

AS Level Mathematics B (MEI)

H630/02 Pure Mathematics and Statistics

Question Set 5

1

Solve the inequality $2x+5 < 6x-3$.

[2]

$$2x + 5 < 6x - 3$$

$$\Rightarrow 2x - 6x < -3 - 5$$

$$\Rightarrow -4x < -8$$

$$\Rightarrow \underline{x > 2}$$

2

Use integration to show that the area bounded by the x-axis and the curve with equation $y = (x-1)^2(x-3)$ is $\frac{4}{3}$ square units. [6]

$$A = \int y \, dx \quad \text{and} \quad y = 0 \Rightarrow x = 1 \quad \text{and} \quad x = 3 \quad (\text{our limits})$$

$$\Rightarrow A = \int_1^3 (x-1)^2(x-3) \, dx = \int_1^3 x^3 - 5x^2 + 7x - 3 \, dx$$

$$\Rightarrow A = \left[\frac{x^4}{4} - \frac{5x^3}{3} + \frac{7x^2}{2} - 3x \right]_1^3 = \left(-\frac{9}{4} \right) - \left(-\frac{11}{12} \right) = -\frac{4}{3}$$

Hence the area is $|- \frac{4}{3}| \Rightarrow A = \underline{\underline{\frac{4}{3}}}$ Squares units as required.

3

In this question you must show detailed reasoning.

A circle has centre $(2, -1)$ and radius 5.

A straight line passes through the points $(1, 1)$ and $(9, 5)$.

Find the coordinates of the points of intersection of the line and the circle.

[8]

Circle: $(2, -1)$ with $r = 5 \Rightarrow (x-2)^2 + (y+1)^2 = 25$ is the equation of our circle.

$$\begin{aligned} \text{Line: } m &= \frac{5-1}{9-1} = \frac{4}{8} = \frac{1}{2} \Rightarrow y-1 = \frac{1}{2}(x-1) \\ &\Rightarrow y = \frac{x}{2} - \frac{1}{2} + 1 \end{aligned}$$

$$\Rightarrow y = \frac{1}{2}x + \frac{1}{2} = \frac{x+1}{2}$$

△ Substitute this in!

$$\Rightarrow (x-2)^2 + (y+1)^2 = 25$$

$$\Rightarrow (x-2)^2 + \left(\frac{1}{2}x + \frac{1}{2} + 1\right)^2 = 25$$

$$\Rightarrow x^2 - 4x + 4 + \frac{x^2 + 6x + 9}{4} = 25$$

$$\Rightarrow 4x^2 - 16x + 16 + x^2 + 6x + 9 = 100$$

$$\Rightarrow 5x^2 - 10x - 75 = 0$$

$$\Rightarrow x^2 - 2x - 15 = 0$$

$$\Rightarrow (x+3)(x-5) = 0$$

$$\Rightarrow x = -3 \quad \text{or} \quad x = 5$$

Then, recalling that $y = \frac{x+1}{2}$, we substitute in our two x -values and we conclude that our points of intersection are:

$$\underline{(-3, -1)} \quad \text{and} \quad \underline{(5, 3)}$$

In this question you must show detailed reasoning.

Solve the equation $3 \cos \theta + 8 \tan \theta = 0$ for $0^\circ < \theta < 360^\circ$, giving your answers correct to the nearest degree. [6]

$$3 \cos \theta + 8 \tan \theta = 0$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$3 \cos \theta + \frac{8 \sin \theta}{\cos \theta} = 0$$

$$\sin^2 \theta + \cos^2 \theta = 1 \Rightarrow \cos^2 \theta = 1 - \sin^2 \theta$$

$$3 \cos^2 \theta + 8 \sin \theta = 0$$

$$3 - 3 \sin^2 \theta + 8 \sin \theta = 0$$

$$\Rightarrow 3 \sin^2 \theta - 8 \sin \theta - 3 = 0.$$

let $\sin \theta = x$ then

$$3x^2 - 8x - 3 = 0$$

$$\Rightarrow 3x^2 - 9x + x - 3 = 0$$

$$\Rightarrow (3x+1)(x-3) = 0$$

$$\Rightarrow x = -\frac{1}{3} \quad \text{and} \quad x = 3$$

$$\Rightarrow \sin \theta = -\frac{1}{3} \quad \text{and} \quad \sin \theta = 3$$

$$\Rightarrow \theta = \sin^{-1}\left(-\frac{1}{3}\right) \quad \begin{array}{c} \Downarrow \\ \text{No Solutions} \end{array}$$

$$\Rightarrow \theta = \underline{-19.47^\circ}$$

$$\Rightarrow \begin{array}{c|c} \checkmark S & \checkmark A \\ \hline \tau & C \end{array}$$

$$\Rightarrow \theta = 180 - (-19.47) = 199.47 \quad \text{and} \quad \theta = 360 - 19.47 = 340.53$$

$$\Rightarrow \theta = \underline{199^\circ} \quad \text{and} \quad \theta = \underline{341^\circ}$$

The equation of a curve is $y = 24\sqrt{x} - 8x^{3/2} + 16$.

(a)

Find $\frac{dy}{dx}$.

