OCR
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Model Solutions

AS Level Mathematics B (MEI)

H630/02 Pure Mathematics and Statistics

Question Set 5

1 Solve the inequality $2x+5 < 6x-3$. [2]

$$2x+5 < 6x-3$$

$$\Rightarrow \quad 2x-6x < -3-5$$

$$\Rightarrow \quad -4x < -8$$

$$\Rightarrow \quad \underline{x > 2}$$

2 Use integration to show that the area bounded by the x-axis and the curve with equation $y=(x-1)^2(x-3)$ is $\frac{4}{3}$ square units. [6]

$A = \int y \, dx$ and $y = 0 \Rightarrow x = 1$ and $x = 3$ (our limits)

$$\Rightarrow \quad A = \int_1^3 (x-1)^2(x-3) \, dx = \int_1^3 x^3 - 5x^2 + 7x - 3 \, dx$$

$$\Rightarrow \quad A = \left[\frac{x^4}{4} - \frac{5x^3}{3} + \frac{7x^2}{2} - 3x\right]_1^3 = \left(-\frac{9}{4}\right) - \left(-\frac{11}{12}\right) = -\frac{4}{3}$$

Hence the area is $\left|-\frac{4}{3}\right| \Rightarrow \underline{A = \frac{4}{3}}$ Squares units as required.

3 **In this question you must show detailed reasoning.**

A circle has centre (2, – 1) and radius 5.

A straight line passes through the points (1, 1) and (9, 5).

Find the coordinates of the points of intersection of the line and the circle. [8]

Circle : $(2,-1)$ with $r=5 \Rightarrow (x-2)^2+(y+1)^2=25$ is the equation of our circle.

Line : $m = \frac{5-1}{9-1} = \frac{4}{8} = \frac{1}{2} \Rightarrow y-1 = \frac{1}{2}(x-1)$

$\Rightarrow y = \frac{x}{2} - \frac{1}{2} + 1$

$\Rightarrow y = \frac{1}{2}x + \frac{1}{2} = \frac{x+1}{2}$

Substitute this in !

$\Rightarrow (x-2)^2 + (y+1)^2 = 25$

$\Rightarrow (x-2)^2 + \left(\frac{1}{2}x+\frac{1}{2}+1\right)^2 = 25$

$\Rightarrow x^2-4x+4 + \frac{x^2+6x+9}{4} = 25$

$\Rightarrow 4x^2-16x+16+x^2+6x+9 = 100$

$\Rightarrow 5x^2-10x-75 = 0$

$\Rightarrow x^2-2x-15 = 0$

$\Rightarrow (x+3)(x-5) = 0$

$\Rightarrow x = -3$ or $x = 5$

Then, recalling that $y = \frac{x+1}{2}$, we substitute in our two x-values and we conclude that our points of intersection are :

$\underline{(-3,-1)}$ and $\underline{(5,3)}$

4 **In this question you must show detailed reasoning.**

Solve the equation $3\cos\theta + 8\tan\theta = 0$ for $0° < \theta < 360°$, giving your answers correct to the nearest degree. [6]

$$3\cos\theta + 8\tan\theta = 0$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$3\cos\theta + \frac{8\sin\theta}{\cos\theta} = 0$$

$$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \cos^2\theta = 1 - \sin^2\theta$$

$$3\cos^2\theta + 8\sin\theta = 0$$

$$3 - 3\sin^2\theta + 8\sin\theta = 0$$

$$\Rightarrow \quad 3\sin^2\theta - 8\sin\theta - 3 = 0.$$

let $\sin\theta = x$ then

$$3x^2 - 8x - 3 = 0$$

$$\Rightarrow \quad 3x^2 - 9x + x - 3 = 0$$

$$\Rightarrow \quad (3x+1)(x-3) = 0$$

$$\Rightarrow \quad x = -\tfrac{1}{3} \quad \text{and} \quad x = 3$$

$$\Rightarrow \quad \sin\theta = -\tfrac{1}{3} \quad \text{and} \quad \sin\theta = 3$$

$\sin\theta = 3 \Rightarrow$ No Solutions

$$\Rightarrow \quad \theta = \sin^{-1}\left(-\tfrac{1}{3}\right)$$

$$\Rightarrow \quad \theta = \underline{-19.47°}$$

$\Rightarrow$

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$$\Rightarrow \quad \theta = 180 - (-19.47) = 199.47 \quad \text{and} \quad \theta = 360 - 19.47 = 340.53$$

$$\Rightarrow \quad \theta = \underline{199°} \quad \text{and} \quad \theta = \underline{341°}$$

5 The equation of a curve is $y = 24\sqrt{x} - 8x^{\frac{3}{2}} + 16$.

(a) Find $\frac{dy}{dx}$. [3]

$y = 24\sqrt{x} - 8x^{3/2} + 16$

$\Rightarrow y = 24x^{1/2} - 8x^{3/2} + 16$

$\Rightarrow \frac{dy}{dx} = 12x^{-1/2} - 12x^{1/2}$

$\Rightarrow \frac{dy}{dx} = \underline{\frac{12}{\sqrt{x}} - 12\sqrt{x}}$

(b) Find the coordinates of the turning point. [3]

