Additional Assessment Materials

Summer 2021

# Pearson Edexcel GCE in As Mathematics

# 8MA0_01 (Public release version)

## Resource Set 1: Topic 7

## Differentiation

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Additional Assessment Materials, Summer 2021

**General guidance to Additional Assessment Materials for use in 2021**

**Context**

- Additional Assessment Materials are being produced for GCSE, AS and A levels (with the exception of Art and Design).
- The Additional Assessment Materials presented in this booklet are an optional part of the range of evidence teachers may use when deciding on a candidate's grade.
- 2021 Additional Assessment Materials have been drawn from previous examination materials, namely past papers.
- Additional Assessment Materials have come from past papers both published (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow teachers to adapt them for use with candidate.

**Purpose**

- The purpose of this resource to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the mapping guidance which will map content and/or skills covered within each set of questions.
- These materials are only intended to support the summer 2021 series.

1.

The curve $C$ has equation

$$y = 2x^2 - 12x + 16$$

Find the gradient of the curve at the point $P$ (5, 6).

*(Solutions based entirely on graphical or numerical methods are not acceptable.)*

**(4)**

$y = 2x^2 - 12x + 16$

$\frac{dy}{dx} = 4x - 12$

$\rightarrow$ at $P(5,6)$ : $\frac{dy}{dx} = 4(5) - 12$

$= 20 - 12$

$= \underline{\underline{8}}$

**(Total for Question 1 is 4 marks)**

2.

A curve has equation

$$y = 2x^3 - 4x + 5$$

Find the equation of the tangent to the curve at the point $P(2, 13)$.

Write your answer in the form $y = mx + c$, where $m$ and $c$ are integers to be found.

**Solutions relying on calculator technology are not acceptable.**

**(5)**

$y = 2x^3 - 4x + 5$

$\frac{dy}{dx} = 6x^2 - 4$

$= 6(2)^2 - 4$

$= 20$

$20 =$ gradient of tangent

$\Rightarrow \quad y - 13 = 20(x - 2)$

$\Rightarrow \quad y - 13 = 20x - 40$

$\Rightarrow \quad \boxed{y = 20x - 27}$

**(Total for Question 2 is 5 marks)**

3.

A curve has equation

$$y = 2x^3 - 2x^2 - 2x + 8$$

(a) Find $\frac{dy}{dx}$ $\qquad$ $\frac{dy}{dx} = 6x^2 - 4x - 2$

**(2)**

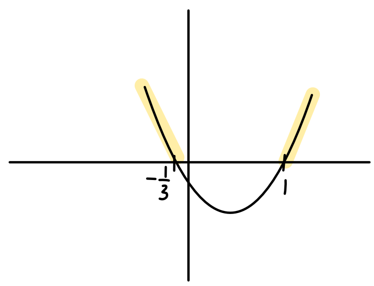
(b) Hence find the range of values of $x$ for which $y$ is increasing.
Write your answer in set notation.

**(4)**

$y$ is increasing when $\frac{dy}{dx} > 0$

$\therefore 6x^2 - 4x - 2 > 0$

$3x^2 - 2x - 1 > 0$




$(x-1)(3x+1) = 0$

$x = 1 \quad x = -\frac{1}{3}$

$x < -\frac{1}{3}$ or $x > 1$ $\qquad$ set notation : $\{x : x < -\frac{1}{3} \text{ or } x > 1\}$

**(Total for Question 3 is 6 marks)**

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4.

A curve has equation

$$y = 3x^2 + \frac{24}{x} + 2 \qquad x > 0$$

(a) Find, in simplest form, $\frac{dy}{dx}$ $\qquad$ $\frac{dy}{dx} = 6x - 24x^{-2}$

**(3)**

(b) Hence find the exact range of values of $x$ for which the curve is increasing.

**(2)**

increasing when $\frac{dy}{dx} > 0$

$6x - 24x^{-2} > 0$

$6x^3 - 24 > 0$

$x^3 > 4$

$\boxed{x > \sqrt[3]{4}}$

**(Total for Question 4 is 5 marks)**

**5.**

Prove, from first principles, that the derivative of $3x^2$ is $6x$.

