Additional Assessment Materials

Summer 2021

Pearson Edexcel GCE in As Mathematics

8MA0_01 (Public release version)

Resource Set 1: Topic 1

Algebra and functions

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Additional Assessment Materials, Summer 2021

General guidance to Additional Assessment Materials for use in 2021

Context

- Additional Assessment Materials are being produced for GCSE, AS and A levels (with the exception of Art and Design).
- The Additional Assessment Materials presented in this booklet are an optional part of the range of evidence teachers may use when deciding on a candidate's grade.
- 2021 Additional Assessment Materials have been drawn from previous examination materials, namely past papers.
- Additional Assessment Materials have come from past papers both published (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow teachers to adapt them for use with candidate.

Purpose

- The purpose of this resource to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the mapping guidance which will map content and/or skills covered within each set of questions.
- These materials are only intended to support the summer 2021 series.

$$g(x) = 2x^3 + x^2 - 41x - 70$$

(a) Use the factor theorem to show that g(x) is divisible by $(x - 5)$.

(2)

a) $f(5) = 2(5)^3 + 5^2 - 41(5) - 70$

$= 2(125) + 25 - 205 - 70$

1. $= 250 + 25 - 205 - 70$

$\Rightarrow f(5) = 0$

(b) Hence, showing all your working, write g(x) as a product of three linear factors.

(4)

b)

$$\begin{array}{r|l} & 2x^2 + 11x + 14 \\ x-5 & 2x^3 + x^2 - 41x - 70 \\ & -(2x^3 - 10x^2) \\ \hline & 11x^2 - 41x - 70 \\ & -(11x^2 - 55x) \\ \hline & 14x - 70 \\ & -(14x - 70) \\ \hline & 0 \end{array}$$

$(2x^2 + 11x + 14)(x-5)$

$(x+2)(2x+7)(x-5)$

$(x+2)(2x+7)(x-5)$

(Total for Question 1 is 6 marks)

2. Find, using algebra, all real solutions to the equation

(i) $16a^2 = 2\sqrt{a}$

(4)

(ii) $b^4 + 7b^2 - 18 = 0$

(4)

i) $16a^2 = 2\sqrt{a}$

$\Rightarrow 16a^2 = 2a^{\frac{1}{2}}$

$\Rightarrow 8a^2 = a^{\frac{1}{2}}$

$8a^{\frac{3}{2}} = 1$

$a^{\frac{3}{2}} = \frac{1}{8}$

$a = \frac{1}{4}$

(Total for Question 2 is 8 marks)

ii) $b^4 + 7b^2 - 18 = 0$

$\boxed{y = b^2}$

$y^2 + 7y - 18 = 0$

$(y+9)(y-2) = 0$

$y = -9, \; y = 2$

$\Rightarrow b^2 = -9, \; b^2 = 2$

No possible solutions to $b^2 = -9$ since b^2 is positive.

So $b^2 = 2$

$\Rightarrow b = \pm\sqrt{2}$

3.

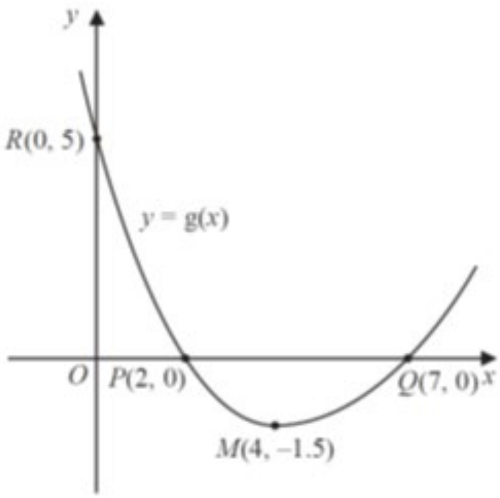


Figure 1

Figure 1 shows a sketch of the curve with equation $y = g(x)$.

The curve has a single turning point, a minimum, at the point $M(4, -1.5)$.
The curve crosses the x-axis at two points, $P(2, 0)$ and $Q(7, 0)$.
The curve crosses the y-axis at a single point $R(0, 5)$.

(a) State the coordinates of the turning point on the curve with equation $y = 2g(x)$.