Turning Point when $\frac{dy}{dx} = 0$; recall that $\frac{dy}{dx} = \frac{12}{\sqrt{x}} - 12\sqrt{x} = 0$

$\Rightarrow 12 - 12\sqrt{x}\sqrt{x} = 0$

$\Rightarrow 12 - 12x = 0$

$\Rightarrow x = 1$, and recall that $y = 24\sqrt{x} - 8x^{3/2} + 16$

$\Rightarrow$ For $x = 1$, $y = 24\sqrt{1} - 8(1)^{3/2} + 16$

$\Rightarrow y = 24 - 8 + 16 = 32$

$\Rightarrow$ Turning Point is $(1, 32)$

(c) Determine the nature of the turning point. [2]

To determine the nature of the turning point we will find the second derivative ; $\frac{dy}{dx} = 12x^{-1/2} - 12x^{1/2} \Rightarrow \frac{d^2y}{dx^2} = -6x^{-3/2} - 6x^{-1/2}$

$\Rightarrow \frac{d^2y}{dx^2} = -\frac{6}{x^{3/2}} - \frac{6}{\sqrt{x}}$ then evaluating this at $x = 1$, $\left.\frac{d^2y}{dx^2}\right|_1 = -12 \leq 0$, hence $(1, 32)$ is a maximum turning point.

6

A car is travelling along a stretch of road at a steady speed of $11\,\mathrm{m\,s^{-1}}$.

The driver accelerates, and t seconds after starting to accelerate the speed of the car, V, is modelled by the formula

$V = A + B(1 - e^{-0.17t})$.

When $t = 3$, $V = 13.8$.

(a) Find the values of A and B, giving your answers correct to 2 significant figures. **[3]**

$V = A + B(1 - e^{-0.17t})$; we know that at $t = 0$, $V = 11\,\mathrm{ms^{-1}}$!

$\Rightarrow \quad 11 = A + B(1 - e^{0})$

$\Rightarrow \quad 11 = A + B(1 - 1) \quad \Rightarrow \quad \underline{A = 11}$

$\Rightarrow \quad V = 11 + B(1 - e^{-0.17t})$ with $t = 3\,\mathrm{s}$ and $V = 13.8\,\mathrm{ms^{-1}}$.

$\Rightarrow \quad 13.8 - 11 = B(1 - e^{-0.51})$

$\Rightarrow \quad B = \dfrac{2.8}{1 - e^{-0.51}} = 7.0086\ldots$

$\Rightarrow \quad \underline{A = 11}$ and $\underline{B = 7.0}$

When $t = 4$, $V = 14.5$ and when $t = 5$, $V = 14.9$.

(b) Determine whether the model is a good fit for these data. **[2]**

Using the model : $V = 11 + 7(1 - e^{-0.17t})$

$t = 4 \quad \Rightarrow \quad V = 11 + 7(1 - e^{-0.17 \times 4}) = 14.45\ldots = \underline{14.5}\,\mathrm{ms^{-1}}$

$t = 5 \quad \Rightarrow \quad V = 11 + 7(1 - e^{-0.17 \times 5}) = 15.008\ldots = \underline{15.0}\,\mathrm{ms^{-1}}$

The model is a good fit as the values match the expected values.
(we put the difference between 14.9 and 15.0 down to rounding errors).

(c) Determine the acceleration of the car according to the model when $t = 5$, giving your answer correct to 3 decimal places. [2]

Acceleration can be found by differentiating the velocity expression.

Recall; $V = 11 + 7\left(1 - e^{-0.17t}\right) = 18 - 7e^{-0.17t}$

$$a = \frac{dV}{dt} = -7 \times -0.17 \times e^{-0.17t} = \underline{1.19e^{-0.17t}}$$

Then at $t = 5$, $a = 1.19e^{-0.17 \times 5} = 0.5086...$

$$\Rightarrow a = \underline{\underline{0.509 ms^{-2}}}$$

The car continues to accelerate until it reaches its maximum speed.

The speed limit on this road is $60\,\text{kmh}^{-1}$. All drivers who exceed this speed limit are recorded by a speed camera and automatically fined £100.

(d) Determine whether, according to the model, the driver of this car is fined £100. [3]

As $t \to \infty$, what happens to V?

$$\lim_{t \to \infty} V = \lim_{t \to \infty} 18 - 7e^{-0.17t} = 18 ms^{-1}.$$

Therefore, the maximum speed will be $18ms^{-1}$ and we must now convert this to kmh^{-1}.

- 18m in km is $\frac{18}{1000} = 0.018$ km.
- 1s in hours is $\frac{1}{60 \times 60} = \frac{1}{3600} \Rightarrow V = \frac{0.018}{\frac{1}{3600}} = 64.8 kmh^{-1}$

$\Rightarrow 64.8 > 60$, hence the driver will be fined $\underline{\underline{\text{£}100}}$.

Total Marks for Question Set 5: 40 marks