Additional Assessment Materials

Summer 2021

Pearson Edexcel GCE in As Mathematics

8MA0_01 (Public release version)

Resource Set 1: Topic 5

Trigonometry

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Additional Assessment Materials, Summer 2021

General guidance to Additional Assessment Materials for use in 2021

Context

- Additional Assessment Materials are being produced for GCSE, AS and A levels (with the exception of Art and Design).
- The Additional Assessment Materials presented in this booklet are an optional part of the range of evidence teachers may use when deciding on a candidate's grade.
- 2021 Additional Assessment Materials have been drawn from previous examination materials, namely past papers.
- Additional Assessment Materials have come from past papers both published (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow teachers to adapt them for use with candidate.

Purpose

- The purpose of this resource to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the mapping guidance which will map content and/or skills covered within each set of questions.
- These materials are only intended to support the summer 2021 series.

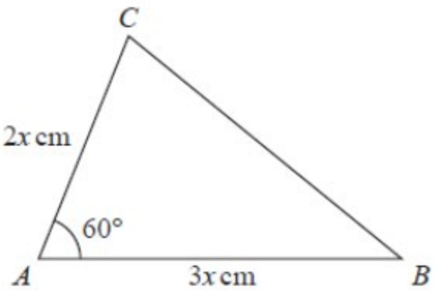


Figure 1

Figure 1 shows a sketch of a triangle ABC with $AB = 3x$ cm, $AC = 2x$ cm and angle $CAB = 60°$

Given that the area of triangle ABC is $18\sqrt{3}$ cm^2

(a) show that $x = 2\sqrt{3}$

(3)

a) $A = \frac{1}{2}ab\sin C$

1.

$18\sqrt{3} = \frac{1}{2}(2x)(3x)\sin(60)$

$18\sqrt{3} = 3x^2\left(\frac{\sqrt{3}}{2}\right)$

$18\sqrt{3} = \frac{3\sqrt{3}}{2}x^2$

$36\sqrt{3} = 3\sqrt{3}x^2$

$\Rightarrow 12 = x^2$

$\Rightarrow x = 2\sqrt{3}$

$BC = 2\sqrt{21}$

(b) Hence find the exact length of BC, giving your answer as a simplified surd.

(3)

b) $c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = (4\sqrt{3})^2 + (6\sqrt{3})^2 - 2(4\sqrt{3})(6\sqrt{3})\cos(60)$

$c^2 = 84$

$\boxed{c = 2\sqrt{21}}$

(Total for Question 1 is 6 marks)

2.

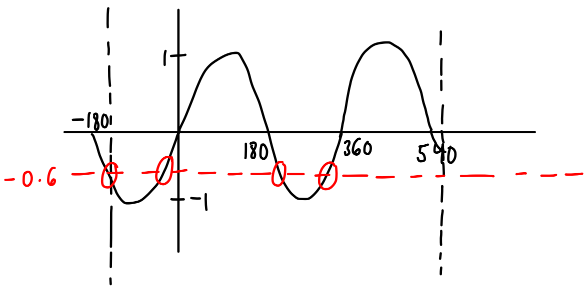
. (i) Solve, for $-90° \leqslant \theta < 270°$, the equation,

$$\sin(2\theta + 10°) = -0.6$$

giving your answers to one decimal place.

(5)

② i) $\sin(2\theta + 10°) = -0.6$



$-90 \leq \theta < 270$

$-180 \leq 2\theta < 540$

$-170 \leq 2\theta + 10° < 550$

$\sin(2\theta + 10) = -0.6$

$2\theta + 10 = \sin^{-1}(-0.6)$

$= -36.9°, 216.9°, 323.1°, -143.1$

$\boxed{\theta = -76.6°, -23.45°, 103.5°, 156.6°}$

(Total for Question 2 is 5 marks)

3

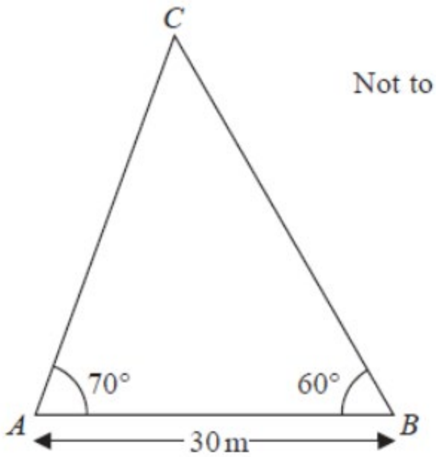


Figure 1

A triangular lawn is modelled by the triangle ABC, shown in Figure 1. The length AB is to be 30 m long.