[3]

$$y = 24\sqrt{x} - 8x^{3/2} + 16$$

$$\Rightarrow y = 24x^{1/2} - 8x^{3/2} + 16$$

$$\Rightarrow \frac{dy}{dx} = 12x^{-1/2} - 12x^{1/2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{12}{\sqrt{x}} - 12\sqrt{x}$$

(b)

Find the coordinates of the turning point.

[3]

Turning Point when $\frac{dy}{dx} = 0$; recall that $\frac{dy}{dx} = \frac{12}{\sqrt{x}} - 12\sqrt{x} = 0$

$$\Rightarrow 12 - 12\sqrt{x}\sqrt{x} = 0$$

$$\Rightarrow 12 - 12x = 0$$

$$\Rightarrow x = 1, \text{ and recall that } y = 24\sqrt{x} - 8x^{3/2} + 16$$

$$\Rightarrow \text{For } x = 1, y = 24\sqrt{1} - 8(1)^{3/2} + 16$$

$$\Rightarrow y = 24 - 8 + 16 = 32$$

\Rightarrow Turning Point is $(1, 32)$

(c)

Determine the nature of the turning point.

[2]

To determine the nature of the turning point we will find

the second derivative; $\frac{dy}{dx} = 12x^{-1/2} - 12x^{1/2} \Rightarrow \frac{d^2y}{dx^2} = -6x^{-3/2} - 6x^{-1/2}$

$\Rightarrow \frac{d^2y}{dx^2} = -\frac{6}{x^{3/2}} - \frac{6}{\sqrt{x}}$ then evaluating this at $x = 1$, $\left. \frac{d^2y}{dx^2} \right|_{x=1} = -12 < 0$,

hence $(1, 32)$ is a maximum turning point.

A car is travelling along a stretch of road at a steady speed of 11 ms^{-1} .

The driver accelerates, and t seconds after starting to accelerate the speed of the car, V , is modelled by the formula

$$V = A + B(1 - e^{-0.17t}).$$

When $t = 3$, $V = 13.8$.

- (a) Find the values of A and B , giving your answers correct to 2 significant figures. [3]

$$V = A + B(1 - e^{-0.17t}) ; \text{ we know that at } t = 0, V = 11 \text{ ms}^{-1}$$

$$\Rightarrow 11 = A + B(1 - e^0)$$

$$\Rightarrow 11 = A + B(1 - 1) \Rightarrow \underline{\underline{A = 11}}$$

$$\Rightarrow V = 11 + B(1 - e^{-0.17t}) \text{ with } t = 3 \text{ s and } V = 13.8 \text{ ms}^{-1}$$

$$\Rightarrow 13.8 - 11 = B(1 - e^{-0.51})$$

$$\Rightarrow B = \frac{2.8}{1 - e^{-0.51}} = 7.0086\dots$$

$$\Rightarrow \underline{\underline{A = 11}} \text{ and } \underline{\underline{B = 7.0}}$$

When $t = 4$, $V = 14.5$ and when $t = 5$, $V = 14.9$.

- (b) Determine whether the model is a good fit for these data. [2]

$$\text{Using the model : } V = 11 + 7(1 - e^{-0.17t})$$

$$t = 4 \Rightarrow V = 11 + 7(1 - e^{-0.17 \times 4}) = 14.45\dots = \underline{\underline{14.5 \text{ ms}^{-1}}}$$

$$t = 5 \Rightarrow V = 11 + 7(1 - e^{-0.17 \times 5}) = 15.008\dots = \underline{\underline{15.0 \text{ ms}^{-1}}}$$

The model is a good fit as the values match the expected values.
(We put the difference between 14.9 and 15.0 down to rounding errors).

- (c) Determine the acceleration of the car according to the model when $t = 5$, giving your answer correct to 3 decimal places. [2]

Acceleration can be found by differentiating the velocity expression.

$$\text{Recall; } V = 11 + 7(1 - e^{-0.17t}) = 18 - 7e^{-0.17t}$$

$$a = \frac{dV}{dt} = -7 \times -0.17 \times e^{-0.17t} = \underline{\underline{1.19e^{-0.17t}}}$$

$$\text{Then at } t = 5, a = 1.19e^{-0.17 \times 5} = 0.5086\dots$$

$$\Rightarrow a = \underline{\underline{0.509 \text{ ms}^{-2}}}$$

The car continues to accelerate until it reaches its maximum speed.

The speed limit on this road is 60 kmh^{-1} . All drivers who exceed this speed limit are recorded by a speed camera and automatically fined £100.

- (d) Determine whether, according to the model, the driver of this car is fined £100. [3]

As $t \rightarrow \infty$, what happens to V ?

$$\lim_{t \rightarrow \infty} V = \lim_{t \rightarrow \infty} 18 - 7e^{-0.17t} = 18 \text{ ms}^{-1}$$

Therefore, the maximum speed will be 18 ms^{-1} and we must now convert this to kmh^{-1} .

• 18 m in km is $\frac{18}{1000} = 0.018 \text{ km}$.

• 1 s in hours is $\frac{1}{60 \times 60} = \frac{1}{3600} \Rightarrow V = \frac{0.018}{\frac{1}{3600}} = 64.8 \text{ kmh}^{-1}$

$\Rightarrow 64.8 > 60$, hence the driver will be fined £100.

Total Marks for Question Set 5: 40 marks

OCR

Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge