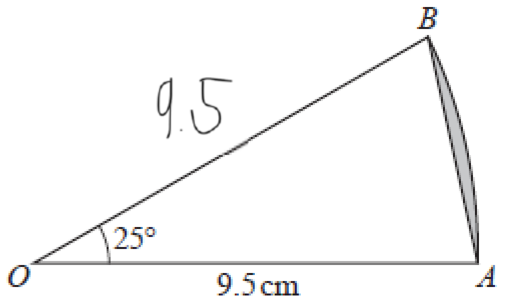
Oxford Cambridge and RSA

AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

Question Set 5

1



The diagram shows a sector AOB of a circle with centre O and radius 9.5 cm. The angle AOB is 25°.

(a) Calculate the length of the straight line AB. **[2]**

$AB^2 = 9.5^2 + 9.5^2 - 2 \times 9.5 \times 9.5 \cos 25$

$AB = 4.11$

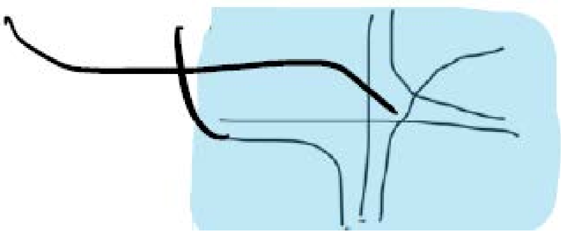
(b) Find the area of the segment shaded in the diagram. **[3]**

$\frac{25}{360} \times \pi \times 9.5^2 - 0.5 \times 9.5^2 \sin 25$

$= 0.619$

2 Two curves have equations $y = \ln x$ and $y = \frac{k}{x}$, where k is a positive constant.

(a) Sketch the curves on a **single** diagram. **[3]**



(b) Explain how your diagram shows that the equation $x \ln x - k = 0$ has exactly one real root. **[2]**

$\frac{dy}{dx} = 2x^{-\frac{1}{2}} - 3$

$\frac{dy}{dx}$ when $x = 4$ $\frac{dy}{dx} = -2$

1 point of intersection

3 **In this question you must show detailed reasoning.**

Find the equation of the normal to the curve $y = 4\sqrt{x} - 3x + 1$ at the point on the curve where $x = 4$. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[7]**

$y = -3$

gradient of tangent $= -2$

g of normal $= +\frac{1}{2}$ $\quad y = +\frac{x}{2} + c \quad -3 = +2 + c \quad c = -5 \quad y = -\frac{1}{2}x - 5 \quad 2y = -x - 10 \quad x + 2y + 10 = 0$

4 **In this question you must show detailed reasoning.**

The cubic polynomial $6x^3 + kx^2 + 57x - 20$ is denoted by $f(x)$. It is given that $(2x-1)$ is a factor of $f(x)$.

(a) Use the factor theorem to show that $k = -37$. **[2]**

$f(\frac{1}{2}) = 0.75 + \frac{k}{4} + \frac{57}{2} - 20 = 0 \quad 0.25k = \frac{37}{4} \quad k = 37$

(b) Using this value of k, factorise $f(x)$ completely. **[3]**

$(2x-1)(3x^2 - 17 + 20)$

$= (2x-1)(x-4)(3x-5)$

(c) **(i)** Hence find the three values of t satisfying the equation $6e^{-3t} - 37e^{-2t} + 57e^{-t} - 20 = 0$. **[2]**

$(2e^{-t} - 1)(e^{-t} - 4)(3e^{-t} - 5)$

$e^{-t} = \frac{1}{2} \quad t = \ln 2 \qquad e^{-t} = 4 \quad t = \ln 0.25 \qquad e^{-t} = \frac{5}{3} \quad t = \ln 0.6$

(ii) Express the sum of the three values found in part (c)(i) as a single logarithm. **[2]**

$\ln 2 + \ln 0.25 + \ln 0.6$

$= \ln 0.3$

5 A curve has equation $y = a(x+b)^2 + c$, where a, b and c are constants. The curve has a stationary point at $(-3, 2)$.

