon using algebra.
His expressions are

$$x°, \quad (x + 40)°, \quad (2x - 30)°, \quad 3(x - 40)° \quad \text{and} \quad 3x°.$$

Show that Ben is incorrect. [6]

$(5-2) \times 180 = 540^\circ$ = sum of interior angles

$(x) + (x+40) + (2x-30) + 3(x-40) + 3x$

$= x + x + 40 + 2x - 30 + 3x - 120 + 3x$

$= 10x - 110$

$10x - 110 = 540$	one angle is $3x$
$10x = 680$	$3(65) = 195^\circ$
$x = 65$	$195^\circ > 180^\circ$

Ben is incorrect because if $x = 65^\circ$ from his expressions not all the internal angles are smaller than 180°.

5. The diagram below shows a plan view of a stage, *ABCD*.
ABC is a sector of a circle, with centre *C*.

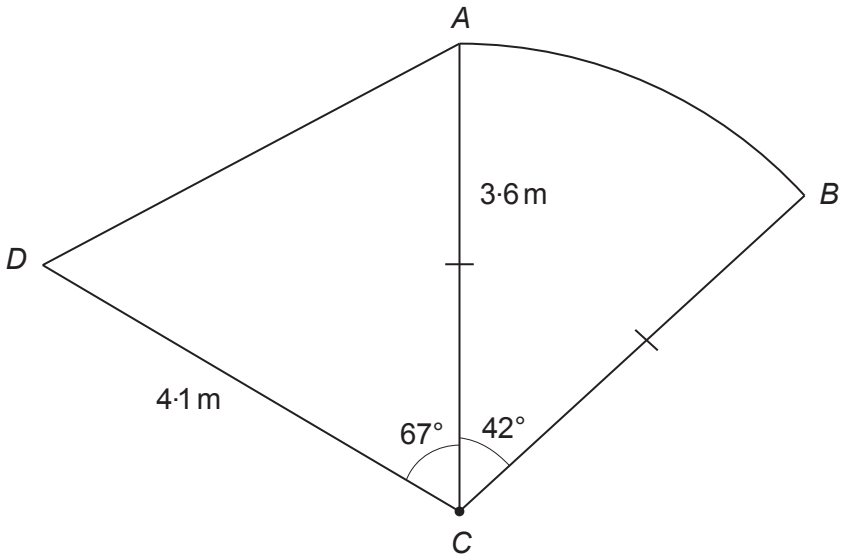


Diagram not drawn to scale

(a) The band *Fredalive* need a stage area of at least 11·5 m^2 to set up equipment and perform.

Is this stage suitable for *Fredalive* to set up equipment and perform?
You must show all your working. [5]

sector ABC area $= \pi \times 3.6^2 \times \frac{42}{360} = 4.75\ m^2$

$\triangle ACD$ area $= \frac{1}{2} \times 4.1 \times 3.6 \times \sin 67$

$= 6.793\ m^2$

Total area $= 4.75 + 6.793 = 11.543$

$11.543 > 11.5$

thus the stage is suitable for Fredalive

Conclusion
The stage is suitable:

Yes ☑ No ☐

(b) *Fredalive* want to place a banner around the perimeter of the stage.

Will a banner of length 14 m fit around this stage without leaving a gap?
You must show all your working. [6]

sector : $3.6 + 2 \times \pi \times 3.6 \times \frac{42}{360} = 4.333\text{m}$

triangle : $4.1 + \overline{AD}$

$c^2 = a^2 + b^2 - 2ab\cos C$

$\overline{AD}^2 = 4.1^2 + 3.6^2 - 2 \times 4.1 \times 3.6 \times \cos 67$

$= 18.236$

$\overline{AD} = 4.27\text{m}$

$4.1 + 4.27 = 8.37\text{m}$

Total perimeter : $4.333 + 8.37 = 12.703\text{m}$

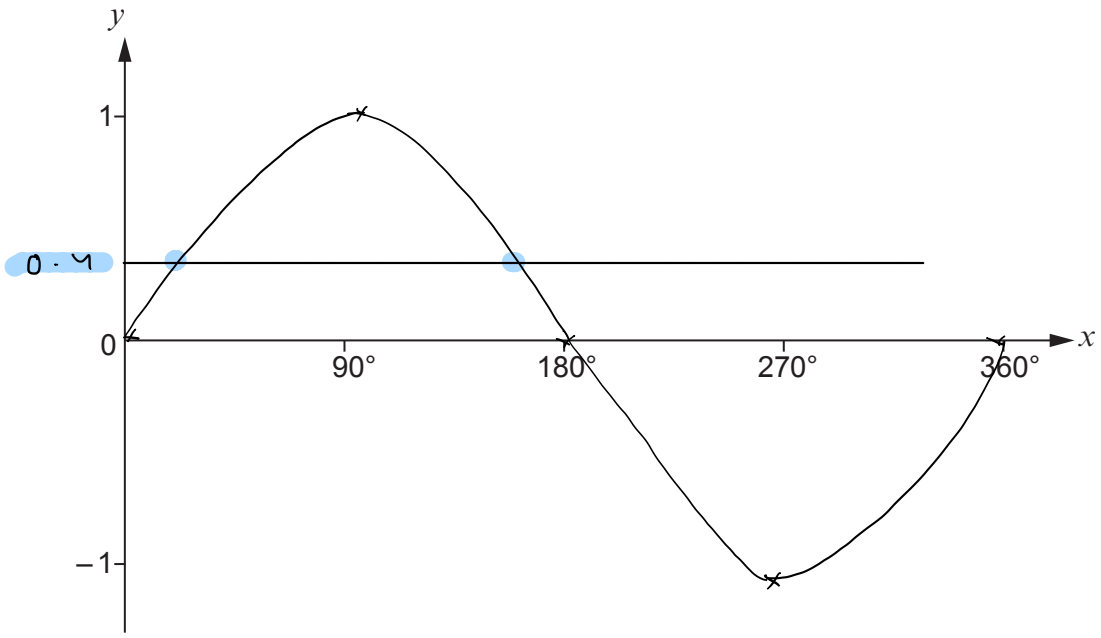
$12.7\text{m} < 14\text{m}$

Thus 14m banner does not fit around the stage.

Conclusion
The banner will fit without leaving a gap:

Yes ☐ No ☑

6. *(a)* Using the axes below, sketch a graph of $y = \sin x$ for values of x from 0° to 360°. [1]



(b) Find all the solutions of the following equation in the range 0° to 360°. [2]

$$5\sin x = 2$$

$5 \sin x = 2$

$\sin x = \frac{2}{5} = 0.4$ (2 solutions from graph)

$x_1 = \sin^{-1} 0.4$

$x_1 = 23.578178$

$x_2 = 180 - x_1 = 156.421821$

$\therefore x = 23.6°, 156.4°$