Given that angle $BAC = 70°$ and angle $ABC = 60°$,

(a) calculate the area of the lawn to 3 significant figures.

(4)

3 a) $\frac{a}{\sin(70)} = \frac{30}{\sin(50)}$

$\Rightarrow \quad a = \frac{30\sin(70)}{\sin(50)} \quad \Rightarrow \quad a = 36.8 \text{ m}$

$A = \frac{1}{2} ab \sin C$

$A = \frac{1}{2}(30)(36.8)\sin 60$

$A = 478 \text{ m}^2$

(b) Why is your answer unlikely to be accurate to the nearest square metre?

(1)

b) the lawn isn't a perfect triangle so the degrees aren't exact

(Total for Question 3 is 5 marks)

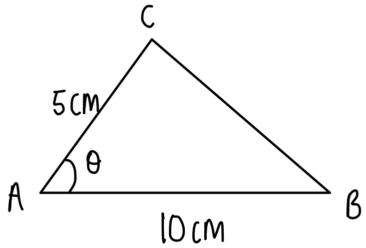
4.

In a triangle ABC, side AB has length 10 cm, side AC has length 5 cm, and angle $BAC = \theta$ where θ is measured in degrees. The area of triangle ABC is 15 cm^2

(a) Find the two possible values of $\cos\theta$

(4)

4 a)



$A = 15 \text{ cm}^2$

$15 = \frac{1}{2} ab \sin\theta$

$15 = \frac{1}{2}(10)(5)\sin\theta \rightarrow \sin\theta = 0.6$

$\theta = 36.87°$

$\sin^2\theta + \cos^2\theta = 1$

$(0.6)^2 + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 0.64$

$\Rightarrow \cos\theta = \pm 0.8$

Given that BC is the longest side of the triangle,

(b) find the exact length of BC.

(2)

b) when $\cos\theta = -0.8$:

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 10^2 + 5^2 - 2(10)(5)(-0.8)$

$c^2 = 205$

$c = \sqrt{205}$ cm

(Total for Question 4 is 5 marks)

5

Solve, for $360° \leqslant x < 540°$,

$$12\sin^2 x + 7\cos x - 13 = 0$$

Give your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

$12\sin^2 x + 7\cos x - 13 = 0$

$12(1 - \cos^2 x) + 7\cos x - 13 = 0$

$12 - 12\cos^2 x + 7\cos x - 13 = 0$

$12\cos^2 x - 7\cos x + 1 = 0$

$(3\cos x - 1)(4\cos x - 1) = 0$

$\cos x = \frac{1}{3}$ or $\cos x = \frac{1}{4}$

$x = \cancel{70.5}, \cancel{289.5}°, 430.5°,$ → in range

$x = \cancel{75.5}°, \cancel{284.5}°, 435.5°$ →

so $x = 430.5°$ and $435.5°$

(Total for Question 5 is 5 marks)

6.

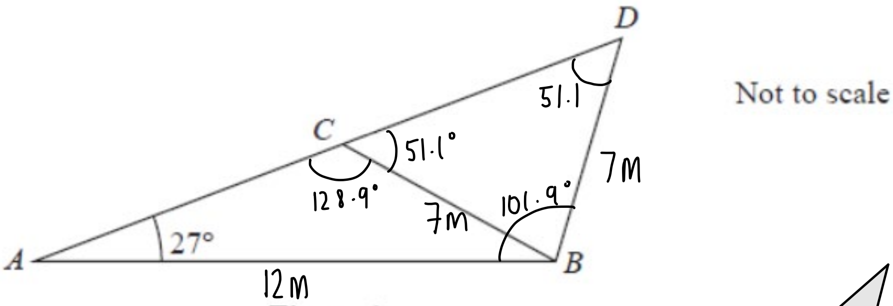


Figure 1

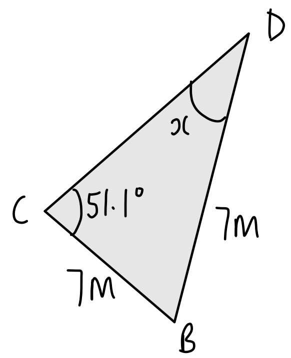


Figure 1 shows the design for a structure used to support a roof.

The structure consists of four steel beams, *AB*, *BD*, *BC* and *AD*.

Given *AB* = 12 m, *BC* = *BD* = 7m and angle *BAC* = 27°

(a) find, to one decimal place, the size of angle *ACB*.