**(4)**

(5) $$\frac{dy}{dx} = \lim_{h \to 0} \left( \frac{f(x+h) - f(x)}{h} \right)$$

$$f(x) = 3x^2$$

$$f(x+h) = 3(x+h)^2$$
$$= 3(x^2 + 2xh + h^2)$$
$$= 3x^2 + 6xh + 3h^2$$

$$\Rightarrow \frac{dy}{dx} = \lim_{h \to 0} \left( \frac{3x^2 + 6xh + 3h^2 - 3x^2}{h} \right)$$

$$= \lim_{h \to 0} (6x + 3h)$$

as h tends to 0, all the terms in h tend to 0.

$$\therefore \frac{dy}{dx}(3x^2) = 6x$$

**(Total for Question 5 is 4 marks)**

6.

Prove, from first principles, that the derivative of $x^3$ is $3x^2$

**(4)**

(6) $$f(x) = x^3$$

$$f(x+h) = (x+h)^3$$
$$= x^3 + 3x^2h + 3xh^2 + h^3$$

$$\frac{dy}{dx} = \lim_{h \to 0} \left( \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} \right)$$

$$\Rightarrow \lim_{h \to 0} (3x^2 + 3xh + h^2)$$

as $h \to 0$, all the terms in h tend to 0

$\therefore$ $3xh \to 0$ and $h^2 \to 0$

$$\therefore \frac{dy}{dx}(x^3) = 3x^2$$

**(Total for Question 6 is 4 marks)**

**7.**

A lorry is driven between London and Newcastle.

In a simple model, the cost of the journey £$C$ when the lorry is driven at a steady speed of $v$ kilometres per hour is

$$C = \frac{1500}{v} + \frac{2v}{11} + 60$$

(a) Find, according to this model,

(i) the value of $v$ that minimises the cost of the journey,

(ii) the minimum cost of the journey.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

a) i) $C = \frac{1500}{V} + \frac{2V}{11} + 60$

minimum cost at $\frac{dC}{dV} = 0$

$\frac{dC}{dV} = \frac{-1500}{V^2} + \frac{2}{11}$

$\frac{2}{11} - \frac{1500}{V^2} = 0$

$2V^2 - 16500 = 0$

$2V^2 = 16500$

$V = \underline{90.8 \text{ kmh}^{-1}}$

ii) minimum cost at $V = 90.8$

$C = \frac{1500}{90.8} + \frac{2(90.8)}{11} + 60$

$C = 93.03$

$\therefore$ minimum cost $= \underline{£93.03}$

(b) Prove by using $\frac{d^2C}{dv^2}$ that the cost is minimised at the speed found in (a)(i).

(2)

b) $\frac{d^2C}{dV^2} = 3000V^{-3}$

$= 3000(90 \cdot 8)^{-3}$

$= 4.0074 \times 10^{-3}$ which is $> 0$ hence

£93·03 is the minimum cost as

$\frac{d^2C}{dV^2} > 0$ at this point.

(c) State one limitation of this model.

(1)

c) it wouldn't be possible to maintain the same speed for the entire journey

**(Total for Question 7 is 9 marks)**

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8.

A curve has equation $y = g(x)$.

Given that

- $g(x)$ is a cubic expression in which the coefficient of $x^3$ is equal to the coefficient of $x$
- the curve with equation $y = g(x)$ passes through the origin
- the curve with equation $y = g(x)$ has a stationary point at $(2, 9)$

(a) find $g(x)$,

(7)

8 a) Since $y = g(x)$ is cubic, $y = ax^3 + bx^2 + cx + d$.

Since the coefficients of $x^3$ and $x$ are equal, $c = a$:

$y = ax^3 + bx^2 + ax + d$.

