OCR
Oxford Cambridge and RSA

Model Solutions

AS Level Mathematics B (MEI)
H630/01 Pure Mathematics and Mechanics

Question Set 2

1. A particle is in equilibrium under the action of three forces in Newtons given by

$$\mathrm{F}_1 = \begin{pmatrix} 8 \\ 0 \end{pmatrix}, F_2 = \begin{pmatrix} 2a \\ -3a \end{pmatrix} \text{ and } \mathrm{F}_3 = \begin{pmatrix} 0 \\ b \end{pmatrix}.$$

Find the values of the constants a and b. **(3)**

Equal to 0 since in equilibrium.

$8 + 2a + 0 = 0$ and $0 - 3a + b = 0$

$a = \frac{-8}{2} = -4$

$-3(-4) = -b$

$12 = -b \Rightarrow b = -12$

2. Fig. 2 shows a block of mass $4m$kg and a particle of mass $4mkg$ connected by a light inextensible string passing over a smooth pulley. The block is on a horizontal table, and the particle hangs freely. The part of the string between the pulley and the block is horizontal. The block slides towards the pulley and the particle descends. In this motion, the friction force between the table and the block is $\frac{1}{2}mgN$.

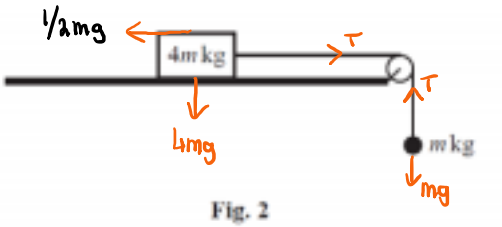


Fig. 2

Find expressions for

- the acceleration of the system
- the tension in the strong **(4)**

$mg - T = ma$

$T - \frac{1}{2}mg = 4ma$

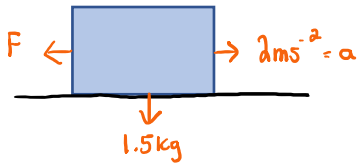
$\frac{1}{2}mg = 5ma \Rightarrow a = \frac{1}{10}g \text{ ms}^{-2}$ and $T = \frac{9}{10}mg\text{N}$

divide by m and rearrange for a

$T = 4ma + \frac{1}{2}mg = \frac{4m}{10}g + \frac{1}{2}mg = \frac{2}{5} + \frac{1}{2}(mg)$

$= \frac{9}{10}$

3. A toy boat of mass $1.5kg$ is pushed across a pond, starting from rest, for 2.5 seconds. During this time, the boat has an acceleration of $2ms^{-2}$. Subsequently, when the only horizontal force acting on the boat is a constant resistance to motion, the boat travels 10m before coming to rest. Calculate the magnitude of the resistance to motion. **(6)**



$S =$
$u = 0ms^{-2}$
$V =$
$a = 2ms^{-2}$
$t = 2.5s$

then $V = u + at$

$V = 2 \times 2.5 = 5ms^{-1}$

Then the boat decelerates from $5ms^{-1}$ to $0ms^{-1}$, over a 10m distance.

$\Rightarrow$ $S = 10m$
$u = 5ms^{-1}$
$V = 0ms^{-1}$
$a =$
$t =$

$\Rightarrow$ $V^2 = u^2 + 2as$

$0 = 25 + 20a$

$\Rightarrow a = -1.25ms^{-2}$ then $F = ma$

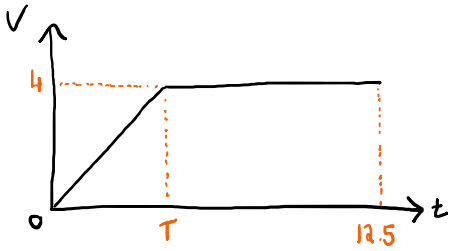
$F = 1.5 \times -1.25 = -1.875N$

$\Rightarrow$ Magnitude of force is $1.875N$.

4. Rory runs a distance of $45m$ in $12.5s$. He starts from rest and accelerates to a speed of $4ms^{-1}$. He runs the remaining distance at $4ms^{-1}$.

Rory proposes a model in which the acceleration is constant until time T seconds.

(i) Sketch the velocity-time graph for Rory's run using this model. **(2)**



We plot velocity against time.
Note the acceleration is constant until $t = T$ then the velocity is constant at $V = 4ms^{-1}$ until $t = 12.5$.

(ii) Calculate T. **(2)**

The total distance equals the area under the graph (45m).

$\Rightarrow 45 = \frac{4 \times T}{2} + (12.5 - T) \times 4$ $\Rightarrow 2T + 50 - 4T = 45$

$2T = 5$

$\Rightarrow T = 2.5s$

(iii) Find an expression for Rory's displacement at time ts for $0 \leq t \leq T$. **(2)**

$a = \frac{V-u}{t} = \frac{4}{2.5} = 1.6ms^{-1}$ then we can find velocity by integrating acceleration

This gives us $V = \int 1.6\,dt = 1.6t + C$ and $C = 0$ since at $t = 0$, $V = 0$, then we can integrate once more to go from velocity to displacement;

$S = \int 1.6t\,dt = \frac{1.6t^2}{2} + D = 0.8t^2 + D$ and $D = 0$ as displacement S at $t = 0$; $S = 0$.

$\Rightarrow$ $S = 0.8t^2$

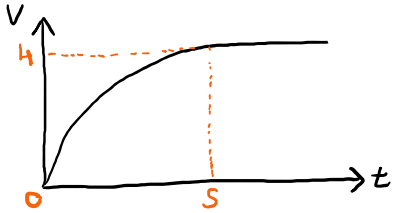
(iv) Use this model to find the time taken for Rory to run the first 4m. **(1)**

$S = 0.8t^2 \Rightarrow t = \sqrt{\frac{4}{0.8}} = \sqrt{5} = 2.24s$

Rory proposes a refined model in which the velocity during the acceleration phase is a quadratic function of t. The graph of Rory's quadratic goes through $(0,0)$ and has its maximum point at $(S, 4)$. In this model the acceleration phase lasts until time S seconds, after which the velocity is constant.

(v) Sketch a velocity-time graph that represents Rory's run using this refined model. **(1)**

This will be similar to our graph in (i) apart from the quadratic function implies that we will have acceleration shown by a curved line with decreasing gradient.



(vi) State, with a reason, whether S is greater than T or less than T. (You are not required to calculate the value of S.) **(1)**

S will be greater than T because the acceleration is no longer constant.

Total Marks for Question Set 2: 22