$(4, -3)$

(1)

(b) State the largest root of the equation

$$g(x + 1) = 0$$

$x = 6$

(1)

(c) State the range of values of x for which $g'(x) \leqslant 0$

$x \leq 4$

(1)

Given that the equation $g(x) + k = 0$, where k is a constant, has no real roots,

(d) state the range of possible values for k.

(1)

$k > 1.5$

(Total for Question 3 is 4 marks)

4. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(i) Solve the equation

$$x\sqrt{2} - \sqrt{18} = x$$

writing the answer as a surd in simplest form.

(3)

$$x(\sqrt{2} - 1) = \sqrt{18}$$

$$x = \frac{\sqrt{18}}{\sqrt{2} - 1}$$

$$x = \frac{\sqrt{18}}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1}$$

$$x = \sqrt{18}(\sqrt{2} + 1)$$

$$x = 3\sqrt{2}(\sqrt{2} + 1)$$

$$\underline{\underline{x = 6 + 3\sqrt{2}}}$$

(ii) Solve the equation

$$4^{3x-2} = \frac{1}{2\sqrt{2}}$$

(3)

$$4^{3x-2} = \frac{1}{2\sqrt{2}}$$

$$\left(\frac{1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}\right)$$

$$\Rightarrow 2^{2(3x-2)} = \frac{\sqrt{2}}{4}$$

$$\Rightarrow 2^{6x-4} = \frac{\sqrt{2}}{4}$$

$$\Rightarrow 2^{6x-4} = \frac{2^{\frac{1}{2}}}{2^2}$$

$$\Rightarrow 2^{6x-4} = 2^{-\frac{3}{2}}$$

$$\Rightarrow 6x - 4 = -\frac{3}{2}$$

$$\Rightarrow 12x - 8 = -3$$

$$\Rightarrow 12x = 5$$

$$x = \frac{5}{12}$$

(Total for Question 4 is 6 marks)

5.

A student was asked to give the exact solution to the equation

$$2^{2x+4} - 9(2^x) = 0$$

The student's attempt is shown below:

$$2^{2x+4} - 9(2^x) = 0$$

$$2^{2x} + 2^4 - 9(2^x) = 0$$

Let $2^x = y$

$$y^2 - 9y + 8 = 0$$

$$(y - 8)(y - 1) = 0$$

$y = 8$ or $y = 1$

So $x = 3$ or $x = 0$

(a) Identify the two errors made by the student.

(2)

a) $2^{2x+4} \neq 2^{2x} + 2^4$

$\rightarrow$ it equals $2^{2x} \times 2^4$

and $2^4 \neq 8$

$\rightarrow$ it equals 16

(b) Find the exact solution to the equation.

(2)

b) $2^{2x+4} - 9(2^x) = 0$

$\Rightarrow 2^{2x} \times 2^4 - 9(2^x) = 0$

$\Rightarrow (2^x)^2 \times 16 - 9(2^x) = 0$

let $y = 2^x$

$\Rightarrow y^2 \times 16 - 9y = 0$

$\Rightarrow 16y^2 - 9y = 0$

$\Rightarrow y(16y - 9) = 0$

$\Rightarrow y = \frac{9}{16}$ or $y = 0$

$\frac{9}{16} = 2^x$

$\boxed{\log_2\left(\frac{9}{16}\right) = x}$

(Total for Question 5 is 4 marks)

6.

$$g(x) = 4x^3 - 12x^2 - 15x + 50$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $g(x)$.

a) $g(-2) = 4(-2)^3 - 12(-2)^2 - 15(-2) + 50$
$= 4(-8) - 12(4) + 30 + 50$
$= -32 - 48 + 80$
$g(-2) = 0$ hence $(x+2)$ is a factor of $g(x)$

(b) Hence show that $g(x)$ can be written in the form $g(x) = (x + 2)(ax + b)^2$, where a and b are integers to be found.

