OCR
Oxford Cambridge and RSA

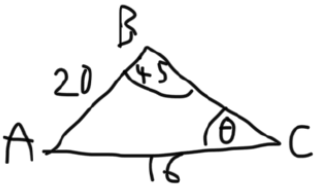
AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

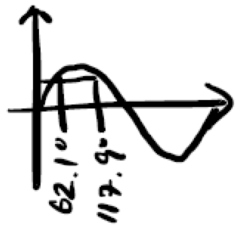
Question Set 1

1 In triangle ABC, $AB = 20$ cm and angle $B = 45°$.

(a) Given that $AC = 16$ cm, find the two possible values for angle C, correct to 1 decimal place. **[4]**



$$\frac{\sin\theta}{20} = \frac{\sin 45}{16}$$



$\sin\theta = \frac{5\sqrt{2}}{8}$ $\theta = 62.1°, 117.9°$

(b) Given instead that the area of the triangle is $75\sqrt{2}$ cm^2, find BC. **[2]**

$0.5 \times BC \times 20\sin 45 = 75\sqrt{2}$

$BC = 15$

2 **(a)** The curve $y = \frac{2}{3+x}$ is translated by four units in the positive x-direction. State the equation of the curve after it has been translated. **[2]**

$y = \frac{2}{3+(x-4)} = \frac{2}{x-1}$ $\quad y = \frac{2}{x-1}$

(b) Describe fully the single transformation that transforms the curve $y = \frac{2}{3+x}$ to $y = \frac{5}{3+x}$. **[2]**

Stretch factor $\frac{5}{2}$ parallel to the y axis

3 In each of the following cases choose one of the statements

a $P \Rightarrow Q$ b $P \Leftarrow Q$ c $P \Leftrightarrow Q$

to describe the relationship between P and Q

(a) P: $y = 3x^5 - 4x^2 + 12x$

Q: $\frac{dy}{dx} = 15x^4 - 8x + 12$ **[1]**

(b) P: $x^5 - 32 = 0$ where x is real Q: $x = 2$ **[1]**

(c) P: $\ln y < 0$
Q: $y < 1$ **[1]**

4 **(a)** Express $4x^2 - 12x + 11$ in the form $a(x+b)^2 + c$. **[3]**

$4(x - \frac{3}{2})^2 + 2$

(b) State the number of real roots of the equation $4x^2 - 12x + 11 = 0$. **[1]**

0 (min is 2)

(c) Explain fully how the value of r is related to the number of real roots of the equation $p(x+q)^2 + r = 0$ where p, q and r are real constants and $p > 0$. **[2]**

if $r > 0$ then no roots $r < 0$, 2 roots
if $r = 0$ 1 root

5 **In this question you must show detailed reasoning.**

The line $x + 5y = k$ is a tangent to the curve $x^2 - 4y = 10$. Find the value of the constant k. **[5]**

$y = -\frac{x}{5} + \frac{k}{5}$

$y = \frac{x^2}{4} - \frac{5}{2}$

$\frac{dy}{dx} = \frac{x}{2}$

$-\frac{1}{5} = \frac{x}{2}$

$x = -\frac{2}{5} \therefore y = -\frac{123}{50}$

$-\frac{123}{50} = \frac{2}{25} + \frac{k}{5}$ $k = -\frac{127}{10}$

6 A pan of water is heated until it reaches 100 °C. Once the water reaches 100 °C, the heat is switched off and the temperature T °C of the water decreases. The temperature of the water is modelled by the equation

$$T = 25 + ae^{-kt},$$

where t denotes the time, in minutes, after the heat is switched off and a and k are positive constants.

(a) Write down the value of a. **[1]**

75

(b) Explain what the value of 25 represents in the equation $T = 25 + ae^{-kt}$. **[1]**

Room temperature

When the heat is switched off, the initial rate of decrease of the temperature of the water is 15 °C per minute.

