Additional Assessment Materials

Summer 2021

Pearson Edexcel GCE in As Mathematics

8MA0_01 (Public release version)

Resource Set 1: Topic 3

Coordinate Geometry

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Additional Assessment Materials, Summer 2021

General guidance to Additional Assessment Materials for use in 2021

Context

- Additional Assessment Materials are being produced for GCSE, AS and A levels (with the exception of Art and Design).
- The Additional Assessment Materials presented in this booklet are an optional part of the range of evidence teachers may use when deciding on a candidate’s grade.
- 2021 Additional Assessment Materials have been drawn from previous examination materials, namely past papers.
- Additional Assessment Materials have come from past papers both published (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow teachers to adapt them for use with candidate.

Purpose

- The purpose of this resource to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the mapping guidance which will map content and/or skills covered within each set of questions.
- These materials are only intended to support the summer 2021 series.

1.

The line l passes through the points $A\ (3, 1)$ and $B\ (4, -2)$.

Find an equation for l.

(3)

gradient $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2-1}{4-3} = \dfrac{-3}{1} = -3$

equation of a line: $y - y_1 = m(x - x_1) \Rightarrow y - 1 = -3(x-3)$

$y - 1 = -3x + 9$

$\boxed{y = -3x + 10}$

(Total for Question 1 is 3 marks)

2.

The line l_1 has equation $2x + 4y - 3 = 0$ $\rightarrow$

$4y = 3 - 2x$

$y = \frac{3}{4} - \frac{1}{2}x$

The line l_2 has equation $y = mx + 7$, where m is a constant.

Given that l_1 and l_2 are perpendicular, $\rightarrow M_1 \times M_2 = -1$ $\rightarrow$

$-\frac{1}{2} \times M = -1$

$\boxed{M = 2}$

(a) find the value of m.

(2)

The lines l_1 and l_2 meet at the point P.

(b) Find the x coordinate of P.

$2x + 4(2x+7) - 3 = 0$

$2x + 8x + 28 - 3 = 0$

$10x + 25 = 0$

$\boxed{x = -2.5}$

(2)

(Total for Question 3 is 4 marks)

3.

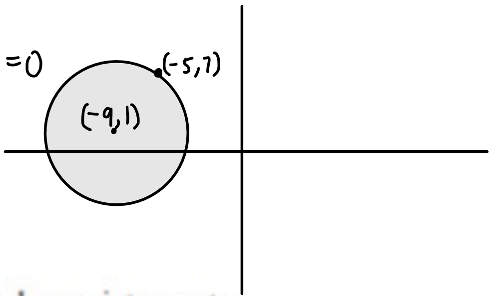
(i) A circle C_1 has equation

$(x+9)^2 - 81 + (y-1)^2 - 1 + 30 = 0$

$(x+9)^2 + (y-1)^2 = 52$

$(-9, 1)$

$$x^2 + y^2 + 18x - 2y + 30 = 0$$



The line l is the tangent to C_1 at the point $P(-5, 7)$.

Find an equation of l in the form $ax + by + c = 0$, where a, b and c are integers to be found.

gradient of radius $= \dfrac{7-1}{-5+9} = \dfrac{6}{4} = \dfrac{3}{2} \Rightarrow$ gradient of tangent $\Rightarrow$ (5) $-\frac{2}{3}$

$y - y_1 = m(x - x_1) \Rightarrow y - 7 = -\frac{2}{3}(x+5)$

$\Rightarrow 3y - 21 = -2x - 10$

$\Rightarrow \boxed{2x + 3y - 11 = 0}$

(Total for Question 3 is 5 marks)

4. A tree was planted in the ground.
Its height, H metres, was measured t years after planting.

Exactly 3 years after planting, the height of the tree was 2.35 metres.
Exactly 6 years after planting, the height of the tree was 3.28 metres.

Using a linear model,

(a) find an equation linking H with t.