(4)

b)
$$\begin{array}{r|l} & 4x^2 - 20x + 25 \\ x+2 & 4x^3 - 12x^2 - 15x + 50 \\ & -(4x^3 + 8x^2) \\ \hline & -20x^2 - 15x + 50 \\ & -(-20x^2 - 40x) \\ \hline \end{array}$$

$$\begin{array}{r} 25x + 50 \\ -(25x + 50) \\ \hline 0 \end{array}$$

$(x+2)(4x^2 - 20x + 25)$

$\boxed{(x+2)(2x-5)^2}$

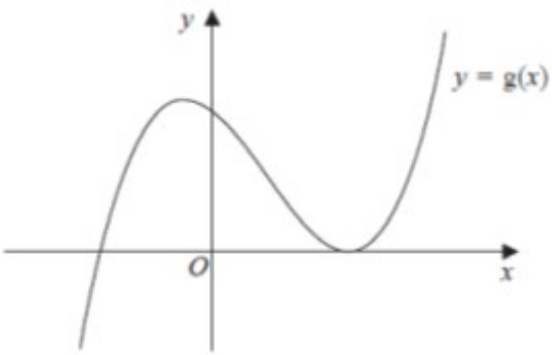


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = g(x)$

(c) Use your answer to part (b), and the sketch, to deduce the values of x for which

(i) $g(x) \leqslant 0$ $\rightarrow$ $x \leq -2$

(ii) $g(2x) = 0$ $\rightarrow$ $x = -1$, $x = \frac{5}{4}$

(3)

(Total for Question 6 is 9 marks)

7. (a) Factorise completely $x^3 + 10x^2 + 25x$

(2)

$\Rightarrow x(x^2 + 10x + 25)$

$\Rightarrow x(x+5)^2$

(b) Sketch the curve with equation

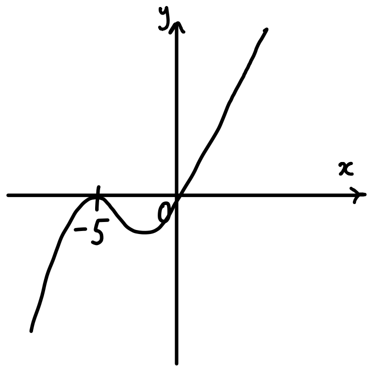
$$y = x^3 + 10x^2 + 25x$$

showing the coordinates of the points at which the curve cuts or touches the x-axis.

(2)

$x = 0$

$x = -5$



The point with coordinates $(-3, 0)$ lies on the curve with equation

$$y = (x + a)^3 + 10(x + a)^2 + 25(x + a)$$

where a is a constant.

(c) Find the two possible values of a.

(3)

c) $x = -3 \quad y = 0$

$0 = (-3 + a)^3 + 10(-3 + a)^2 + 25(-3 + a)$

$0 = a^3 - 9a^2 + 27a - 27 + 10(a^2 - 6a + 9) + 25a - 75$

$0 = a^3 + a^2 - 8a - 12$

$\boxed{a = 3 \text{ or } a = -2}$

(Total for Question 7 is 7 marks)

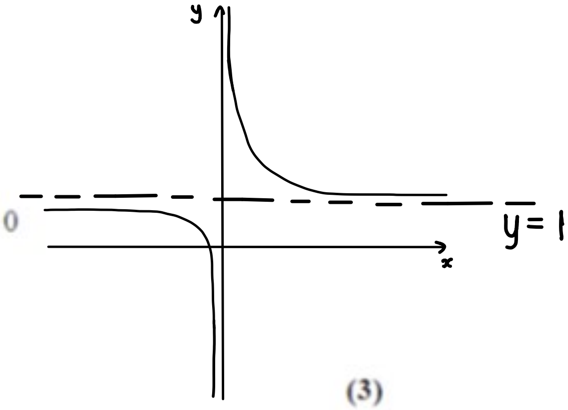
8.

The curve C has equation

$$y = \frac{k^2}{x} + 1 \qquad x \in \mathbb{R},\ x \neq 0$$

where k is a constant.