(a) State the values of b and c. **[2]**

$b = 3 \quad c = 2$

When the curve is translated by $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$ the transformed curve passes through the point $(3, -18)$.

(b) Determine the value of a. **[3]**

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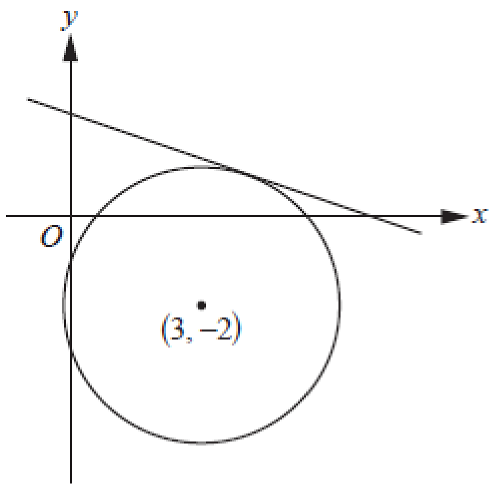
$y = a(x-4+3)+2$
$-18 = a(2)+2$ $a = -10$

6 **In this question you must show detailed reasoning.**



The diagram shows the line $3y + x = 7$ which is a tangent to a circle with centre $(3, -2)$.

Find an equation for the circle. [6]

$(x-3)^2 + (y+2)^2 = r^2$

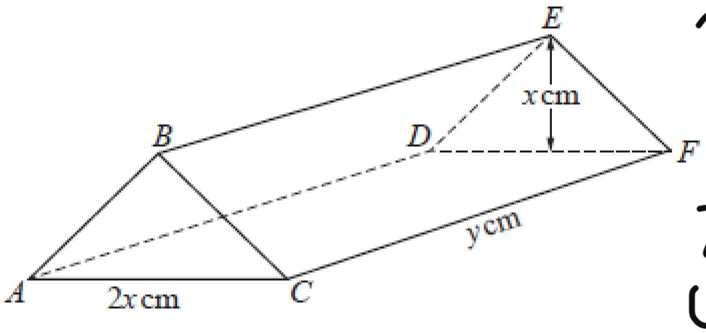
$y = \frac{-x}{3} + \frac{7}{3}$

normal ⇒ $y = 3x + c$
$-2 = 9 + c$
$c = -11$
$y = 3x - 11$

$3x - 11 = -\frac{x}{3} + \frac{7}{3}$
$x = 4$ $y = 1$

$(4-3)^2 + (1+2)^2 = r^2$
$r^2 = 10$
$(x-3)^2 + (y-2)^2 = (\sqrt{10})^2$

7



$2\times0.5\times2x\times x=2x^2$
$3\times y\times 2x=6xy$
$2x^2+6xy=600$
$y=\frac{100}{x}-\frac{x}{3}$

The diagram shows a model for the roof of a toy building. The roof is in the form of a solid triangular prism $ABCDEF$. The base $ACFD$ of the roof is a horizontal rectangle, and the cross-section ABC of the roof is an isosceles triangle with $AB = BC$.

The lengths of AC and CF are $2x$ cm and y cm respectively, and the height of BE above the base of the roof is x cm. $V=0.5\times2x\times y\times x$ $V=x^2y$

The total surface area of the five faces of the roof is 600 cm^2 and the volume of the roof is $V\text{ cm}^3$.

(a) Show that $V = kx(300 - x^2)$, where $k = \sqrt{a} + b$ and a and b are integers to be determined. [6]

$V=x^2\left(\frac{100}{x}-\frac{x}{3}\right)$ $V=\frac{x}{3}(300-x^2)$ $a=0$ $b=\frac{1}{3}$

(b) Use differentiation to determine the value of x for which the volume of the roof is a maximum. [4]

$\frac{dV}{dx}=100-x^2=0$ $x=10$

(c) Find the maximum volume of the roof. Give your answer in cm^3, correct to the nearest integer. [1]

$V=\frac{2000}{3}\text{ cm}^3$

(d) Explain why, for this roof, x must be less than a certain value, which you should state. [2]

Width would be greater than Length

Total Marks for Question Set 5: 50