(3)

a) gradient $= \dfrac{3.28 - 2.35}{6 - 3} = 0.31$

$y - 2.35 = 0.31(x - 3)$

$y - 2.35 = 0.31x - 0.93$

$(3, 2.35)$

$(6, 3.28)$

$y = 0.31x + 1.42$

$\boxed{H = 0.31t + 1.42}$

(1.4m) at $t = 0$

The height of the tree was approximately 140 cm when it was planted.

(b) Explain whether or not this fact supports the use of the linear model in part (a).

(2)

$H = 0.31(0) + 1.42 \Rightarrow H = 1.42$

$\therefore$ the fact does support the linear model as $1.4 \approx 1.42$

(Total for Question 4 is 5 marks)

5.

$4y = 3x + 10$

$y = \frac{3}{4}x + \frac{5}{2}$

gradient $= \dfrac{8+1}{-1-5} = \dfrac{9}{-6} = -\dfrac{3}{2}$

The line l_1 has equation $4y - 3x = 10$

The line l_2 passes through the points $(5, -1)$ and $(-1, 8)$.

Determine, giving full reasons for your answer, whether lines l_1 and l_2 are parallel, perpendicular or neither.

$-\frac{3}{2} \times \frac{3}{4} = -\frac{9}{8}$ so they're not perpendicular as $m_1 \times m_2 \neq -1$.

(4)

Not parallel because the gradients aren't equal. Therefore they are neither.

(Total for Question 5 is 4 marks)

6.

A circle C has equation

$$x^2 + y^2 - 4x + 8y - 8 = 0$$

$(x-2)^2 - 4 + (y+4)^2 - 16 - 8 = 0$

$(x-2)^2 + (y+4)^2 = 28$

(a) Find

(i) the coordinates of the centre of C, $\rightarrow$ $(2, -4)$

(ii) the exact radius of C. $\rightarrow$ $\sqrt{28} = \boxed{2\sqrt{7}}$

(3)

The straight line with equation $x = k$, where k is a constant, is a tangent to C.

(b) Find the possible values for k.

(2)

$k^2 + y^2 - 4k + 8y - 8 = 0$

$y^2 + 8y + (k^2 - 4k - 8) = 0$

$b^2 - 4ac = 0$

$8^2 - 4(1)(k^2 - 4k - 8) = 0$

$64 - 4k^2 + 16k + 32 = 0$

$4k^2 - 16k - 96 = 0$

$k^2 - 4k - 24 = 0$

$k = 2 + 2\sqrt{7}$ or $2 - 2\sqrt{7}$

Total for Question 6 is 5 marks)

7.

A tank, which contained water, started to leak from a hole in its base.

The volume of water in the tank 24 minutes after the leak started was 4m^3 $(24, 4)$

The volume of water in the tank 60 minutes after the leak started was 2.8m^3 $(60, 2.8)$

The volume of water, $V\text{m}^3$, in the tank t minutes after the leak started, can be described by a linear model between V and t.

(a) Find an equation linking V with t.

gradient $= \frac{2.8-4}{60-24} = -\frac{1}{30}$

$y - 4 = -\frac{1}{30}(x - 24)$

$y - 4 = -\frac{1}{30}x + \frac{4}{5}$

$y = -\frac{1}{30}x + 4.8$

$\boxed{V = -\frac{1}{30}t + 4.8}$

(4)

Use this model to find

(b) (i) the initial volume of water in the tank, → $V = 4.8\ \text{m}^3$

(ii) the time taken for the tank to empty. → $0 = -\frac{1}{30}t + 4.8 \Rightarrow t = 144$ minutes

(3)

(c) Suggest a reason why this linear model may not be suitable.

(1)

Assumes that the hole doesn't get bigger.

Total for Question 7 is 8 marks)

8.

$(x-2)^2 + (y-5)^2 = r^2$

A circle C has centre (2, 5). Given that the point $P(-2, 3)$ lies on C.

(a) find an equation for C.