Since $y = g(x)$ passes through the origin:

$0 = 0 + 0 + 0 + d$

$\Rightarrow d = 0$

$\Rightarrow y = ax^3 + bx^2 + ax$

Turning point at $(2,9)$: $\frac{dy}{dx} = 3ax^2 + 2bx + a$

$3a(2)^2 + 2b(2) + a = 0$

$\Rightarrow 13a + 4b = 0$ ①

Graph passes through $(2,9)$: $9 = a(2)^3 + b(2)^2 + 2a$

$\Rightarrow 10a + 4b = 9$ ②

Solve ① and ② simultaneously: ② − ①: $-3a = 9 \Rightarrow a = -3$

$b = \frac{9 - 10a}{4} \Rightarrow b = \frac{39}{4}$

Hence, $\boxed{y = -3x^3 + \frac{39}{4}x^2 - 3x.}$

(b) prove that the stationary point at (2, 9) is a maximum.

**(2)**

b) $\frac{d^2y}{dx^2} = -18x + \frac{39}{2}$. At (2,9), $\frac{d^2y}{dx^2} = -\frac{33}{2} < 0$ so (2,9) is a maximum.

**(Total for Question 8 is 9 marks)**

**9.**

Diagram not drawn to scale

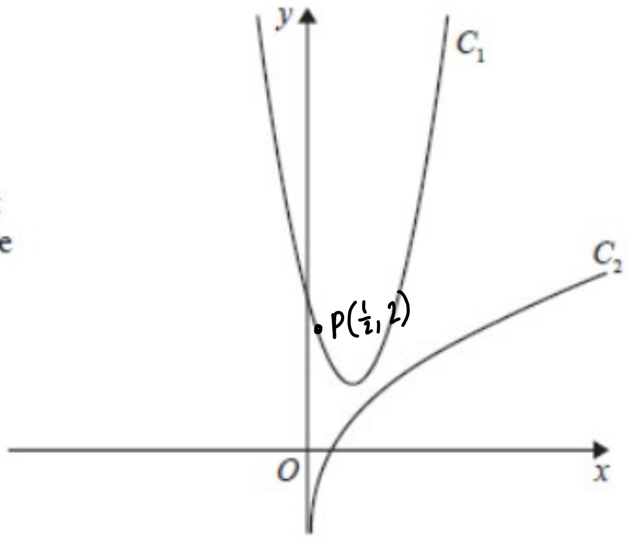



**Figure 3**

The curve $C_1$, shown in Figure 3, has equation $y = 4x^2 - 6x + 4$.

The point $P\left(\frac{1}{2}, 2\right)$ lies on $C_1$

The curve $C_2$, also shown in Figure 3, has equation $y = \frac{1}{2}x + \ln(2x)$.

The normal to $C_1$ at the point $P$ meets $C_2$ at the point $Q$.

Find the exact coordinates of $Q$.

*(Solutions based entirely on graphical or numerical methods are not acceptable.)*

**(8)**

First find the gradient of the tangent at P:

$$y = 4x^2 - 6x + 4 \implies \frac{dy}{dx} = 8x - 6$$

$$\text{at } P\left(\tfrac{1}{2}, 2\right), \ \frac{dy}{dx} = 8\left(\tfrac{1}{2}\right) - 6 = -2 \quad \therefore \text{ gradient of normal } = \frac{1}{2}$$

equation of normal to $C_1$ at P:

$$y - 2 = \frac{1}{2}\left(x - \frac{1}{2}\right)$$

$$\implies \boxed{y = \frac{1}{2}x + \frac{7}{4}}$$

Now substitute equation of normal to $C_1$ into equation of $C_2$:

$$\cancel{\tfrac{1}{2}x} + \ln(2x) = \cancel{\tfrac{1}{2}x} + \frac{7}{4}$$

$$\implies \ln(2x) = \frac{7}{4}$$

$$\implies e^{\frac{7}{4}} = 2x \implies x = \frac{1}{2}e^{\frac{7}{4}}$$

$$y = \frac{1}{2}x + \ln(2x)$$

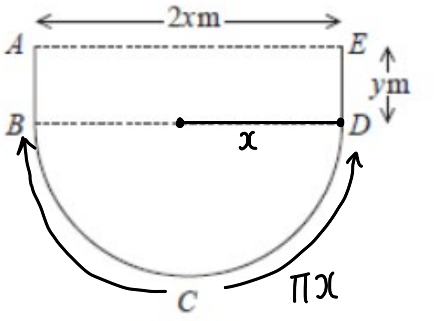
$$y = \frac{1}{2}\left(\frac{1}{2}e^{7/4}\right) + \ln\left(2 \times \frac{1}{2}e^{7/4}\right)$$

$$y = \frac{1}{4}e^{7/4} + \frac{7}{4}$$

$$\boxed{Q\left(\frac{1}{2}e^{7/4}, \frac{1}{4}e^{7/4} + \frac{7}{4}\right)}$$

**(Total for Question 9 is 8 marks)**

10.




circumference of semicircle

$= 2\pi r \div 2$

$= 2\pi(x) \div 2$

$= 2\pi x \div 2$

$\boxed{= \pi x}$

**Figure 4**

Figure 4 shows the plan view of the design for a swimming pool.

The shape of this pool $ABCDEA$ consists of a rectangular section $ABDE$ joined to a semicircular section $BCD$ as shown in Figure 4.

Given that $AE = 2x$ metres, $ED = y$ metres and the area of the pool is $250\,m^2$,

(a) show that the perimeter, $P$ metres, of the pool is given by

$$P = 2x + \frac{250}{x} + \frac{\pi x}{2}$$

(4)

a) Area $= 250m^2$

$250 = 2xy + \frac{\pi x^2}{2}$

$2xy = 250 - \frac{\pi x^2}{2}$

$2y = \frac{250}{x} - \frac{\pi x}{2}$

$P = 2x + 2y + \pi x$

$P = 2x + \frac{250}{x} - \frac{\pi x}{2} + \pi x$

$\therefore P = 2x + \frac{250}{x} + \frac{\pi x}{2}$

(b) Explain why $0 < x < \sqrt{\frac{500}{\pi}}$

(2)

b) • as $x > 0$ and $y > 0 \Longrightarrow 250 - \frac{\pi x^2}{2} > 0$ so $\frac{\pi x^2}{2} < 250$

$\pi x^2 < 500$

$x^2 < \frac{500}{\pi}$

$\therefore x < \sqrt{\frac{500}{\pi}}$

• $x$ and $y$ are both positive since they are distances

Hence, $0 < x < \sqrt{\frac{500}{\pi}}$.

(c) Find the minimum perimeter of the pool, giving your answer to 3 significant figures.
(4)

c) minimum perimeter occurs when $\frac{dP}{dx} = 0$ and $\frac{d^2P}{dx^2} > 0$

$$P = 2x + \frac{250}{x} + \frac{\pi x}{2}$$

$$\frac{dP}{dx} = 2 - 250x^{-2} + \frac{\pi}{2} = 0$$

$$\frac{250}{x^2} = 2 + \frac{\pi}{2}$$

$$250 = \left(2 + \frac{\pi}{2}\right)x^2$$

$$\Rightarrow x^2 = \frac{250}{2 + \frac{\pi}{2}}$$

$$x^2 = \frac{500}{4+\pi}$$

$$x = \sqrt{\frac{500}{4+\pi}}$$

Check it's a minimum:

$$\frac{d^2P}{dx^2} = 500x^{-3}$$

$500\left(\sqrt{\frac{500}{4+\pi}}\right)^{-3} > 0$ so it is a minimum at $x = \sqrt{\frac{500}{4+\pi}}$

$$\text{minimum } P = 2\left(\sqrt{\frac{500}{4+\pi}}\right) + \frac{250}{\sqrt{\frac{500}{4+\pi}}} + \frac{\pi\left(\sqrt{\frac{500}{4+\pi}}\right)}{2}$$

$$\boxed{\therefore P = 59.8 \text{ to } 3 \text{ s.f.}}$$

**(Total for Question 10 is 10 marks)**