(c) Calculate the value of k. **[3]**

$\frac{dT}{dt} = -75ke^{-kt}$ at $t=0$ $-15 = -75k$

$k = 0.2$

(d) Find the time taken for the temperature of the water to drop from 100 °C to 45 °C. **[3]**

$45 = 25 + 75e^{-0.2t}$ $\frac{20}{75} = e^{-0.2t}$ $\ln\frac{20}{75} = -0.2t$ $t = 6.61 =$ 6min 37s

(e) A second pan of water is heated, but the heat is turned off when the water is at a temperature of less than 100 °C. Suggest how the equation for the temperature as the water cools would be modified by this. **[1]**

decrease the value of a

7 **(a)** Show that the equation

$$2\sin x \tan x = \cos x + 5$$

can be expressed in the form

$$3\cos^2 x + 5\cos x - 2 = 0.$$ **[3]**

$2\sin x \frac{\sin}{\cos} = \cos x + 5$

$2\sin^2 x = \cos^2 x + 5\cos x$

$2(1 - \cos^2 x) = \cos^2 x + 5\cos x$

$3\cos^2 x + 5\cos x - 2 = 0$

(b) Hence solve the equation

$$2\sin 2\theta \tan 2\theta = \cos 2\theta + 5,$$

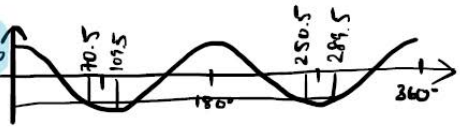
giving all values of θ between $0°$ and $180°$, correct to 1 decimal place. **[5]**

$3\cos^2 2\theta + 5\cos 2\theta - 2 = 0$

$(3\cos 2\theta - 1)(\cos 2\theta + 2)$

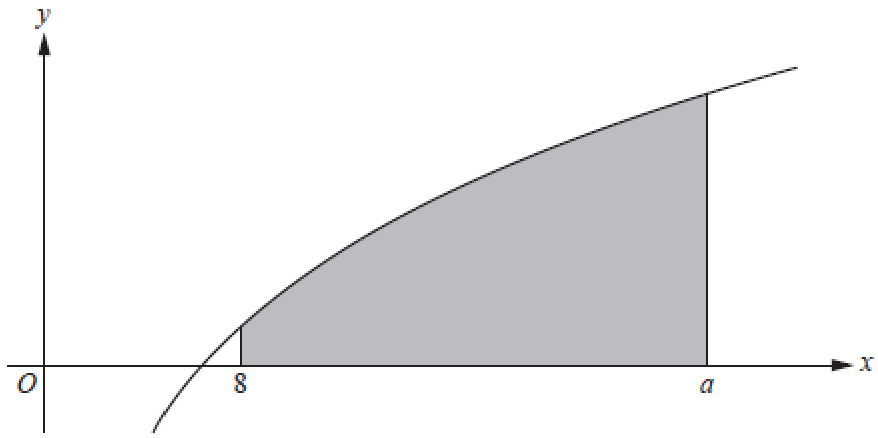
$\cos 2\theta = \frac{1}{3}$

$2\theta = 70.5, 109.5°, 250.5, 289.5$ $\theta = 35.3°, 54.8°, 125.3°, 144.8°$



8 **In this question you must show detailed reasoning.**

The diagram shows part of the graph of $y = 2x^{\frac{1}{3}} - \frac{7}{x^{\frac{1}{3}}}$. The shaded region is enclosed by the curve, the x-axis and the lines $x = 8$ and $x = a$, where $a > 8$.



Given that the area of the shaded region is 45 square units, find the value of a. **[9]**

$\int_8^a 2x^{\frac{1}{3}} - 7x^{-\frac{1}{3}} = 45$

$\left[\frac{3x^{\frac{4}{3}}}{2} - \frac{21}{2}x^{\frac{2}{3}}\right]_8^a = 45$

$\left(\frac{3a^{\frac{4}{3}}}{2} - \frac{21}{2}a^{\frac{2}{3}}\right) - (-18) = 45$

$(a^{\frac{4}{3}} - 7a^{\frac{2}{3}}) = (18)$

a needs to a whole number as equation = 18 ∴ needs to be integer when cube rooted

∴ $a = 27$

Total Marks for Question Set 1: 50