(a) Sketch C stating the equation of the horizontal asymptote. **(3)**



The line l has equation $y = -2x + 5$

(b) Show that the x coordinate of any point of intersection of l with C is given by a solution of the equation

$$2x^2 - 4x + k^2 = 0$$

$$-2x + 5 = \frac{k^2}{x} + 1$$
$$\Rightarrow -2x^2 + 5x = k^2 + x$$
$$\Rightarrow 2x^2 - 4x + k^2 = 0$$

(2)

(c) Hence find the exact values of k for which l is a tangent to C. **(3)**

▸ when l is a tangent, there is one real root for the equation of the intersection.

$$b^2 - 4ac = 0$$
$$\therefore\ (-4)^2 - 4(2)(k^2) = 0$$
$$\Rightarrow 16 - 8k^2 = 0$$
$$\Rightarrow 8k^2 = 16$$
$$k^2 = 2$$
$$\boxed{k = \sqrt{2} \text{ or } k = -\sqrt{2}}$$

Total for Question 8 is 8 marks)

9.

The equation $kx^2 + 4kx + 3 = 0$, where k is a constant, has no real roots.

Prove that

$$0 \leqslant k < \frac{3}{4}$$

(4)

no real roots means $b^2 - 4ac < 0$

$$\therefore\ (4k)^2 - 4(k)(3) < 0$$
$$\Rightarrow 16k^2 - 12k < 0$$

▸ $k(16k - 12) < 0$

$$\Rightarrow 4k(4k - 3) < 0$$
$$\Rightarrow k = 0 \text{ or } k = \frac{3}{4}$$
$$\Rightarrow 0 < k < \frac{3}{4}$$

For $k=0$, the equation becomes $3=0$ so there are clearly no real roots in this case either.

So, no real roots for $0 \leq k < \frac{3}{4}$.

Total for Question 9 is 4 marks)

10.

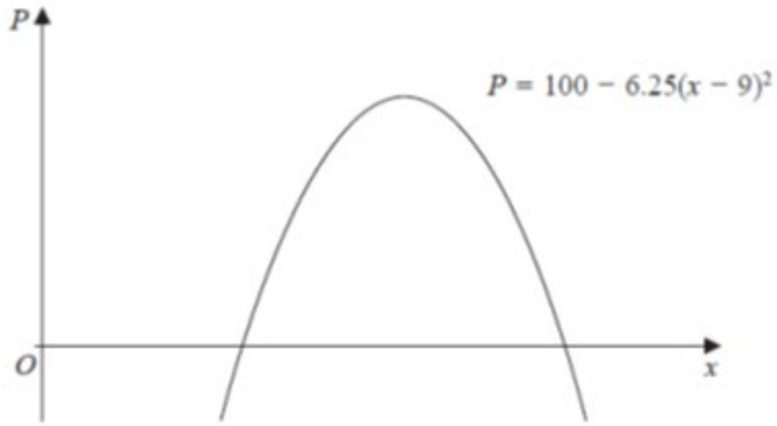


Figure 1

A company makes a particular type of children's toy.

The annual profit made by the company is modelled by the equation

$$P = 100 - 6.25(x - 9)^2$$

where P is the profit measured in thousands of pounds and x is the selling price of the toy in pounds.

A sketch of P against x is shown in Figure 1.

Using the model,

(a) explain why £15 is not a sensible selling price for the toy.

(2)

when $x = 15$

$p = 100 - 6.25(15 - 9)^2$

$\Rightarrow p = 100 - 6.25(36) \Rightarrow p = -125$

so the company would lose money if the toy was £15

Given that the company made an annual profit of more than £80 000

(b) find, according to the model, the least possible selling price for the toy.

(3)

$p > 80$

$\therefore 100 - 6.25(x-9)^2 > 80$

$\Rightarrow 20 > 6.25(x-9)^2$

$\Rightarrow 3.2 > (x-9)^2$

$\Rightarrow x - 9 = \pm\sqrt{3.2}$

$\Rightarrow x = \sqrt{3.2} + 9$ or $x = -\sqrt{3.2} + 9$

$\Rightarrow$ least possible selling price is £7.21

The company wishes to maximise its annual profit.

State, according to the model,

(c) (i) the maximum possible annual profit,

(ii) the selling price of the toy that maximises the annual profit.

(2)

c) i) maximum annual profit = £100000

ii) $100 = 100 - 6.25(x-9)^2$

$6.25(x-9)^2 = 0$

$(x-9)^2 = 0$

$x - 9 = 0$

$x = 9$

$\Longrightarrow$ selling price that maximises the profit = £9

Total for Question 10 is 7 marks)