$(-2-2)^2 + (3-5)^2 = r^2$

$16 + 4 = r^2 \Rightarrow r^2 = 20$

equation of C : $(x-2)^2 + (y-5)^2 = 20$

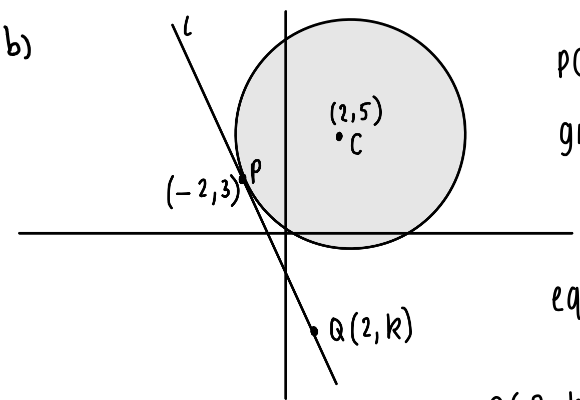
(3)

The line l is the tangent to C at the point P. The point $Q(2, k)$ lies on l.

(b) Find the value of k.

(5)

b)



PC and PQ are perpendicular.

gradient of PC $= \frac{5-3}{2+2} = \frac{2}{4} = \frac{1}{2}$

gradient of $l = -2$

equation of l : $y - 3 = -2(x+2)$

$\boxed{y = -2x - 1}$

$Q(2, k)$ lies on l:

↳ $k = -2(2) - 1$

$k = -4 - 1$

$\boxed{k = -5}$

Total for Question 8 is 8 marks)

9.

The circle C has equation

$$x^2 + y^2 - 6x + 10y + 9 = 0$$

$(x-3)^2 + (y+5)^2 + 9 - 9 - 25 = 0$

$(x-3)^2 + (y+5)^2 = 25$

(a) Find

(i) the coordinates of the centre of C → $C:(3,-5)$

(ii) the radius of C → $\sqrt{25} = 5$

(3)

The line with equation $y = kx$, where k is a constant, cuts C at two distinct points.

(b) Find the range of values for k.

(6)

$(x-3)^2 + (kx+5)^2 = 25$

$(x^2 - 6x + 9) + (k^2x^2 + 10kx + 25) = 25$

$(k^2+1)x^2 + (10k-6)x + 9 = 0$

$b^2 - 4ac > 0$ if it cuts at 2 distinct points.

$(10k-6)^2 - 4(k^2+1)(9) > 0$

$100k^2 - 120k + 36 - 36k^2 - 36 > 0$

$64k^2 - 120k > 0$

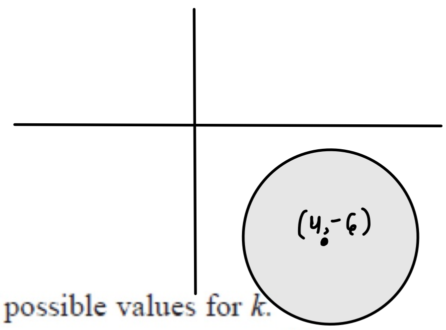
$64k - 120 > 0$

$k > \frac{15}{8}$

Total for Question 9 is 9 marks)

10.

$c(4,-6)$



(ii) A different circle C_2 has equation

$$x^2 + y^2 - 8x + 12y + k = 0$$

where k is a constant.

(positive x, negative y)

Given that C_2 lies entirely in the 4th quadrant, find the range of possible values for k.

(4)

$(x^2 - 8x) + (y^2 + 12y) + k = 0$

$(x-4)^2 - 16 + (y+6)^2 - 36 + k = 0$

$(x-4)^2 + (y+6)^2 + k - 52 = 0$

$(x-4)^2 + (y+6)^2 = 52 - k$

Radius $R = \sqrt{52-k} \Rightarrow 52 - k \geq 0 \Rightarrow \underline{k \leq 52}$

By diagram: $R < 4 \Rightarrow \sqrt{52-k} < 4$

$\Rightarrow 52 - k < 16$

$\Rightarrow k > 36$

So, $\boxed{36 < k \leq 52}$

Total for Question 10 is 4 marks)