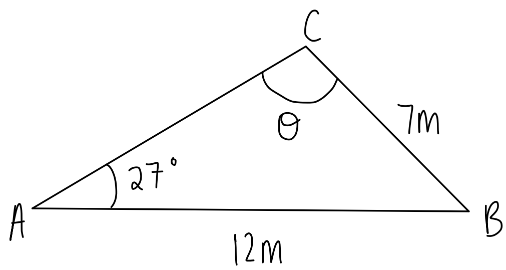
(3)

a) $\frac{7}{\sin 27} = \frac{12}{\sin\theta}$

$\sin\theta = \frac{12\sin 27}{7}$

$\theta = 51.1°$ or $128.9°$

$\boxed{\theta = 128.9°}$



The steel beams can only be bought in whole metre lengths.

(b) Find the minimum length of steel that needs to be bought to make the complete structure.

(3)

b) $AD^2 = 12^2 + 7^2 - 2(12)(7)\cos(101.9°)$

$AD^2 = 144 + 49 - 2(84)\cos(101.9°)$

$AD^2 = 227.6$

$AD = 15.1$ m

total length of steel

$= 12 + 7 + 7 + 16$

$\boxed{= 42\text{ m}}$

Total for Question 6 is 6 marks)

7.

(ii) (a) A student's attempt at the question

"Solve, for $-90° < x < 90°$, the equation $7\tan x = 8\sin x$"

is set out below.

$$7\tan x = 8\sin x$$
$$7 \times \frac{\sin x}{\cos x} = 8\sin x$$
$$7\sin x = 8\sin x\cos x$$
$$7 = 8\cos x$$
$$\cos x = \frac{7}{8}$$
$$x = 29.0° \text{ (to 3 sf)}$$

Identify two mistakes made by this student, giving a brief explanation of each mistake.

(2)

a) 1. shouldn't have cancelled out the $\sin x$ in the 4th step as now some solutions will be missing. should have factored out the $\sin x$ instead.

2. $29°$ is not the only solution; another solution would be $-29.0°$.

(b) Find the smallest positive solution to the equation

$$7\tan(4\alpha + 199°) = 8\sin(4\alpha + 199°)$$

(2)

b) Set $x = 4\alpha + 199$

$7\sin x - 8\sin x\cos x = 0$

$\sin x(7 - 8\cos x) = 0$

$\sin x = 0$ or $\cos x = \frac{7}{8}$

$x = 0°$ or $x = 29°, -29°, 331°$

$4\alpha + 199 = 331$

$4\alpha = 132$

$\alpha = 33.0°$

smallest positive solution $= 33.0°$

Total for Question 7 is 4 marks)

8.

(a) Show that

$$\frac{10\sin^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta} \equiv 4 - 5\cos\theta$$

(4)

a) $\dfrac{10\sin^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta}$

$= \dfrac{10 - 10\cos^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta}$

$= \dfrac{-10\cos^2\theta - 7\cos\theta + 12}{3 + 2\cos\theta}$

$= \dfrac{-(5\cos\theta - 4)(\cancel{2\cos\theta + 3})}{\cancel{3 + 2\cos\theta}}$

$= -(5\cos\theta - 4)$

$= 4 - 5\cos\theta \equiv \text{RHS}$

(b) Hence, or otherwise, solve, for $0 \leqslant x < 360°$, the equation

$$\frac{10\sin^2 x - 7\cos x + 2}{3 + 2\cos x} = 4 + 3\sin x$$

(3)

b) $4 - 5\cos x = 4 + 3\sin x$

$3\sin x = -5\cos x$

$\tan x = -\dfrac{5}{3}$

$x = \cancel{-59.04°}, 120.96°, 301°$

$x = \underline{121°}$ and $\underline{301°}$

Total for Question 8 is 7 marks)

9.

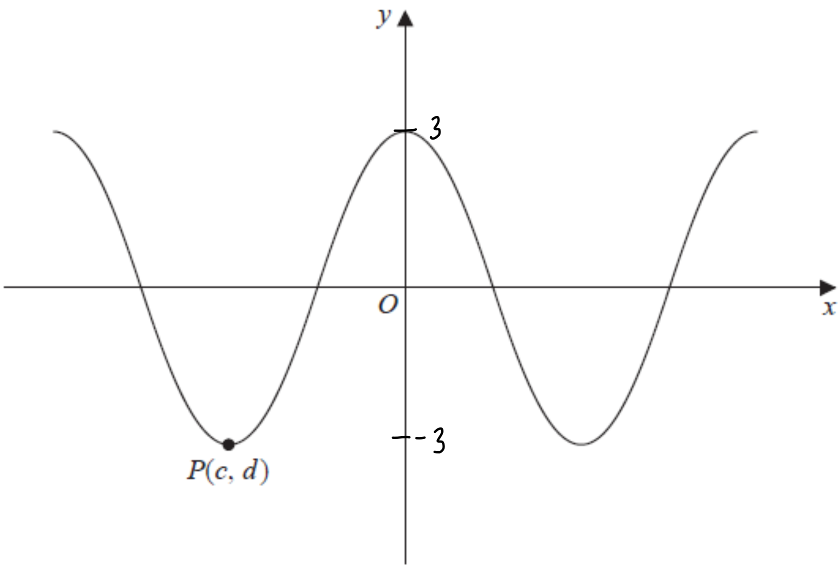


Figure 3

Figure 3 shows part of the curve with equation $y = 3\cos x°$.

The point $P(c, d)$ is a minimum point on the curve with c being the smallest negative value of x at which a minimum occurs.

(a) State the value of c and the value of d.

$c = -180 \quad d = -3$

(1)

(b) State the coordinates of the point to which P is mapped by the transformation which transforms the curve with equation $y = 3\cos x°$ to the curve with equation

(i) $y = 3\cos\left(\frac{x°}{4}\right)$ $\quad P(-720, -3)$

(ii) $y = 3\cos(x - 36)°$ $\quad P(-144, -3)$

(2)

(c) Solve, for $450° \leqslant \theta < 720°$,

$$3\cos\theta = 8\tan\theta$$

giving your solution to one decimal place.

In part (c) you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(5)

$$3\cos\theta = \frac{8\sin\theta}{\cos\theta}$$

$$3\cos^2\theta = 8\sin\theta$$

$$3(1-\sin^2\theta) = 8\sin\theta$$

$$3 - 3\sin^2\theta = 8\sin\theta$$

$$3\sin^2\theta + 8\sin\theta - 3 = 0$$

$$(3\sin\theta - 1)(\sin\theta + 3)$$

$\Rightarrow \sin\theta = \frac{1}{3}$ or $\cancel{\sin\theta = -3}$

not in range

$\theta = \cancel{19.5°}, \cancel{160.5°}, \underline{379.5°}, \underline{520.5°},$

$\theta = \underline{379.5°}$ and $\underline{520.5°}$

10.

. (a) Show that the equation

$$4\cos\theta - 1 = 2\sin\theta\tan\theta$$

can be written in the form

$$6\cos^2\theta - \cos\theta - 2 = 0$$

(4)

(b) Hence solve, for $0 \leqslant x < 90°$ $\longrightarrow$ $0 \leq 3x < 270°$

$$4\cos 3x - 1 = 2\sin 3x\tan 3x$$

giving your answers, where appropriate, to one decimal place.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

a) $4\cos\theta - 1 = 2\sin\theta\tan\theta$

$4\cos\theta - 1 = 2\sin\theta\left(\frac{\sin\theta}{\cos\theta}\right)$

$4\cos^2\theta - \cos\theta = 2\sin^2\theta$

$4\cos^2\theta - \cos\theta = 2(1-\cos^2\theta)$

$4\cos^2\theta - \cos\theta = 2 - 2\cos^2\theta$

$\Longrightarrow 6\cos^2\theta - \cos\theta - 2 = 0$

b) $6\cos^2(3x) - \cos(3x) - 2 = 0$

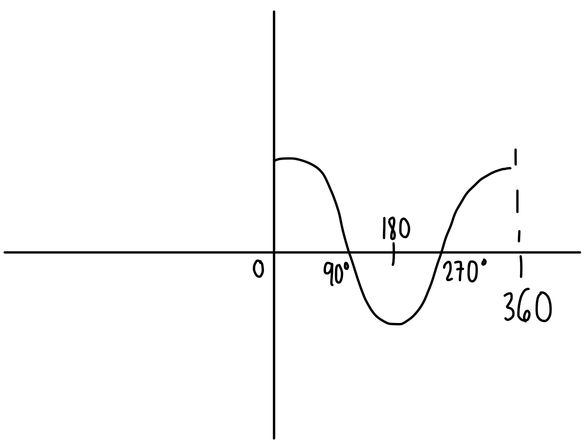
$(3\cos 3x - 2)(2\cos 3x + 1) = 0$

$\Rightarrow \cos 3x = \frac{2}{3}$ $\qquad \cos 3x = -\frac{1}{2}$

$3x = 48.2°$ $\qquad 3x = 120°, 240°$

$x = 16.1°$ $\qquad x = 40°, 80°$

$x = \underline{16.1°}, \underline{40°}, \underline{80°}$



Total for Question 10 is 8 marks)